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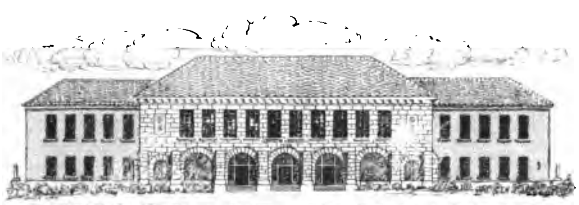

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WILLIAMS & ROGERS SERIES

NEW
COMMERCIAL ARITHMETIC

BY

JOHN H. MOORE

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NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

An attempt has been made to make the treatment of the whole subject highly educative, but methods and topics distinctively utilitarian in their value have received due attention. Arithmetical puzzles and improbable conditions have been studiously avoided, and a feature is made of concrete business problems from the outset. Particular attention has been devoted to the subject of Addition. The group method is carefully developed through a series of oral and written drills. The exercises on tabulation and all the exercises calling for vertical and horizontal additions are especially valuable.

Only small common fractions are introduced; they are the only ones used in ordinary business. In connection with this subject special care has been devoted to the topics Quantity, Price, and Cost, and Bills and Accounts. The methods developed and the forms illustrated in this part of the book are especially helpful and practical. In the chapter on Denominate Numbers a feature is made of the subject Practical Measurements. In the preparation of this portion of the book the author consulted mechanics, contractors, and business men, thoroughly versed in their several departments, in order to get at current, practical usages. In the chapter on Percentage and its Applications, the subjects Commercial Discounts, Interest, Bank Discount, and Customhouse Business have been especially emphasized because they are so closely connected with modern business transactions. In the chapter on Sharing, the subject Partnership has been thoroughly covered. All the problems given in this work are treated from the accountant's standpoint, and are entirely free from all unusual conditions. In the preparation of all the subjects, business men have been consulted freely.

In connection with many of the subjects a great deal of valuable information is given. Numerous business forms are also introduced, and made the basis of a series of problems.

Some of the problems given have been taken from the Williams and Rogers's Commercial Arithmetic, by Oscar F. Williams; but the majority of them are new.

Acknowledgment is due to Professor C. D. Clarkson of the Department of Commerce in Drexel Institute, Philadelphia, for valuable assistance in perfecting the volume.

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NEW COMMERCIAL ARITHMETIC

SIMPLE NUMBERS

PRELIMINARY DEFINITIONS

1. **Arithmetic** is the science of numbers and the art of computing by them.

2. A **unit** is a single thing, or a definite quantity regarded as a single thing.

In selling cloth by the yard the unit is one yard of cloth; in measuring lands by the acre the unit is one acre of land; in counting the number of students in a class the unit is one student; in buying bricks by the thousand the unit is one thousand bricks; in selling posts by the hundred the unit is one hundred posts.

3. An **integral unit** is *one*, or a whole thing.

4. A **decimal unit** is one of the parts obtained by dividing an integral unit into tenths, hundredths, and so on.

5. A **fractional unit** is one of the parts obtained by dividing an integral unit into any number of equal parts.

6. A **number** is a unit or two or more units.

7. An **integer** is an integral unit or two or more integral units.

8. An **abstract number** is a number not associated with any particular thing or quantity; as, 2, 7, 11.

9. A **concrete number** is a number associated with some particular thing or quantity; as, 11 men, 6 cords of wood.

10. A **denominate number** is a concrete number expressing standard money value, or standard measure or weight; as, 1 dollar; 5 gallons; 6 pounds, 4 ounces.

11. **Like numbers** are numbers that have the same unit value; as, 2, 6, 9; 3 houses, 7 houses, 5 houses; 2 years, 9 years, 20 years.

12. **Unlike numbers** are numbers that have different unit values; as, 12, 16 days, 4 boys, 2 hours.

13. A **simple number** is a number consisting of a unit or a collection of units of the same kind; as 2, 12 men, 8 pounds.

14. A **compound number** is a number consisting of two or more denominations of the same unit; as, 7 bushels, 3 pecks, 1 quart; 6 pounds, 3 ounces.

15. A **problem** is a question to be solved.

16. A **principle** is a general law used as a basis for computations.

17. A **rule** is a concise outline of the steps to be taken in the performance of a computation.

ORAL EXERCISE

1. All denominate numbers are concrete. Are all concrete numbers denominate? Explain.

2. All the following numbers are concrete. Are they denominate? Explain. 16 pounds, 12 men, 4 rods, 7 dollars, 9 houses.

3. State clearly the difference between a concrete number and a denominate number.

4. Give an example of a compound number.

5. Is there any difference between a compound number and a denominate number? Explain.

6. What is the unit of 16? of 75 barrels of molasses? of $7500? of $\frac{9}{20}$ of a week? of $2\frac{1}{2}$ dozen?

7. Give an example of a simple abstract number; of a simple concrete number.

8. Name two like numbers; two unlike numbers.

NOTATION AND NUMERATION

18. **Notation** is the art of writing numbers.

19. Numbers are generally expressed by figures or letters, but they may also be expressed by words.

20. **Numeration** is the art of giving oral expression to numbers.

21. The two methods of notation in use are the *Arabic* and the *Roman*.

ARABIC NOTATION

22. The **Arabic Method of Notation, first used** by the Arabs, comprises ten characters or figures, as follows:

0	1	2	3	4	5	6	7	8	9
Naught	**One**	**Two**	**Three**	**Four**	**Five**	**Six**	**Seven**	**Eight**	**Nine**

The figures 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *digits*, and the figure 0 is called *zero*, *naught*, or *cipher*.

23. The **value of a digit** is determined (1) by its name, and (2) by its position in a number. A digit, when standing alone, always equals the number of units which its name indicates; when combined with other digits its value is determined by the place which it occupies in the number.

24. The value of a digit in any given number increases from right to left, and decreases from left to right in a tenfold ratio.

Thus, in the number eleven, expressed 11, the second 1 from the right has a value ten times as great as the first 1.

25. Orders of Units. The place which a figure occupies in a number is called its *order*. The ones of a number are called *units of the first order;* the tens, *units of the second order;* the hundreds, *units of the third order;* the thousands, *units of the fourth order;* and so on.

Ten units of any given order are equal to one unit of the next higher order.

26. Periods. Numbers containing four figures or more are, for convenience, separated by the comma into periods of three figures each. Beginning at the right, the first group is the *period of units;* the second, the *period of thousands;* the third, the *period of millions;* the fourth, the *period of billions;* and so on.

One thousand units of any given period are equal to one unit of the next higher period.

The left-hand period of any number may consist of one, two, or three figures.

Numeration Table

Period	Number	Numeration	Order
7th	153	Hundreds of quintillions Tens of quintillions Quintillions	21st 20th 19th
6th	679	Hundreds of quadrillions Tens of quadrillions Quadrillions	18th 17th 16th
5th	161	Hundreds of trillions Tens of trillions Trillions	15th 14th 13th
4th	068	Hundreds of billions Tens of billions Billions	12th 11th 10th
3d	917	Hundreds of millions Tens of millions Millions	9th 8th 7th
2d	835	Hundreds of thousands Tens of thousands Thousands	6th 5th 4th
1st	284	Hundreds Tens Units	3d 2d 1st

ORAL EXERCISE

1. Read the following numbers: 5,005; 1,925; 3,036; 4,569; 260; 715.

Note. In reading numbers, always express them in the shortest way possible. Thus, 1520 should be read *fifteen hundred twenty*, not *one thousand five hundred twenty*. This is important in writing amounts in checks, notes, and drafts, where the space is often limited.

Do not read *and* between periods or between hundreds and units. Thus, 16,725 should be read *sixteen thousand, seven hundred twenty-five*, not *sixteen thousand and seven hundred and twenty-five*. This distinction is of the utmost importance in connection with the writing of decimals.

2. What is the name of the second period of notation? the third? the fourth? the fifth? the seventh?

3. What are the names of the successive periods expressed by seven figures? by eleven figures?

4. How many figures are required to write millions? trillions?

5. How many units of the first order in the second period of 9,321?

6. Read the following numbers: 246,920,460; 750,861,432,120; 9,246,921,006; 1,269,247,268,490,621; 700,600,070,000,000,002.

7. Express in figures three units of the fourth order, three of the third, nine of the second, and seven of the first.

8. Distinguish between an order and a period as related to numbers.

ROMAN NOTATION

27. The **Roman Method of Notation** is used extensively in numbering volumes, chapters, sections, and the other important divisions of books; also in numbering dials and tabular outlines. It employs seven characters or letters, as follows:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

28. Roman Values. The value of Roman characters is twofold.

1. Each character when standing alone has a definite value, as above.

2. Each character also has a varying value when written in varying positions in combination with other Roman numerals.

29. General Principles. 1. Repeating a letter repeats its value.

Thus, II represents two; XX, twenty; CCC, three hundred.

2. When a letter of less value is placed before one of greater value the number indicated is the difference between the values of such numbers.

Thus, IX represents nine; XC, ninety.

3. When a letter of less value is placed after one of greater value, the number indicated is the sum of the values of such letters.

Thus, CX represents one hundred ten; LXIV, sixty-four.

4. A bar placed over a letter multiplies the value of the letter by one thousand.

Thus, $\overline{V}$ represents five thousand; $\overline{C}$, one hundred thousand.

5. A letter should not be repeated more than three times in expressing numbers.

6. A bar is never placed over the letter I.

TABLE OF ROMAN NUMERALS WITH ARABIC EQUIVALENTS

I	1	XII	12	L	50	DCC	700
II	2	XIII	13	LX	60	DCCC	800
III	3	XIV	14	LXX	70	CM	900
IV	4	XV	15	LXXX	80	M	1000
V	5	XVI	16	XC	90	MM	2000
VI	6	XVII	17	C	100	$\overline{V}$	5000
VII	7	XVIII	18	CC	200	$\overline{X}$	10000
VIII	8	XIX	19	CCC	300	$\overline{L}$	50000
IX	9	XX	20	CD	400	$\overline{C}$	100000
X	10	XXX	30	D	500	$\overline{D}$	500000
XI	11	XL	40	DC	600	$\overline{M}$	1000000

ORAL EXERCISE

Read the following expressions:

XCII; XXVII; XXIX; CCXVII; $\overline{DLXX}$; $\overline{DCC}$; MDCCCLIII; MMDXLIV; MCDLXX.

ADDITION

30. **Addition** is the process of combining several numbers into one equivalent number.

31. The **sum** or **amount** is the result obtained by addition.

32. The sign + signifies addition and is read *plus.*

33. The sign = signifies equality and is read *equals.*

34. **General Principles.** 1. Only the same orders of units of like numbers can be added.

2. The sum always expresses units of the same name as the several numbers to be added.

3. The sum of two or more numbers is the same in whatever order the numbers may be added.

ORAL EXERCISE

1. Beginning at 7 count by 7's to 98; by 6's to 79.
2. Beginning at 31 count by 9's to 112; by 8's to 95.
3. Beginning at 17 count by 4's to 117; by 12's to 77.
4. Beginning at 49 count by 6's to 139; by 8's to 113.
5. Beginning at 29 count by 8's to 93; by 7's to 78.
6. Beginning at 72 count by 5's to 117; by 9's to 108.
7. Beginning at 0 count by 13's to 156; by 11's to 121.
8. Beginning at 0 count by 14's to 126; by 12's to 108.
9. Beginning at 0 count by 15's to 135; by 17's to 153.
10. Beginning at 0 count by 16's to 144; by 9's to 144.
11. Beginning at 29 count by 15's to 104; by 9's to 110.
12. Beginning at 37 count by 11's to 136; by 7's to 86.
13. Beginning at 3 count by 12's to 147; by 19's to 60.
14. Beginning at 4 count by 18's to 94; by 17's to 106.

35. Example. Find the sum of 945, 626, 924, and 726.

$$\begin{array}{r} 945 \\ 626 \\ 924 \\ 726 \\ \hline 3221 \end{array}$$

SOLUTION. Since only units of the same order can be added, write units under units, tens under tens, and hundreds under hundreds, and draw a line beneath. Beginning at the right-hand, or units' column, and adding downwards, the sum is 21 units, or 2 tens and 1 unit. Write 1 in the units' column and add 2 to the tens' column, obtaining as a result 12 tens, or 1 hundred and 2 tens. Write 2 in the tens' column and add 1 to the hundreds' column, obtaining 32. Write this entire result to the left of the numbers before written. The required result is 3221.

36. To insure accuracy in addition all figures should be: (1) uniformly spaced; (2) legibly written; (3) of a uniform size.

RAPID ADDITION

37. The secret of rapid addition lies mainly in the ability to group series of figures with facility. In reading the words of a sentence we do not look at the individual letters, but rather at groups of letters which make words; so in attempting to add columns rapidly we should not think of the individual figures, but of the results of groups of figures.

Thus, in adding 6, 9, 2, 3, 1, 2, and 5, we should not say 6 and 9 are 15 and 2 are 17 and 3 are 20 and 1 are 21 and 2 are 23 and 5 are 28; but 15 (6 + 9), 21 (15 + $\overline{2 + 3 + 1}$), 28 (21 + $\overline{2 + 5}$).

38. The ability to group numbers readily may be acquired by intelligent, persistent practice. By constantly aiming to form groups in adding, we gradually become master of a vocabulary of groups which will eventually serve us to advantage in all numerical operations which we may perform. We should commence by grouping two figures, then three, and so on until we can take in at a glance from two to four figures in all work.

39. Addition is one of the most important and one of the most frequently used operations of arithmetic. It is the key to all rapid business calculations, and should be thoroughly mastered before any of the other principles of commercial arithmetic are attempted.

ORAL EXERCISE

In the following exercise the student should make combinations or groups of two figures in finding the totals. The work should be done rapidly, the student speaking aloud the successive results.

Thus, in problem 1, below, beginning at the top and adding downwards, results should be named as follows: 8, 18, 24, 30, 9, 49. Only the unit figure should be pronounced when the amount continues in the same tens.

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*	*9.*	*10.*
2	3	2	4	7	3	2	7	3	9
6	2	1	3	2	2	1	8	2	2
7	1	7	1	1	7	7	2	4	4
3	2	2	2	4	9	2	4	2	5
1	6	9	3	6	2	6	6	3	1
5	7	3	6	3	1	3	2	6	2
4	3	1	2	5	4	4	7	2	6
2	2	6	1	1	8	1	2	5	3
8	4	3	5	6	3	5	8	5	5
1	6	5	3	4	3	7	2	4	5
5	2	2	2	7	1	2	3	5	7
5	7	1	5	2	2	5	7	3	4

11.	*12.*	*13.*	*14.*	*15.*	*16.*	*17.*	*18.*	*19.*	*20.*
3	6	3	3	3	2	3	2	2	7
7	2	2	4	2	8	2	4	5	4
2	4	8	7	8	1	1	3	4	3
5	3	6	6	6	7	4	1	1	5
3	2	1	2	1	2	6	7	7	6
2	1	2	5	2	4	2	4	2	7
1	7	7	8	7	5	8	9	9	3
7	2	5	3	5	4	4	3	3	5
3	8	4	7	4	6	5	7	7	9
2	4	5	3	6	4	3	9	3	2
7	2	7	4	5	8	7	4	1	9
3	1	5	6	5	2	3	6	9	5

Drill on the foregoing and similar combinations until you can make groups of two figures each and combine them in a total as rapidly as you can count 1, 2, 3, etc. Next use the foregoing and similar exercises in drilling upon adding by groups of three figures each.

WRITTEN EXERCISE

Copy or write from dictation and find the sum of:

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*
8481	4615	4521	3146	2610	1652	1431
2341	9184	6210	7214	3115	1748	2115
4678	8632	1940	1431	4221	2631	6211
3444	1531	7249	1625	1635	4217	2542
1234	3116	2614	3126	1724	2724	1625
5678	4227	1837	1847	1142	1925	1143
9212	1328	9246	2932	2416	1839	2748
3456	2014	2143	1621	1345	4114	1932
3231	9126	3214	4217	1621	1028	1647
1645	3214	9125	2114	1942	1686	4212

NOTE. Any of the above or similar problems may be copied on the board and each student in turn required to add aloud, making groups of from two to four figures. The student should begin with groups of two figures and gradually work up to groups of three or four figures. He should be required to speak results only. Thus, in problem 7, grouping two figures, he should say, 6, 9, 17, 27, 36, in adding the first column; 7, 12, 18, 25, 30, in adding the second column; 8, 15, 22, 38, 46, in adding the third column; 7, 15, 17, 20, 25, in adding the fourth column. The rate of naming the successive results may be slow at first, but it should be gradually quickened as facility is attained.

ORAL EXERCISE

1. Beginning at 37 count by 8's to 77; by 9's to 73.

2. Beginning at 19 count by 7's to 47; by 8's to 51.

3. Beginning at 29 count by 12's to 77; by 14's to 71.

4. Beginning at 26 count by 15's to 86; by 9's to 62.

5. Beginning at 37 count by 8's to 101; by 9's to 118.

6. Beginning at 13 count by 8's to 173; by 7's to 160.

7. Beginning at 14 count by 13's to 66; by 14's to 84.

8. Beginning at 52 count by 9's to 133; by 16's to 180.

DRILL EXERCISE

Pronounce *at sight* the totals of the following combinations:

3	7	5	3	8	9	6	9	9	8	9	6
5	2	4	7	2	1	6	9	7	5	6	5

7	9	4	2	6	6	9	4	5	7	8	5
6	2	7	9	9	8	3	9	9	8	9	7

42	58	65	44	78	56	95	57	62	48	21	48
9	7	9	8	9	7	6	8	7	9	6	9

32	39	46	34	63	48	52	65	85	95	79	49
7	8	9	7	8	8	9	9	8	7	8	8

7	4	7	7	6	8	9	7	9	3	4	6
3	9	8	6	8	4	2	3	3	2	1	2
2	6	5	7	6	8	9	9	8	5	5	2

8	7	4	5	4	7	5	7	9	5	8	9
5	2	2	2	3	1	3	8	4	7	7	8
2	1	4	3	3	2	2	3	7	3	5	2

3	8	6	6	8	4	4	8	9	7	2	4
9	9	7	4	8	6	8	4	7	9	5	7
7	3	7	8	4	6	3	6	4	4	6	9

27	45	72	86	45	39	48	67	54	59	75	93
6	8	9	3	6	7	5	5	4	8	2	5
6	2	1	6	4	3	5	6	6	2	8	7

65	84	93	82	49	29	56	87	76	86	45	52
9	7	6	5	3	4	7	8	7	2	9	6
3	9	5	7	2	3	2	3	4	4	1	4

NOTE. Ten and twenty practically add themselves to any number; hence in adding columns of figures an advantage is always secured by finding groups aggregating ten or twenty. To form these groups it is sometimes advisable to take up the figures of a column in irregular order.

Thus in adding 3, 8, 7, 2, 5, and 8, if 3 and 7 are combined first, the group 18 is instantly seen; then if 2 and 8 are next combined, the group 15 and the total 33 is quickly obtained.

In adding the following numbers, form groups of ten and twenty wherever possible.

57	62	47	38	52	68	58	57	62	79	52	74
3	2	1	4	5	2	2	8	7	1	1	2
4	3	4	1	1	7	1	1	2	3	3	4
3	5	5	5	4	3	8	1	1	4	6	9

59	31	37	45	29	39	14	25	35	37	25	36
1	2	1	2	6	1	7	6	5	4	9	3
4	1	8	3	1	6	6	9	6	7	2	8
6	7	1	1	3	3	7	5	9	9	9	9

4	1	4	3	2	7	6	9	8	4	6	2
2	2	2	1	2	2	4	1	2	3	7	5
7	3	3	4	2	4	3	7	9	6	5	9
7	4	1	2	4	8	7	7	1	7	4	4

7	9	8	6	6	8	3	2	9	6	3	5
5	4	8	1	8	4	6	8	6	7	5	2
3	8	9	5	4	2	5	4	7	8	5	6
4	9	5	9	3	9	7	8	8	9	9	5

12	11	13	14	15	41	47	56	41	37	59	68
3	2	7	4	3	3	3	4	6	2	9	6
7	8	3	2	9	6	9	2	5	9	1	8
5	6	1	8	4	9	8	6	5	7	1	2
5	4	9	4	4	2	2	9	9	2	9	4

40. Horizontal Addition. Numbers, when written in horizontal lines, as on invoices and other business forms, may be added without being rewritten in vertical columns.

41. In adding numbers horizontally, add from left to right and then verify all results by adding from right to left. The group method may be employed to advantage where numbers are written horizontally. The ability to add horizontally saves a great deal of time in making out bills and in performing other commercial operations.

ORAL EXERCISE

Add from left to right and review from right to left the following:

1. 9, 9, 2, 5, 4, 3, 1, 6, 2.

2. 42, 21, 46, 32, 14, 21.

3. 52, 46, 35, 72, 68, 50.

4. 21, 26, 32, 34, 81, 63, 45, 90, 31.

5. 66, 31, 41, 18, 41, 62, 59, 35, 45.

6. 21, 32, 40, 82, 56, 30.

7. 31, 18, 28, 36, 45, 21.

8. 67, 61, 60, 63, 62, 65.

9. 51, 67, 34, 58, 56, 29.

10. 62, 60, 51, 28, 35, 62.

11. How many days in the summer months?

12. Find the sum of the four numbers that may be expressed by the figures 2 and 3; 4 and 5; 6 and 7.

13. Find the sum of all the even numbers from 6 to 12 inclusive.

NOTE. When figures to be added appear in consecutive order, and there is an odd number of them, the total may be found by multiplying the middle figure by the number of consecutive figures.

Thus, $3 + 4 + 5 + 6 + 7 = 5 \times 5 = 25.$

When *any* numbers appear in consecutive order their total may be found by multiplying one half the sum of the first and last numbers by the number of consecutive numbers.

Thus, $14 + 15 + 16 + 17 + 18 = 16 \times 5 = 80.$

14. Find the sum of all the numbers from 7 to 19 inclusive.

15. Find the sum of all the numbers from 1 to 9 inclusive.

16. Find the sum of all the numbers from 3 to 19 inclusive; of all the numbers from 5 to 13 inclusive.

17. Find the sum of 42, 42, 42, 42, 75.

NOTE. When a number is repeated several times in any addition, the work may be shortened by multiplication.

18. A man who was born in 1853 died when he was forty-nine years old. In what year did he die?

19. What is the sum of 15, 23, 36, 18, 26, 92? of 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31?

WRITTEN EXERCISE

Drill on the following and similar problems until correct results can be obtained in twenty seconds or less.

1.	*2.*	*3.*	*4.*	*5.*	*6.*
24264264	47257386	27452462	56319217	72519218	57264592
62462148	52472164	87950241	48263547	67482153	87492165
64292862	83492752	20724065	62519546	72186349	48576901
56259421	26534721	86957447	38641948	39256258	66875465
62462962	23425625	72757786	95722618	78295416	52163441
52462564	32462813	77777777	77554286	87596357	10205211
62469264	42612542	88888888	62496246	21111016	93758617
62462942	78955473	22222222	62462942	20407030	58759218

Drill on the following and similar exercises until correct results may be obtained in twenty seconds or less.

7.	*8*	*9.*	*10.*	*11.*	*12.*
2714	4052	4032	3146	1487	1846
2652	6021	5061	4219	2116	1092
1493	1473	4728	2614	4574	1531
7510	5687	3214	9743	6589	1675
1126	7214	6010	6478	3752	1832
4251	9386	5271	2592	1678	1645
6859	7521	2642	7286	7593	1729
3114	4268	5537	4924	9164	1011
7996	7821	6214	6214	7386	4010
4216	5275	9146	7585	9552	6020
3114	3942	3910	2137	7829	5190
6996	4728	1120	7214	3687	1786
7245	3659	2110	2110	2014	2405
3865	2854	1640	1016	1730	7216
5125	7529	2114	4032	3019	4520
6219	2110	1431	2016	2170	7121
4346	1011	5214	6147	2590	2514

13. Show the totals of the following columns downwards and from left to right. Prove the results by adding the vertical and horizontal totals.

6249	2145	2592	6014	2172	4592	——
4625	1687	1649	5019	1645	7126	——
1872	1421	3145	2041	1392	5218	——
4124	3652	1650	6215	1746	9041	——
3635	1926	1722	9018	7592	7592	——
4215	4521	1490	7016	6219	6218	——
3417	1725	7518	4110	5764	7527	——
1641	1686	2041	6211	2047	2692	——
4356	4035	4250	2140	6211	1420	——
——	——	——	——	——	——	——

14. Complete the following table by showing the totals of the columns vertically and horizontally. Prove the work by adding the vertical and horizontal totals.

DEPARTMENTAL SALES FOR THE WEEK ENDING NOV. 15, 1903

DAYS	CLOTHING	DRY GOODS	FURNISHINGS	MILLINERY	HOUSEHOLD UTENSILS	TOTAL
Monday	$790.50	$988.40	$126.50	$256.85	$496.80	
Tuesday	640.18	890.50	90.18	420.62	841.62	
Wednesday	960.70	950.40	75.60	398.40	462.50	
Thursday	490.18	960.80	214.90	425.60	521.90	
Friday	930.50	720.50	126.70	396.80	762.80	
Saturday	840.15	989.72	215.20	459.65	925.54	
Total						

42. The Two-column Method of Addition. Some accountants are very partial to the two-column method of addition, claiming that it is more rapid and accurate.

43. In adding two columns at once, combine first the tens of the numbers and then the units.

Thus, in adding 75 and 32 think of 105 (75 + 30) and 2, or 107.

To illustrate this method of addition, take the accompanying example. Beginning with the number 46 at the top of the column, add first the tens and then the units of the successive numbers, as follows:

46	
32	$46 + 30 = 76$; $76 + 2 = 78$
65	$78 + 60 = 138$; $138 + 5 = 143$
51	$143 + 50 = 193$; $193 + 1 = 194$
26	$194 + 20 = 214$; $214 + 6 = 220$
220	

In making computations in this manner name results only. Thus, beginning at the top of the accompanying example and adding downwards, read 76, 8, 138, 143, 193, 4, 214, 220.

ORAL EXERCISE

1. Add the following by double columns as explained above:

45	24	52	39	28	57	62	27	33	41	12	13	19	57	92
39	26	58	56	52	31	34	48	45	37	43	56	38	14	12

2. Announce the totals of the above combinations *at sight* from left to right and from right to left. Thus, 84, 50, etc.

NOTE. Require that this work be done rapidly. Drill on the above and similar combinations until the student can announce the totals as rapidly as he can count 1, 2, 3, etc.

3. Add the following by double columns, naming results only:

61	63	82	21	43	19	42	37	51	28	46	27	43	19	24
39	17	19	23	17	31	24	33	25	52	25	32	27	41	26
43	41	40	36	64	44	16	28	72	11	10	21	11	16	21

4. Add the numbers in problem 3 horizontally by the two-column method. Add from left to right and verify the work by adding from right to left.

5. Name results only in determining the totals of the following by the two-column method:

28	14	64	48	37	51	45	16	24	81	59	72	27	45	52
46	26	81	52	43	42	92	41	36	47	31	16	52	92	41
25	42	95	13	94	18	61	72	33	16	73	41	95	27	92
92	18	62	24	26	36	43	86	37	52	87	64	65	46	68
51	32	51	37	51	41	86	14	42	19	49	17	84	85	72
28	16	28	82	28	35	91	92	48	37	51	86	76	41	14

44. Proving Addition. The simplest way to test the correctness of addition is to add the columns a second time in reverse order.

45. Accountants who have to add very long columns of figures frequently begin at the right-hand column and write on a piece of waste paper the full sum of each column added, and then, to verify the work, begin at the left side and add the columns in reverse order, again writing the full sums on a piece of waste paper. If the sum of the totals shown by the first addition is the same as the sum of the totals shown by the second addition, the work is assumed to be correct.

46. The accountant not infrequently has to perform his work amid more or less confusion. In adding long columns, if this method is employed, he can be interrupted or can leave his work for a time and resume it again without examining in detail the columns which have already been completed.

47. This method of proving addition is illustrated in the following example and solution:

SOLUTION. Beginning with the right-hand column and adding downwards, the total is 28. Write 28 to the right of the numbers added, or on a piece of waste paper; without carrying add the next column, and the total is 15, which should be written as shown in the accompanying illustration; add the next column without carrying, and the total is 17; add the next column, and the total is 19. The sum of these totals is 20878.

19	4225	28
17	6248	15
15	5419	17
28	2856	19
20878	2130	20878
	20878	

VERIFICATION. Beginning with the left-hand column and adding upwards, the total is 19. Write this to the left of the figures to be added or on a piece of waste paper, as shown in the accompanying illustration; without carrying add the next column, and the total is 17, which write as shown in the illustration; add the next column, and the total is 15; the next, and the total is 28. The sum of these totals is 20878, or the same as found by the first addition; hence it is assumed that the work is correct.

WRITTEN REVIEW

1. In the following statement add the columns downwards and from left to right, and then prove the work by adding the vertical and horizontal totals.

State Assessments

Year	Armories	Metropolitan Sewer	Abolition of Grade Crossings	Metropolitan Water	Highways	Total
1895–1896	$21,498.29	$59,702.19	$25,811.94	$285,600.54	$161.67	
1896–1897	28,056.27	119,321.10	45,583.53	211,901.92	100.45	
1897–1898	28,056.27	95,421.14	62,677.92	199,900.41	571.94	
1898–1899	34,223.15	75,753.81	56,854.31	258,990.00	153.23	
1899–1900	34,223.15	129,773.27	71,662.03	411,861.54	101.82	
1900–1901	34,223.15	12,625.73	131,074.00	578,696.96	68.78	
Total						

2. Complete the following sales sheet. Prove the work by adding the vertical and horizontal totals.

Summary of Daily Sales

July 2	Shoes	Gloves	Hats	Dress Goods	Clothing	Total
A to D Ledger	$237.31	$126.92	$132.16	$263.64	$423.09	
E to H Ledger	228.80	140.75	110.25	357.18	387.75	
I to L Ledger	238.84	231.78	106.35	676.83	627.71	
M to P Ledger	143.54	157.57	161.69	382.55	541.23	
Q to T Ledger	848.49	657.02	510.45	510.59	551.45	
U to Z Ledger	556.51	213.19	388.54	811.82	680.29	
Total						

3. The sales of a dry goods house for the week ending Nov. 22, 1903, were as follows: Monday, domestics, $ 540.10; notions, $325.85; woolens, $864.98; dress goods, $325.78. Tuesday, domestics, $995.85; notions, $ 419.62; woolens, $ 919.10; dress goods, $146.84. Wednesday, domestics, $ 975.89; notions, $ 853.64; woolens, $ 1659.89; dress goods, $ 1259.89. Thursday, domestics, $856.74; notions, $459.13; woolens, $756.85; dress goods, $588.74. Friday, domestics, $862.47; notions, $817.39; woolens, $1249.86; dress goods, $1560.84. Saturday, domestics, $1529.84; notions, $915.62; woolens, $958.22; dress goods, $1079.54.

Arrange these facts in tabular form, in six columns, with proper headings. Show (*a*) the total sales for each department, (*b*) the total daily sales, and (*c*) the total sales for the week.

4. The records of a city post office show the following mail for one week: Monday, registered letters, 725; ordinary letters, 15,279; postal cards, 2147; book packets, 963; parcels, 184; newspapers, 26,419. Tuesday, registered letters, 461; ordinary letters, 12,365; postal cards, 2011; book packets, 395; parcels, 416; newspapers, 21,936. Wednesday, registered letters, 369; ordinary letters, 16,285; postal cards, 1989; book packets, 618; parcels, 365; newspapers, 23,162. Thursday, registered letters, 8490; ordinary letters, 14,317; postal cards, 416; book packets, 562; parcels, 213; newspapers, 23,164. Friday, registered letters, 959; ordinary letters, 25,162; postal cards, 2116; book packets, 475; parcels, 163; newspapers, 22,790. Saturday, registered letters, 416; ordinary letters, 11,259; postal cards, 659; book packets, 384; parcels, 175; newspapers, 21,218.

Arrange these facts in tabular form, in eight columns, with proper headings. Find (*a*) the total number of separate pieces of mail for each day, (*b*) the total number of pieces of each class, and (*c*) the total number of pieces for the week.

Copy or write from dictation and find the sums of the following:

5.	*6.*	*7.*	*8.*
92451826	24164290	32169528	12345678
40159061	72154031	62169528	28968457
52192165	16941762	62195437	10475631
87965421	15304693	65954370	20047509
74926587	31462845	65109011	33715586
59346599	32168492	52416011	88475621
92657788	11141017	10401721	78991047
65945876	21411731	41627428	74839101
92517496	17283142	31426357	10108765
93479491	65493762	21407110	56461086
59627488	58911476	11169042	77562345
95178654	72491368	25172825	87653421
72958649	72159072	41627598	24683157
58721985	21311510	65901080	36912141
58759271	21411631	72164010	11354678
21864925	47293742	69957788	10019087
17264592	40171650	28521654	98798778
18259015	21101670	29364124	76453111

SUBTRACTION

48. **Subtraction** is the process of finding the difference between two numbers.

49. The **subtrahend** is the number to be subtracted.

50. The **minuend** is the number from which the subtrahend is to be subtracted.

51. The **remainder** or **difference** is the number obtained by subtraction.

52. The sign – signifies subtraction and is read *minus* or *less*.

When the sign of subtraction is placed between two numbers, it indicates that the number written *after* it is to be taken from the one written *before* it.

53. Numbers written within a **parenthesis** ( ), under a **vinculum** $\overline{\quad}$, or separated by the **sign of multiplication** ×, are to be considered together.

Thus, $16-(4+2)$ or $16-\overline{4+2}$ signifies that the sum of 4 and 2 is to be subtracted from 16; $16-4\times 2$ signifies that the product of 4 and 2 is to be subtracted from 16.

54. **General Principles.** 1. Only the same orders of units of like numbers can be subtracted.

2. The sum of the subtrahend and remainder is equal to the minuend.

ORAL EXERCISE

1. Subtract by 4's from 44 to 0; from 39 to 3.
2. Subtract by 6's from 49 to 1; from 78 to 0.
3. Subtract by 5's from 135 to 0; from 121 to 1.
4. Subtract by 7's from 38 to 3; from 64 to 1; from 44 to 2.
5. Subtract by 8's from 91 to 3; from 55 to 7; from 37 to 5.
6. Subtract by 9's from 131 to 23; from 57 to 12; from 95 to 5.
7. Subtract by 15's from 90 to 0; from 120 to 0; from 76 to 1.
8. Subtract by 13's from 41 to 2; from 57 to 5; from 63 to 11.
9. Subtract by 11's from 88 to 0; from 72 to 6; from 91 to 3.
10. Subtract by 12's from 54 to 6; from 128 to 8; from 145 to 1.

55. Example. **Find the difference between 348 and 185.**

$$\begin{array}{r}348\\ \underline{185}\\ 163\end{array}$$

SOLUTION. Since only units of the same order can be subtracted, write units under units, tens under tens, and hundreds under hundreds, and draw a line beneath. Beginning at the right, 5 units from 8 units leaves 3 units. Write 3 under the column of units. Since 8 tens cannot be taken from 4 tens, transform 1 of the 3 hundreds into 10 tens, and add it to the 4 tens, making 14 tens; then, 8 tens from 14 tens leaves 6 tens, which write under the column of tens. Since 1 of the 3 hundreds has been taken, there are only 2 hundreds remaining; then, 1 hundred from 2 hundreds leaves 1 hundred. The difference between the two numbers given is, therefore, 163.

In practice think only of results and write them without hesitating. Thus, in the above problem write or think only 3, 6, 1.

ORAL EXERCISE

1. From what number must we subtract $2.54 to have $7.46 remaining?

2. If I pay $375 for a carriage and sell it at a loss of $73.75, how much do I receive for it?

3. The smaller of two numbers is 96; their difference is 46. What is the larger number?

4. Pronounce at sight the difference between the numbers in each of the following groups.

79	62	56	85	67	78	89	98	67
34	27	47	19	21	49	13	14	12

105	107	108	106	127	168	99	119	121
97	53	38	56	77	48	38	47	69

5. Pronounce at sight the difference between the numbers in each of the following groups.

The subtrahend is placed above the minuend in order to give practice in finding the difference between numbers that are so arranged. If one is not able to subtract in this manner, he is frequently required to rearrange the numbers on separate paper to subtract them. This is a waste of time, since by a little practice one can readily subtract numbers that are not regularly arranged.

79	58	72	29	47	57	62	49	26
92	164	149	132	98	124	126	93	78
18	123	93	47	244	169	158	137	214
54	246	186	94	488	239	248	267	264

SHORT METHODS

56. The **complement** of a number is the difference between such number and a unit of the next higher order.

Thus, the complement of 6 is 4, since 4 is the difference between 6 and 10, or 1 *ten*, a unit of the next higher order than 6; the complement of 83 is 17, since 17 is the difference between 83 and 100, or 1 *hundred*, a unit of the next higher order than 83.

57. Two numbers whose sum is equal to a unit of the next higher order are called **complementary numbers**.

Thus, 209 and 791 are complementary numbers, since their sum is equal to 1000; 2467 and 7533 are complementary numbers since their sum is equal to 10000.

58. If two numbers of more than one figure are complementary numbers, the sum of their units figures is 10, and of each of their corresponding higher orders, 9.

Thus, 642 and 358 are complementary numbers; the sum of the units figures is 10, and the sum of the figures in the corresponding higher orders is 9.

59. The foregoing principle may be applied to advantage in making change. Since we read numbers from left to right, it is generally best in making change to begin at the left to subtract. In beginning at the left to subtract, take 1 from the number of units of the highest order in the minuend, and regard each of the lower orders as 9 except the last, which must be regarded as 10.

60. Example. A gave a twenty-dollar bill in payment for an account of $14.72. How much change should he receive?

$20
14.72
$ 5.28

SOLUTION. Begin at the left to subtract. 1 from the highest order in the minuend leaves 1. 1 from 1 leaves 0. 4 from 9 leaves 5, which write in the units' column. 7 from 9 leaves 2, which write in the tenths' column. 2 from 10 leaves 8, which write in the hundredths' column. The result is $5.28.

DRILL EXERCISE

By *inspection*, find the difference between the following numbers:

400	600	300	700	900	1000	100	200	300	500
132	175	86	263	458	532	52	31	57	138

$1.00	$2.00	$3.00	$4.00	$5.00	$8.00	$7.00	$8.00
.39	1.15	1.17	1.58	2.21	2.39	5.36	1.37

$20.00	$20.00	$10.00	$30.00	$30.00	$40.00	$50.00
2.59	8.76	5.72	28.61	29.57	25.86	6.78

Subtract each of the following numbers from $2.00: 27¢, 52¢, 89¢, $1.52, $1.13, $1.41, $1.59, 85¢, 41¢, 37¢, 56¢, 18¢, 97¢,

If $100 is offered in payment for each of the following accounts, what amount of change should be returned? $25.95, $85.67, $37.54, $92.18, $65.51, $87.75, $69.52, $18.75, $37.58, $88.13, $71.15, $41.30, $39.18, $25.72.

Exercises similar to the above should be continued until correct results can be given without a moment's hesitation.

61. Frequently an accountant finds it desirable to take the sum of several numbers from the sum of several other numbers without transferring the totals from the books of record to separate paper. The following explanations will be found suggestive of short cuts which may be employed to advantage in such cases.

62. Examples. *1.* From 24,794 subtract the sum of 4159, 6490, and 4462.

24794
4159
6490
4462
9683

SOLUTION. For convenience write the numbers under each other with the minuend set off from the subtrahend by a straight line. Beginning at the right and adding the units of the subtrahend the sum is 11, which, subtracted from 14 (the next higher number ending with 4), leaves 3, the units of the required result.

The sum of the figures in the tens' column plus 1 (the number of tens added to the minuend in the previous subtraction) is 21, which, subtracted from 29 (the next higher number ending with 9), leaves 8, the tens' figure of the required result.

The sum of the figures in the hundreds' column plus 2 (the number of hundreds added to the minuend in the previous subtraction) is 11, which, subtracted from 17, leaves 6, the hundreds' figure of the required result.

The sum of the figures in the thousands' column plus 1 (the number of thousands added to the minuend in the previous subtraction) is 15, which, subtracted from 24, leaves 9, or the thousands' figure of the required result.

2. The gross weights and tares of 5 barrels of sugar are as follows: 319–19, 322–21, 311–17, 322–19, 329–21 pounds. Find the net weight.

Solution. In billing, the above numbers would be written horizontally as follows: 319–19, 322–21, 311–17, 322–19, 329–21.

The minuend is the gross weight, and the subtrahend is the tare. Adding the units of the subtrahend horizontally the sum is 27, which, subtracted from the next higher order of units (30), leaves 3. 3 added to the units of the minuend equals 26. Write 6 as the units of the net weight. Since the tens of the subtrahend are 1 more than the tens of the minuend, add 1 to the next higher order of the subtrahend, or subtract 1 from the next higher order of the minuend. Add 1 to the tens of the subtrahend, and the result is 8, which, subtracted from the next higher order of units (10), leaves 2. Adding 2 to the tens of the minuend, the result is 10. Write 0 as the tens of the net weight. Since the tens of the minuend are the same as the tens of the subtrahend, there is nothing to carry. Adding the hundreds of the minuend, the result is 15. Write 15 as the hundreds of the net weight. The net weight is then 1506 pounds.

Note. In billing where items are listed as gross weight and tare the above process will be found especially helpful. Sufficient practice should be required to give the student facility in making the extensions properly.

This principle may also be used to advantage in finding the balances of ledger accounts.

WRITTEN EXERCISE

Note. In the first four problems below the gross weight in pounds is written to the left of the hyphen and the tare in pounds to the right of the hyphen. Find the net weight as explained in Example 2, above.

1. 10 casks of hams, 392–67, 412–71, 402–71, 411–67, 408–68, 425–71, 400–69, 399–70, 398–71, 426–68.

2. 6 baskets pork loins, 312–49, 301–56, 297–48, 415–43, 312–49, 314–56.

3. 4 tubs of lard, 71–14, 70–15, 69–14, 62–15.

4. 8 casks shoulders, 428–19, 322–21, 327–19, 311–17, 314–17, 315–18, 317–21, 342–24.

By either of the short methods explained in **62** find the balances of the following accounts.

5.

Dr. **The Union Bank,** in account with Jas. W. Macy Jr. *Cr.*

1903				1903			
Feb. 7	Dep. chs.	1426	95	Feb. 28		401	65
24	„ cy. & chs.	920	52	28		76	95
28	„ chs.	146	95	28		320	01
				28		76	42
				28		190	66

6.

Dr. **The Union Bank,** in account with C. W. Canan & Co. *Cr.*

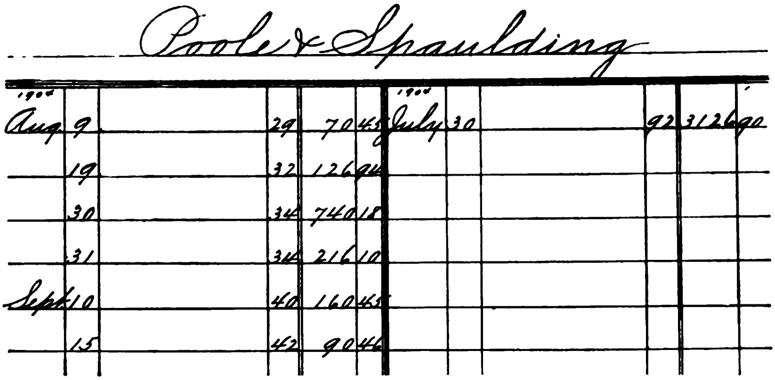
1903.				1903			
Mar. 7	Dep. cy.	102	90	Mar. 31		260	80
12	„ cy. & chs.	9241	64	31		401	90
20	„ chs.	720	40	31		640	82
				31		1241	86
				31		90	60

7.

R. D. Simpson

1904.						1904.					
June	2		21	1942	16	June	15		14	260	40
							29		18	100	65
						July	6		19	90	25
							9		19	10	72
							12		20	104	72
							14		21	720	45

8.



Poole & Spaulding

1904						1904					
Aug	9		29	70	45	July	30		92	3126	90
	19		32	126	94						
	30		34	740	18						
	31		34	216	10						
Sept	10		40	160	45						
	15		42	90	46						

Find the balances of the following bank accounts without using pen or pencil except to write the results.

9. Balance in bank June 1, $ 650.40. Checks from June 1 to July 1, $ 145.20, $ 14.90, $ 60.50, $ 20.40. What is the balance in bank July 1?

10. Balance in bank Aug. 15, $ 695.40. Deposit Aug. 17, $ 65.98. Checks from Aug. 15 to Sept. 1, $ 146.20, $ 90.50, $ 60.95. What is the balance in bank Sept. 1?

11. Balance in bank Jan. 15, $ 460.40. Deposit Jan. 20, $ 152.65. Checks from Jan. 15 to Feb. 1, $ 172.40, $ 14.90, $ 16.95, $ 40.65. What is the balance in bank Feb. 1?

63. Combining Addition and Subtraction in One Process. When a number, or the total of several numbers, is to be taken away from the total of several other numbers, the two processes may be combined as in **62**, or as shown in the following examples.

64. Examples. *1.* From the sum of 12 and 6 take 3.

SOLUTION. Adding 12 to 6 we have 18. It is self-evident that 18 − 3 is equivalent to 18 + (10 − 3) − 10.

When a number is both added to and subtracted from any quantity, the value of the quantity is not changed.

Applying this principle in solving the above problem, mentally take 3 from 10, add the difference to 6 and 12, and subtract 10 from the result. Thus, 7, 25, 15, the required result.

2. From the sum of 827 and 534 subtract 356.

Solution. Arranging the numbers horizontally and adding the units, by naming the results only, we have from the right 4 (10 − 6), 8, 15, 5 (15 − 10) to write as the first figure of the required result. Adding the tens, we have 5 (10 − 5), 8, 10, 0 (10 − 10) to write as the tens of the required result. Adding the hundreds, we have 7 (10 − 3), 12, 20, 10 (20 − 10) to write as the hundreds of the required result. The final result is, therefore, 1005.

3. From the sum of 729 and 642 subtract 211.

Solution. 9, 11, 20, or 0 to write and 1 to carry to the minuend. 10 (9 + 1 carried), 14, 16, or 6 to write. 8, 14, 21, or 11 to write. The final result is, therefore, 1160.

4. From the sum of 321 and 811 subtract 369.

Solution. 1, 2, 3, or 3 to write and 1 to subtract from the tens of the minuend. 3 (4 − 1), 4, 6, or 6 to write and 1 to subtract from the minuend. 6 (7 − 1), 14, 17, or 7 to write. The result is, therefore, 763.

Note. In this class of work grouping may be used to advantage. To show every step in the process the results in the above solutions were determined without grouping.

65. Hence the following rule:

Take each order of units in the subtrahend from 10, add the difference to the same order of units in the minuend, and deduct 10 from the result obtained.

In adding any order of units if the result is less than 20, there is nothing to carry to the next higher order in the minuend; if the sum is 20 or more, there is always something to carry to the next higher order in the minuend; if the sum is less than 10, there is 1 to subtract from the next higher order in the minuend.

Thus, if the sum of any order of units is a number from 10 to 19 inclusive, carry nothing; a number from 20 to 29 inclusive, carry 1; a number from 1 to 9 inclusive, subtract 1.

66. Individual Ledger Balances. The above method is particularly helpful in making extensions on a banking individual ledger.

67. Example. Find the balance to the credit of D. Roe in the following bank account:

Names	Balances		Checks		Deposits		Balances	
D. Roe	692	85	146	25	625	42		

Solution. The first column shows the balance on deposit, the second the amount withdrawn by checks, and the third the amount deposited. The sum of the old balance and the deposits is therefore the minuend, and the sum of the checks the subtrahend. Employing the principles just explained, the balance of the foregoing account may be determined mentally, as follows:

2, 7 $(2+\overline{10-5})$, 12. Write 2 in the new balance column.

4, 12 $(4+\overline{10-2})$, 20. Write 0 in the new balance column and carry 1.

6 (5 + 1 carried), 10 $(6+\overline{10-6})$, 12. Write 2 in the new balance column.

2, 8 $(2+\overline{10-4})$, 17. Write 7 in the new balance column.

6, 15 $(6+\overline{10-1})$, 21. Write 11 in the new balance column.

The new balance is, therefore, $1172.02.

WRITTEN EXERCISE

1. Copy or write from dictation the following individual ledger accounts and find the new balances without using pen or pencil except to write the results. After extending the new balances add the old balances, checks, deposits, and new balances, respectively. Prove the work. The sum of the total old balances and the total deposits minus the total checks should equal the total new balances.

Names	Balances		Checks		Deposits		Balances	
Allen, E. W.	962	59	421	65	875	90	—	—
Briggs, C. W.	725	42	126	42	215	95	—	—
Comer, L. M.	826	54	217	47	421	66	—	—
Day, O. D.	924	54	413	86	966	75	—	—
Emery, A. L.	592	87	436	58	297	52	—	—
Foley, B. E.	726	88	315	92	496	87	—	—
Good, J. I.	925	43	413	86	575	94	—	—
Hall, L. O.	1426	88	613	92	726	48	—	—
Irwin, Chas. E.	1217	95	214	86	926	45	—	—
Jones, Chas. H.	725	77	216	54	818	72	—	—
	—	—	—	—	—	—	—	—

2. In the following account find (*a*) the total checks, and (*b*) the new balances. Prove the work.

NAMES	BALANCES		CHECKS IN DETAIL		TOTAL CHECKS		DEPOSITS		BALANCES	
Ames, D. T.	9241	10	126	95	—	—	1400	00	—	—
Ballou, M. T.	6418	40	200 216	00 10	—	—	700 900	00 00	—	—
Collins, W. T.	1421	19	500 417	00 40	—	—	920 1240	00 10	—	—
Dorman & Co.	2146	11	200 711	00 40	—	—	1750	92	—	—
Evans & Son	1492	20	400 240	00 10	—	—	1120	00	—	—
Farley Bros.	1742	20	410 920	00 19	—	—	1750	00	—	—
Grant & Snow Co.	2114	90	750	00	—	—	8710 2500	00 00	—	—
Hall & Smith	6218	10	1200	00	—	—	1100 1460	09 41	—	—
Irwin, J. T.	1721	10	200 1140	00 80	—	—	1400	62	—	—
Jamison, M. L	4216	91	600	00	—	—	1721	42	—	—
	—	—	—	—	—	—	—	—	—	—

ORAL REVIEW

1. From 100 take 15; 25; 42; 16; 73; 81; 19; 16; 14; 22; 33; 45; 55; 65; 72; 87; 64; 47; 35; 51; 17.

2. I gave a fifty dollar bill in payment for an account of $23.45. How much change should I receive?

NOTE. In making change it is always advisable to determine the amount by subtraction and then to verify the result by addition. Thus, if $10 is received in payment for a bill of $7.42, by inspection, determine the balance and prove the result by adding to the amount purchased the change counted out. If the amount of the payment is obtained by such addition, the result is assumed to be correct.

3. $\$10 - \$4.17 = ?$

4. $\$15 - \$2.77 = ?$

5. $\$20 - \overline{\$1.75 + \$2.16} = ?$

6. $\$20 - \$2.95 = ?$

7. $\$30 - \$22.79 = ?$

8. $\$15 - \$11.68 = ?$

9. $\$25 - \$23.75 = ?$

10. $\$5 - \overline{\$1.03 + \$2.17} = ?$

11. From $3 take 89¢; 74¢; 67¢; 83¢; 74¢; 18¢; 24¢; 36¢; 43¢; 58¢; 38¢; 27¢; 52¢.

12. In an account with Charles Spencer a payment of $17 which he makes is erroneously charged, instead of credited, to his account. What is the error in the balance of his account? Explain.

13. By inspection, find the difference between the following:

900	400	2000	800	5000	3000
9	52	172	24	57	127
$100.00	$200.00	$400.00	$50.00	$100.00	$200.00
14.65	15.65	17.24	5.21	11.45	15.65
$500.00	$300.00	$600.00	$700.00	$1000.00	$200.00
65.95	11.42	18.52	17.25	127.50	18.50
$100.00	$7000.00	$100.00	$3000.00	$10.00	$150.00
19.00	18.59	7.92	15.49	7.92	1.92

WRITTEN REVIEW

1. A has $950, which is $275 more than I have, and I have $300 more than B. How much have we together?

2. A and B together owe me $9275; A owes me $3150. After paying me $1900 on account, how much does B still owe me?

3. A produce dealer bought 200 barrels of apples for $415. Had he received $75 more in selling them his gain would have been equal to the amount originally paid for the apples. What amount was received from the sale of the apples?

4. A furniture dealer bought a stock of goods amounting to $5216. After selling goods amounting to $4917, he took an account of stock and found that he had furniture on hand amounting to $1937. Did he gain or lose, and how much?

5. A retail hardware dealer bought merchandise amounting to $1249. After selling from this stock articles amounting to $842, he took an account of the stock remaining unsold and found that it was worth $311. Did he gain or lose, and how much?

6. At the close of the business, July 1, a merchant had cash in the safe amounting to $314. July 2 he received from sales $526; on account, $435; the cash in the safe at the close of July 2 amounted to $219. What were the total disbursements for July 2?

7. A father divided his farm, consisting of 675 acres, among his three sons, Harvey, William, and Albert. Harvey received 75 more acres than William, who received 225 acres, and to Albert was given the remainder. How many acres were given to Albert?

MULTIPLICATION

68. **Multiplication** is the process of taking one of two numbers as many times as there are units in the other.

69. The **multiplicand** is the number that is to be taken a required number of times.

70. The **multiplier** is the number which indicates how many times the multiplicand is to be taken or multiplied.

71. The **product** is the number obtained by multiplication.

72. The sign × signifies multiplication, and is read *times* or *multiplied by.*

The sign × is read *times* when the multiplier precedes the multiplicand and *multiplied by* when the multiplicand precedes the multiplier.

73. **General Principles.** 1. The multiplier always signifies a *number of times,* and is an abstract quantity.

2. The multiplicand may be either an abstract or a concrete number.

3. The product always has the same name as the multiplicand.

74. **Factors** are the numbers used in obtaining a product.

75. The **numerical result** of one number by another is the same whichever factor is regarded as the multiplier. The above general principles are to be recognized only in explanations of work done.

For illustration, take the following example:

If one barrel of apples cost $3, what will 125 barrels cost?

Solution: Since 1 barrel of apples cost $3, 125 barrels will cost 125 times $3, which is $375.

We cannot multiply 125 by $3, but since 3 times 125 times is equal to 125 times 3, we may interchange the factors and have 3 times $125. The product, it will be observed, is the same in either case. Hence,

An interchange of the factors in any multiplication does not affect the product.

76. In multiplying one number by another, always use the smaller quantity as the multiplier. It should be remembered, however, that the product always has the same name as the *true* multiplicand.

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	82	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	256	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

77. In the accompanying table take the multiplicand in the figures arranged horizontally at the top or the bottom, and the multiplier in the column at the left; the product is then the number under or above the multiplicand and opposite the multiplier.

78. Examples. *1.* Find the product of 2418 × 7.

$$\begin{array}{r} 2418 \\ \underline{7} \\ 16926 \end{array}$$

SOLUTION. Write the multiplier 7 below the unit figure of the multiplicand as shown in the margin, and begin at the right to multiply. 7 times 8 units equals 56 units, or 5 tens and 6 units. Write 6 units in the place of units, and reserve 5 tens to add to the product of tens. 7 times 1 ten equals 7 tens, and adding the 5 tens reserved gives 12 tens, or 1 hundred and 2 tens. Write 2 in the place of tens, and reserve 1 to add to the product of hundreds. 7 times 4 hundreds equals 28 hundreds, and adding the 1 hundred reserved gives 29 hundreds, or 2 thousands and 9 hundreds. Write 9 in the place of hundreds, and reserve 2 to add to the product of thousands. 7 times 2 thousands equals 14 thousands, and adding the 2 thousands reserved gives 16 thousands. Write this entire sum to the left of the figures already written, thus completing the multiplication and obtaining as a result 16,926.

2. Find the product of 417 × 352.

SOLUTION. Write the multiplier below the multiplicand in the same unit order from the right. The multiplier is composed of 2 units, 5 tens, and 3 hundreds, or 2 + 50 + 300. Multiplying the multiplicand by 2, 50, and 300 respectively, and adding the products, the results are as follows:

(*a*) FULL PARTIAL PRODUCTS		(*b*) ABBREVIATED PARTIAL PRODUCTS
417		417
352		352
834	Partial Product by 2	834
20850	Partial Product by 50	2085
125100	Partial Product by 300	1251
146784	Complete Product	146784

The ciphers at the right of the partial products are of no value in finding the complete product, and they should be omitted in practice as shown in (*b*). Observe that the first right-hand figure of each partial product is then always directly under the figure of the multiplier used.

CONTRACTIONS IN MULTIPLICATION

79. **To multiply any number by 10, 100, 1000, etc.**

80. Every cipher that is annexed to a number moves each digit one place to the left, or converts units into tens, tens into hundreds, and so on.

81. Hence to multiply a number by 10, 100, 1000, etc.,

To one factor annex as many ciphers as there are ciphers in the other factor.

82. **To multiply by any number of 10's, 100's, 1000's, etc.**

83. Since 50 is 5 tens, 500 5 hundreds, etc., to multiply a number by 50, 1600, 7000, etc.,

Omit the ciphers on the right of the factors, multiply the remaining part of the multiplicand by the remaining part of the multiplier, and to the product thus obtained annex the ciphers omitted.

ORAL EXERCISE

By inspection, find the products of:

1. 34,000 × 600.
2. 216,000 × 300.
3. 5400 × 70.
4. 5120 × 2000.
5. 710 × 6000.
6. 72 × 3000.
7. 714,000 × 200.
8. 28,000 × 20.
9. 716 × 20.
10. 160 × 20.
11. 805 × 2000.
12. 43,007 × 100

84. To multiply any number by 11.

85. Examples. *1.* Find the product of 72 × 11.

(*a*) Full Operation

$$\begin{array}{r} 72 \\ \underline{11} \\ 72 \\ \underline{72} \\ 792 \end{array}$$

(*b*) Contracted Operation

Write 2 in the place of the units of the product. $2 + 7 = 9$. Write 9 in the place of the tens of the product. Bring down 7 for the place of the hundreds of the product.

The completed product is therefore 792.

Solution. By glancing at (*a*) it will be observed that:

1. The partial product by the units of the multiplier contains the same figures as the partial product by the tens.
2. The first figure in the partial product by tens of the multiplier falls under the second figure of the partial product by units.

Making practical use of these observations, we have the abbreviated operation as in (*b*).

2. Find the product of 89 × 11.

Solution. Write 9 as the first figure of the product. $9 + 8 = 17$. Write 7 as the second figure of the product, and carry 1. $8 + 1 = 9$. Write 9 as the third figure of the product, thus completing the multiplication and obtaining as a result 979.

3. Find the product of 195 × 11.

Solution. Write 5 as the first figure in the product. $5 + 9 = 14$. Write 4 as the second figure in the product, and carry 1. $9 + 1 + 1 = 11$. Write 1 as the third figure in the product, and carry 1. $1 + 1 = 2$. Write 2 as the fourth figure in the product, thus completing the multiplication and obtaining as a result 2145.

86. Hence the rule:

Write as the first figure of the product the first figure of the multiplicand.

Beginning at the right of the multiplicand add the units and tens, the tens and hundreds, the hundreds and thousands, and so on.

Finally, bring down the left-hand figure of the multiplicand as the left-hand figure of the product. Carry when necessary.

ORAL EXERCISE

By inspection, find the products of:

1. 62 × 11.
2. 51 × 11.
3. 11 × 71.
4. 45 × 11.
5. 11 × 29.
6. 11 × 75.
7. 11 × 84.
8. 11 × 91.
9. 86 × 11.
10. 32 × 11.
11. 37 × 11.
12. 11 × 43.
13. 59 × 11.
14. 11 × 78.
15. 81 × 11.
16. 21 × 11.
17. 24 × 11.
18. 11 × 26.
19. 11 × 34.
20. 11 × 47.
21. 52 × 11.
22. 85 × 11.
23. 11 × 93.
24. 11 × 68.
25. 121 × 11.
26. 132 × 11.
27. 11 × 141.
28. 11 × 164.
29. 11 × 214.
30. 216 × 11.
31. 225 × 11.
32. 11 × 428.
33. 11 × 927.
34. 11 × 728.
35. 726 × 11.
36. 487 × 11.
37. 1926 × 11.
38. 11 × 1726.
39. 11 × 2814.
40. 5419 × 11.

87. To multiply by any number of 11's; as, 22, 33, etc.

88. Examples. *1.* Multiply 24 by 22.

Solution. 4 × 2 = 8. Write 8 as the first figure of the product. 6(4 + 2) × 2 = 12. Write 2 as the second figure of the product, and carry 1. 2 × 2 + 1 = 5. Write 5 as the third figure of the product, thus completing the multiplication and obtaining as a result 528.

2. Find the product of 121 × 77.

Solution. Write 7 as the first figure of the product. 3(1 + 2) × 7 = 21. Write 1 as the second figure of the product and carry 2. 3(2 + 1) × 7 + 2 = 23. Write 3 as the third figure of the product and carry 2. 1 × 7 + 2 = 9. Write 9 as the fourth figure of the product, thus completing the multiplication, and obtaining as a result 9317.

It should be observed that this method is practically the same as the method of multiplying by 11.

WRITTEN EXERCISE

Find the products of:

1. 21 × 22.
2. 44 × 36.
3. 12 × 22.
4. 66 × 215.
5. 33 × 425.
6. 42 × 33.
7. 66 × 71.
8. 214 × 33.
9. 125 × 88.
10. 22 × 1214.
11. 44 × 74.
12. 25 × 88.
13. 146 × 55.
14. 22 × 216.
15. 151 × 44.
16. 66 × 125.

89. To multiply by any number, one part of which is contained a certain number of times in another part.

90. Examples. *1.* Find the product of 254 × 357.

$$\begin{array}{r} 254 \\ \underline{357} \\ 1778 \\ \underline{8890} \\ 90678 \end{array}$$

Solution. Observe that the tens and hundreds (35) of the multiplier make a number five times the units (7). Multiply 254 by 7 in the usual manner. Multiply the resulting partial product by 5 (the partial product by 7 multiplied by 5 equals the partial product by 35), and complete the multiplication by adding.

2. Find the product of 12,121 × 12,816.

$$\begin{array}{r} 12121 \\ \underline{12816} \\ 193936 \\ \underline{1551488} \\ 155342736 \end{array}$$

Solution. Multiply 12,121 by 16, obtaining 193,936. Multiply 193,936 by 8 (128 is just 8 times 16), and obtain 1,551,488. Add the two partial products and obtain 155,342,736, thus completing the multiplication.

WRITTEN EXERCISE

1. If a page contains 1864 ems, how many ems in a book of 794 pages?

2. If 735 men can dig a canal in 9328 days, how many men would be required to complete the work in 1 day?

3. How many links in 639 chains, each chain having 8471 links?

4. The Boston "Boot Maker" will enable a workman to make 324 pairs of boots daily. How many can be made with this machine in 328 days?

5. What must be paid for grading a railroad 1809 miles long at $1288 a mile?

6. What will 248 acres of land cost at $217 per acre?

7. If the circulation of the city library is 27,126 books daily, how much would it be in 168 days?

8. A dray horse can draw 10 loads, of 1569 pounds each, per day. How many pounds can 749 horses draw in 1 day at the same rate?

9. A barrel of flour weighs 196 pounds. What is the weight of 639 barrels?

10. A railway is 1449 miles in length, and was completed at an average cost of $106,775 per mile. What was the total cost of constructing it?

91. Cross Multiplication. The possibilities of what is known as cross multiplication are almost without end. The method is particularly helpful in making mental extensions on invoices. By it the product of any two numbers of two figures each may be ascertained mentally, and the product of any number by any other number of two figures may be obtained by simply writing the completed product. By intelligent, persistent practice any number may be multiplied by any other number of three or four figures without writing any of the partial products that are ordinarily written in multiplying one number by another.

92. Examples. *1.* Find the product of 74 × 23.

$$\begin{array}{r}74\\ \underline{23}\\ 1702\end{array}$$

Solution. 4 × 3 = 12. Write 2 as the first figure of the product and carry 1. 7 × 3 + 1 (carried) + 8 (4 × 2) = 30. Write 0 as the second figure of the product and carry 3. 7 × 2 + 3 (carried) = 17. Write 17 to the left of the figures already written in the product, thus completing the multiplication and obtaining a product of 1702.

2. Find the product of 124 × 62.

$$\begin{array}{r}124\\ \underline{62}\\ 7688\end{array}$$

Solution. 4 × 2 = 8. Write 8 as the first figure of the product. 2 × 2 + 24 (4 × 6) = 28. Write 8 as the second figure of the product and carry 2. 1 × 2 + 12 (2 × 6) + 2 (carried) = 16. Write 6 as the third figure of the product and carry 1. 1 × 6 + 1 (carried) = 7. Write 7 as the fourth figure of the product, thus completing the multiplication and obtaining a product of 7688.

3. Find the product of 2146 × 32.

$$\begin{array}{r}2146\\ \underline{32}\\ 68672\end{array}$$

Solution. 6 × 2 = 12. Write 2 and carry 1. 4 × 2 + 1 (carried) + 18(6 × 3) = 27. Write 7 and carry 2. 1 × 2 + 2 (carried) + 12(4 × 3) = 16. Write 6 and carry 1. 2 × 2 + 1 (carried) + 3(1 × 3) = 8. Write 8. 2 × 3 = 6. Write 6, thus completing the multiplication and obtaining a product of 68672.

4. Find the product of 214 × 236.

$$\begin{array}{r}214\\ \underline{236}\\ 50504\end{array}$$

Solution. 4 × 6 = 24. Write 4 and carry 2. 1 × 6 + 2 + 12(4 × 3) = 20. Write 0 and carry 2. 2 × 6 + 2 + 3(1 × 3) + 8(4 × 2) = 25. Write 5 and carry 2. 2 × 3 + 2 + 2(1 × 2) = 10. Write 0 and carry 1. 2 × 2 + 1 = 5. Write 5, thus completing the multiplication and obtaining a product of 50504.

93. The method of cross multiplication can hardly be covered by a set rule, since it includes such a wide range of numbers. In attempting to make practical application of this method the following principles are important.

1. Units multiplied by units equal the units of the product.

2. Tens multiplied by units, plus units multiplied by tens equal the tens of the product.

3. Hundreds multiplied by units plus tens multiplied by tens plus units multiplied by hundreds equal hundreds of the product.

4. Thousands multiplied by units plus hundreds multiplied by tens plus tens multiplied by hundreds plus units multiplied by thousands equal thousands of the product.

WRITTEN EXERCISE

Find the product in each of the following problems. Do not use pen or pencil except to write the product.

1. 24 × 32.
2. 41 × 35.
3. 2115 × 32.
4. 127 × 23.
5. 53 × 42.
6. 2144 × 36.
7. 1121 × 42.
8. 116 × 45.
9. 47 × 26.
10. 37 × 48.
11. 1174 × 26.
12. 181 × 59.
13. 125 × 34.
14. 36 × 58.
15. 1215 × 57.
16. 2125 × 64.
17. 164 × 32.
18. 172 × 27.
19. 34 × 51.
20. 1217 × 42.
21. 241 × 36.
22. 142 × 28.
23. 45 × 91.
24. 3215 × 42.

94. To multiply any number by the numbers from 101 to 109 inclusive.

95. Examples. *1.* Find the product of 64 × 102.

$$\begin{array}{r} 64 \\ \underline{102} \\ 6528 \end{array}$$

SOLUTION. 64 × 2 = 128. Write 28 and carry 1. 64 × 1 + 1 = 65. Write 65 to complete the product.

2. Find the product of 215 × 102.

$$\begin{array}{r} 215 \\ \underline{102} \\ 21930 \end{array}$$

SOLUTION. 15 × 2 = 30. Write 30 for the first figures of the product. 2 × 2 + 5 = 9. Write 9 as the third figure of the product. 21 × 1 = 21. Write 21 to complete the product.

3. Find the product of 2265 × 104.

$$\begin{array}{r} 2265 \\ \underline{104} \\ 235560 \end{array}$$

SOLUTION. 5 × 4 = 20. Write 0 and carry 2. 6 × 4 + 2 = 26. Write 6 and carry 2. 2 × 4 + 2 + 5 = 15. Write 5 and carry 1 2 × 4 + 1 + 6 = 15. Write 5 and carry 1. 22 × 1 + 1 = 23. Write 23 to complete the product.

NOTE. The above method of multiplication may be used to advantage in billing where the price is $1.02 and $1.03, etc.

ORAL EXERCISE

By inspection, find the product of:

1. 32 × 102.
2. 103 × 47.
3. 39 × 104.
4. 53 × 105.
5. 72 × 106.
6. 114 × 105.
7. 58 × 102.
8. 104 × 32.
9. 106 × 58.
10. 105 × 94.
11. 71 × 102.
12. 88 × 101.

96. To multiply by any number of three figures, the tens of which is a cipher.

97. Examples. *1.* Find the product of 126 × 302.

$$\begin{array}{r} 126 \\ \underline{302} \\ 38052 \end{array}$$

SOLUTION. 26 × 2 = 52. Write 52 as the first two figures of the product. 1 × 2 + 6 × 3 = 20. Write 0 as the third figure of the product and carry 2. 12 × 3 + 2 = 38. Write 38 to complete the product.

2. Find the product of 1215 × 304.

$$\begin{array}{r} 1215 \\ \underline{304} \\ 369360 \end{array}$$

SOLUTION. 15 × 4 = 60. Write 60 as the first two figures of the product. 2 × 4 + 5 × 3 = 23. Write 3 as the third figure of the product, and carry 2. 1 × 4 + 2 (carried) + 1 × 3 = 9. Write 9 as the fourth figure of the product. 12 × 3 = 36. Write 36 to complete the product.

It will be observed from the above solutions that this method is practically the same as 94.

WRITTEN EXERCISE

Find the product in each of the following problems without using pen or pencil except to write the figures of the completed product.

1. 121 × 202.
2. 116 × 403.
3. 151 × 304.
4. 165 × 405.
5. 305 × 408.
6. 431 × 309.
7. 918 × 201.
8. 725 × 402.
9. 413 × 301.
10. 365 × 308.
11. 413 × 503.
12. 936 × 405.
13. 123 × 407.
14. 218 × 905.
15. 721 × 801.
16. 718 × 203.

98. To square any number of two figures.

99. To square any number of two figures the method of cross multiplication may be used, or the work may be further contracted as shown in the following example.

100. Example. Square 72.

$$\begin{array}{r} 72 \\ \underline{72} \\ 5184 \end{array}$$

SOLUTION. 2 × 2 = 4. Write 4 as the first figure of the product. 14 (7 + 7) × 2 = 28. Write 8 as the second figure of the product, and carry 2. 7 × 7 + 2 = 51. Write 51 to complete the product; or,

2 × 2 = 4. Write 4 as the first figure of the product. 4 (2 + 2) × 7 = 28. Write 8 as the second figure of the product, and carry 2. 7 × 7 + 2 = 51. Write 51 to complete the product.

101. Therefore the following rule:

Multiply the units of the multiplicand by the units of the multiplier and write the result in the product, carrying as usual.

Add the tens in the multiplier to the tens of the multiplicand and multiply by the units of the multiplier; or, add the units of the multiplier to the units of the multiplicand and multiply the sum by the tens of the multiplier. Write the result in the product, carrying as usual.

Multiply the tens of the multiplicand by the tens of the multiplier, and write the full result in the product.

WRITTEN EXERCISE

1. Find the sum of the squares of the following numbers.

24, 36, 32, 34, 67, 84, 92, 76, 89, 47, 39, 38, 43,
56, 75, 88, 95, 83, 94, 71, 29, 44, 59, 65, 73.

2. Square the numbers written below, writing the results horizontally; then find the sum of the squares.

37, 48, 68, 62, 98, 26, 27.

3. Square the numbers written below, and from the sum of the first five squares subtract the sum of the second five.

63, 61, 49, 76, 81, 41, 33, 23, 35, 37.

ORAL REVIEW

By inspection, find the cost of each of the following items:

1. 215 pounds of butter at 22¢; 102 pounds at 28¢.
2. 104 bushels of garden corn at $1.75; 204 bushels at 52¢.
3. 125 bushels of wheat at $1.02; 115 bushels at $1.05.
4. 102 barrels of apples at $1.48; 103 barrels at $1.57.
5. 33 bushels of pears at $1.39; 55 bushels at $1.15.
6. 65 bushels of potatoes at 65¢; 58 bushels at 52¢.
7. 44 baskets of peaches at $1.23; 52 baskets at 87¢.
8. 64 barrels of flour at $5.15; 33 barrels at $6.50.
9. 26 bags of bran at $1.03; 32 bags at $1.05.

10. 125 pounds of coffee at 22¢; 164 pounds at 33¢.

11. 56 gallons of molasses at 36¢; 84 gallons at 34¢.

12. 43 pounds of chocolate at 43¢; 52 pounds at 52¢.

13. 23 sacks of pancake flour at 42¢; 45 sacks at 45¢.

14. 62 boxes of ice cream salt at 62¢; 53 boxes at 51¢.

WRITTEN REVIEW

1. A manufacturer sold 171 corn shellers at $23 each. How much did he receive for them?

2. There are 5280 feet in a mile. How many feet are there in 104 miles?

3. What will 462 barrels of petroleum cost at $1.08 per barrel?

4. In freighting, lime and flour are each estimated to weigh 200 pounds per barrel; pork and beef, each 32 pounds; apples and potatoes, each 150 pounds; cider, whisky, and vinegar, each 150 pounds. What will be the weight of the freight in a car containing 22 barrels of each of these products?

5. If a bushel of barley weighs 48 pounds, of clover seed 60 pounds, of flax seed 55 pounds, of beans 60 pounds, of buckwheat 48 pounds, of rye 56 pounds, of corn 56 pounds, of oats 32 pounds, of potatoes 60 pounds, of timothy seed 45 pounds, of wheat 60 pounds, what will be the total weight of 77 bushels of each product?

6. A man rented a farm of 132 acres of grain land, 76 acres of pasture land, and 45 acres of meadow land; paying for the grain land $7 per acre, for the pasture land $4 per acre, and for the meadow land $11 per acre. He produced 61 bushels of oats per acre on 45 acres, 32 bushels of barley per acre on 30 acres, 75 bushels of corn per acre on 15 acres, 150 bushels of potatoes per acre on 9 acres, 28 bushels of buckwheat per acre on 20 acres, and 24 bushels of beans per acre on the remainder of the grain ground. He relet the pasture land for $200, and on the meadow cut 2 tons per acre of hay, worth $13 per ton. If he paid $695 for labor and $467 for other expenses, and sold the oats at 26¢ per bushel, the barley at 65¢, the corn at 40¢, the potatoes at 33¢, the buckwheat at 60¢, and the beans at $2, did he gain or lose, and how much?

DIVISION

102. Division is the process of finding how many times one number is contained in another.

103. The **dividend** is the number to be divided.

104. The **divisor** is the number by which the dividend is to be divided.

105. The **quotient** is the result obtained by division.

106. Division is **exact** when all the dividend is divided and the quotient is an integer.

107. The **remainder** is the part left undivided when the division is not exact.

108. The sign ÷ signifies division and is read *divided by.*

Thus 24 ÷ 8 = 3 is read *24 divided by 8 equals 3.*

The dividend in division corresponds to the product in multiplication, and the divisor and quotient to the multiplicand and multiplier, respectively.

109. General Principles. 1. Multiplying the dividend or dividing the divisor multiplies the quotient.

2. Dividing the dividend or multiplying the divisor divides the quotient.

3. Multiplying or dividing both the dividend and divisor by the same number does not change the quotient.

4. When the divisor and dividend are like numbers, the quotient is an abstract number.

5. When the divisor is an abstract number, the dividend and quotient are like numbers.

110. When the divisor is so small that the division may be performed mentally, the process is called **short division.**

111. Example. Divide 3713 by 8.

$$\begin{array}{r} 464\tfrac{1}{8} \\ \hline 8)3713 \end{array}$$

Solution. Write the divisor at the left of the dividend with a curved line between them.

8 is not contained in 3 thousands, therefore divide 37 hundreds by 8. 8 is contained in 37 hundreds 4 times with a remainder 5. Write 4 in the quotient in the place of hundreds. Reducing 5 hundreds to tens and adding the one ten of the dividend, the result is 51. 8 is contained 6 times in 51 with a remainder 3. Write 6 in the quotient in the

place of tens. Reducing the 3 tens to units and adding the 3 units of the dividend, the result is 33 units. 8 is contained in 33 units 4 times with a remainder 1. Write 4 in the place of units and place the remainder over the divisor with a line between; thus, $\frac{1}{8}$. The complete quotient is $464\frac{1}{8}$.

112. When the divisor is so large that each step in the division must be written, the process is called **long division.**

113. Example. Divide 5207 by 98.

$$\begin{array}{r} 53\frac{13}{98} \\ 98\overline{)5207} \\ \underline{490} \\ 307 \\ \underline{294} \\ 13 \end{array}$$

Solution. Since 98 is more than 52, it is necessary to take 520 for the first partial dividend. The nearest number of tens represented by the divisor is 10; take 10, therefore, for the trial divisor. The number of tens in the partial dividend is 52. 10 is contained 5 times in 52. Write 5 in the quotient over the right-hand figure of the partial dividend as shown in the margin. Multiplying the exact divisor by 5 and subtracting the product from the partial dividend, the remainder is 30, to which annex the 7 units of the dividend, and the second partial dividend is 307. The nearest number of tens represented by the second partial dividend is 31, which will contain 10 3 times. Write 3 in the quotient. Multiplying the exact divisor by 3 and subtracting the product, the remainder is 13, to be written in the form of a fraction and annexed to the quotient. The complete quotient is $53\frac{13}{98}$.

ORAL EXERCISE

1. The quotient is 61. If the dividend and divisor were each multiplied by 4, what would the quotient be?

2. The quotient is 53. If the dividend and divisor were each divided by 3, what would the quotient be?

3. If the divisor were 4 times what it is, the quotient would be 1606. What is the quotient?

4. The quotient of one number divided by another is 12. What would the quotient be if the divisor were multiplied by 3? divided by 3?

5. How many 15's must we add together to get 4590?

WRITTEN EXERCISE

1. $21,735 was received from the sale of a farm at $35 an acre. How many acres did the farm contain?

2. What number must be added to 21,786 that it may be exactly divisible by 168?

3. The remainder is 14, the quotient 5041, and the divisor 15. What is the dividend?

4. The remainder is 7, the quotient 19,023, and the dividend 247,306. What is the divisor?

5. If 8 men can do a piece of work in 24 days, in how many days can 12 men do the same work?

6. I sell my village home for $3250, my store for $5000, my stock of goods for $11,250, receiving in part payment $8775, and for the remainder, Iowa prairie land at $15 per acre. How many acres should I receive?

7. If there are 128 cubic feet in 1 cord, how many cords in 141,492 cubic feet?

8. If 93 be added to a certain number, it will contain 648 twenty-five times. What is the number?

9. What number must be subtracted from 3476 that it may be exactly divisible by 155?

10. A man bought 490 acres of land at $40 an acre and after paying $2900 for improvements sold it for $25,000. Did he gain or lose, and how much?

CONTRACTIONS IN DIVISION

114. **To divide any number by 10, 100, 1000, etc.**

115. By the decimal system of notation numbers increase in value from right to left and decrease from left to right in a tenfold ratio; hence to divide a number by 10, 100, 1000, etc.:

From the right in the dividend point off as many places as the divisor contains ciphers.

The figures so cut off express the remainder, to be written in fractional form.

ORAL EXERCISE

By inspection, find the complete quotients of:

1. 759 ÷ 10.

2. 7527929 ÷ 1000.

3. 29 ÷ 100.

4. 4997 ÷ 10000.

5. 75627 ÷ 10.

6. 8967 ÷ 100.

7. 297249 ÷ 10.

8. 759 ÷ 10000.

9. 4627490 ÷ 1000.

116. **To divide by any number of 10's, 100's, 1000's, etc.**

117. **Example.** Find the quotient of 14,131 ÷ 4000.

$$\begin{array}{r}
3\frac{2131}{4000} \\
4|000\overline{)14|131} \\
\underline{12} \\
\frac{2131}{4000}
\end{array}$$

Solution. **Mark off as many figures in the dividend as there are ciphers in the divisor, thus dividing by 1000. The first quotient is then 14 with a first remainder 131. Dividing 14 by 4 gives 3 as the final quotient with a second remainder 2. Multiplying the second remainder by 1000 to obtain its true value and adding the first remainder, the result is the true remainder, 2131, to be written in fractional form and placed by the side of the integral quotient. The completed quotient is then $3\frac{2131}{4000}$.**

ORAL EXERCISE

By inspection, find the complete quotients of:

1. 1627 ÷ 400.
2. 571119 ÷ 19000.
3. 48887 ÷ 40.
4. 6427 ÷ 80.
5. 9687 ÷ 120.
6. 7879 ÷ 390.
7. 19249 ÷ 4000.
8. 762179 ÷ 190000.
9. 51295 ÷ 17000.
10. 6439 ÷ 160.
11. 19279 ÷ 160.
12. 15597 ÷ 5000.
13. 21259 ÷ 700.
14. 72899 ÷ 24000.
15. 7849 ÷ 260.
16. 8479 ÷ 280.
17. 9579 ÷ 190.
18. 125265 ÷ 250.
19. 16219 ÷ 4000.
20. 39379 ÷ 13000.
21. 4179 ÷ 1200.

PROPERTIES OF NUMBERS

118. **Properties of numbers** are those qualities which belong to and are inseparable from them.

119. All integral numbers are: (1) *odd* or *even;* (2) *prime* or *composite.*

120. An **odd number** is a number that cannot be exactly divided by 2; as, 5, 9, 23.

121. An **even number** is a number that can be exactly divided by 2; as, 6, 8, 44.

122. **Factors of numbers** are those numbers the continued product of which will produce the number.

123. A **prime number** is a number that cannot be resolved into two or more factors; or,

it is a number that has no integral factors except unity and itself.

Thus, 23, 59, 11, and 13 are prime numbers. 2 is the only even number that is prime.

124. A **composite number** is a number that can be resolved into two or more factors; or,

it is a number which is the product of two or more integral factors.

125. A **prime factor** is a prime number used as a factor.

126. A **composite factor** is a composite number used as a factor.

127. An **exact divisor** of a number is any integral factor of that number.

128. A **common divisor** of two or more numbers is any exact divisor of those numbers.

129. The **greatest common divisor** of two or more numbers is the greatest exact divisor common to those numbers.

130. Numbers having no common divisor or factor are said to be **relatively prime.**

131. Tests of Divisibility. In arithmetical computations it is frequently necessary to determine whether one number is divisible by another or not. In dividing numbers the following tests of divisibility will be found helpful.

1. When a number is even, it is divisible by 2.
2. When the sum of the digits of any number is divisible by 3 or 9, the whole number is divisible by 3 or 9.
3. When the right-hand figure of any number is 5 or 0, the whole number is divisible by 5.
4. When the right-hand figure is 0, the whole number is divisible by 10.
5. When the number expressed by the two right-hand figures of a number is divisible by 4, the whole number is divisible by 4.
6. When a number is even and divisible by 3, it is also divisible by 6.
7. When the number expressed by the three right-hand figures of a number is divisible by 8, the whole number is divisible by 8.

132. A **multiple** of a number is one or more times the number; or, it is that product of which the given number is an exact divisor.

133. A **common multiple** of two or more numbers is that product of which the given numbers is an exact divisor.

134. The **least common multiple** of two or more numbers is the least product of which each of the given numbers is an exact divisor.

FACTORING

135. Factoring is the process of separating or dissolving a composite number into factors.

136. General Principles. 1. Any composite number is divisible by each of its several factors successively.

2. A composite number is equal to the product of all its prime factors.

137. To find the prime factors of a composite number.

138. Example. Find the prime factors of 4290.

$$\begin{array}{r|l} 5 & 4290 \\ \hline 2 & 858 \\ \hline 3 & 429 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

Solution. The given number ends with a 0, hence is exactly divisible by 5. Dividing by 5, the quotient 858 is obtained. 858, being an even number, is exactly divisible by 2. Dividing by 2, the quotient 429 is obtained. The sum of the digits in 429 is divisible by 3; therefore the whole number is divisible by 3. Dividing by 3, the quotient 143 is obtained. 143 is exactly divisible by 11. Dividing by 11, the quotient 13 is obtained. The several divisors, 5, 2, 3, 11, and the last quotient, 13, are the prime factors required.

139. Therefore the following **rule:**

Divide the given number by any prime factor.

Divide the successive quotients in the same manner until a quotient that is prime is obtained.

The several divisors and the last quotient are the prime factors required.

WRITTEN EXERCISE

Find the prime factors of:

1. 144.
2. 124.
3. 924.
4. 289.
5. 135.
6. 25785.
7. 1575.
8. 252.
9. 951.
10. 1527.

GREATEST COMMON DIVISOR

140. **To find the greatest common divisor of two or more numbers.**

141. **Example.** Find the greatest common divisor of 42, 66, and 84.

$$\begin{array}{r|l} 2 & 42 - 66 - 84 \\ \hline 3 & 21 - 33 - 42 \\ \hline & \;\;7 - 11 - 14 \end{array}$$

Solution. Arrange the numbers as shown in the margin. The first prime factor that will divide all the numbers is 2. Divide by 2, obtaining 21, 33, and 42 as the quotients. The only prime factor that is common to these numbers is 3. Divide by 3, obtaining as the quotient 7, 11, and 14. There is no factor common to all of these numbers. Since 2 will divide all the given numbers, and 3 will divide the resulting quotients, the product of 2 × 3, or 6, is the greatest common divisor of 42, 66, and 84.

142. Hence the following rule:

Write the numbers in a horizontal line, separating them by dashes.

Divide by any prime number that will divide all the given numbers without a remainder, writing the quotients in a line below. Continue the process until the quotients have no common factor.

Multiply together the several divisors, and the result is the greatest common divisor.

143. Sometimes the series of numbers of which the greatest common divisor is to be found cannot be factored by inspection, and a method similar to the following is employed.

144. **Example.** Find the greatest common divisor of 697 and 779.

$$\begin{array}{l} 697)779(1 \\ \;\;\underline{697} \\ \;\;\;\;82)697(8 \\ \;\;\;\;\;\;\;\;\underline{656} \\ \;\;\;\;\;\;\;\;\;\;41)82(2 \\ \;\;\;\;\;\;\;\;\;\;\;\;\;\;\underline{82} \end{array}$$

Solution. Divide the greater number by the less, the divisor by the remainder, and so continue until there is no remainder. The last divisor is the greatest common divisor. Therefore the greatest common divisor of 697 and 779 is 41.

WRITTEN EXERCISE

Find the greatest common divisor of:

1. 22, 55, and 99.

2. 24, 36, 60, and 96.

3. 679 and 1869.

4. 32, 48, 80, 112, and 144.

5. A farmer has a triangular piece of land which he wishes to inclose with a board fence so that the boards may be of the greatest length possible and no fractional lengths used. If the sides of the tract of land are 84, 96, and 108 feet, respectively, what is the length of the longest board that can be used in making the inclosure?

6. How many boards will inclose, without waste, a rectangular garden, 98 feet long by 70 feet wide, the fence being straight and 5 boards high, if the boards be of equal length and the longest possible?

LEAST COMMON MULTIPLE

145. General Principles. 1. The product of two or more numbers, or any number of times their product, is a common multiple of those numbers.

2. Two or more numbers may have any number of common multiples but only one least common multiple.

3. A multiple of a number contains all the prime factors of that number.

4. A common multiple of two or more numbers contains all the prime factors of each of the numbers.

5. The least common multiple of two or more numbers is the least number that will contain all the prime factors of the given numbers.

146. To find the least common multiple of two or more numbers.

147. Example. Find the least common multiple of 12, 16, 63, and 90.

(*a*)

$12 = 2 \times 2 \times 3$

$16 = 2 \times 2 \times 2 \times 2$

$63 = 3 \times 3 \times 7$

$90 = 3 \times 3 \times 2 \times 5$

$90 \times 7 \times 2 \times 2 \times 2 = 5040$

Solutions. (*a*) Since no number less than 90 can be divided by 90, it is evident that the least common multiple cannot be less than that number; hence, it must contain the prime factors 3, 3, 2, and 5 in order to be divisible by 90; it must contain 7 as a factor in order to be divisible by 63; and it must contain 2 as a factor three more times in order to be divisible by 16; hence, the product of the factors 3, 3, 2, 5, 7, 2, 2, and 2, or 5040, must be the least common multiple of the numbers 12, 16, 63, and 90. Or,

(b)

$$\begin{array}{r|l} 2 & 12-16-63-90 \\ \hline 2 & 6-\ 8-63-45 \\ \hline 3 & 3-\ 4-63-45 \\ \hline 3 & 1-\ 4-21-15 \\ \hline & 1-\ 4-\ 7-\ 5 \end{array}$$

(b) First divide by 2; 63 not being divisible by 2, bring it to the lower line and divide again by 2; neither 63 nor 45 being divisible by 2, bring both to the quotient line. Next divide by 3; 4 not being divisible by 3, bring it to the quotient line and divide again by 3. The remaining numbers, 4, 7, and 5, being relatively prime, should be taken together with the prime divisors 2, 2, 3, and 3 as factors of the least common multiple. Their product is 5040, the same as found in 147.

NOTE. The latter method of determining the least common multiple will, in the majority of cases, be found the most convenient for practical purposes.

When one of the given numbers is a factor of another, reject the smaller one.

148. Therefore the following rule:

Write the numbers in a horizontal line, separating them by dashes.

Divide by any factor common to all the numbers, or by any prime factor common to two or more of them.

In the same manner divide the quotients obtained, and continue the process until the quotients are relatively prime.

The product of the several divisors and the undivided numbers is the least common multiple.

WRITTEN EXERCISE

Find the least common multiple of:

1. 12, 20, and 32.

2. 25, 90, and 225.

3. 6, 16, and 26.

4. A, B, and C are traveling men. A makes a visit to Boston every four months, B every three months, and C every two months. If they are all in Boston on January 1, 1903, when will they all be in that city together again for the first time?

5. What is the least number of acres that a piece of land can contain to be exactly divided into lots of 12, 14, and 18 acres respectively?

CANCELLATION

149. Cancellation is the process of shortening the operation of division, or the combined operations of multiplication and division, by omitting or striking out equal factors from the dividend and divisor.

150. General Principles. 1. Canceling a factor from a number has the effect of dividing the number by that factor.

2. Canceling a factor from both dividend and divisor does not affect the value of the quotient.

151. Example. Divide 18 × 36 by 3 × 32.

(*a*)

$$\frac{\overset{9}{\cancel{18}} \times \overset{\overset{3}{\cancel{12}}}{\cancel{36}}}{\cancel{3} \times \underset{\underset{4}{\cancel{8}}}{\cancel{32}}} = \frac{27}{4} = 6\tfrac{3}{4}$$

(*b*)

$$\begin{array}{r|l} & 9 \\ \cancel{3} & \cancel{18} \\ \cancel{32} & \cancel{36} \\ \cancel{8} & \cancel{12} \\ \underline{4} & \underline{3} \\ 4 & 27 = 6\tfrac{3}{4} \end{array}$$

Solution. Indicate the process by either of the methods shown in the margin.

3 and 36 contain the common factor 3, which cancel and write 12 in the dividend. 12 and 32 contain the common factor 4, which cancel from both and write the factors 3 and 8 in the dividend and divisor, respectively. 18 and 8 contain the common factor 2, which cancel from both and write the factors 9 and 4 in the dividend and divisor, respectively. The product of the remaining factors of the dividend, divided by the remaining factors of the divisor, is the required quotient, or 6¾.

152. Hence the following rule:

Indicate the operation in convenient form.

Cancel from the dividend and divisor all factors common to both.

Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

The result obtained is the required quotient.

WRITTEN EXERCISE

1. Divide the product of 9, 8, 12, and 24 by the product of 2, 14, and 8.

2. What is the quotient of 35 × 75 ÷ 7 × 5 × 3?

3. Multiply together 18, 42, and 64, and divide the product by the product of 6, 16, and 32.

4. $\frac{12 \times 3 \times 35 \times 24 \times 2}{6 \times 9 \times 5 \times 6} = ?$

5. How many bushels of potatoes at 60¢ per bushel will pay for 450 pounds of sugar at 6¢ per pound?

6. A farmer traded 4 hogs weighing 325 pounds at 6¢ per pound for sugar at 5¢ per pound. How many entire barrels of 312 pounds each should the farmer have received?

7. How many yards of cloth at 15¢ per yard should be given for 9 barrels of pork, each barrel containing 200 pounds, at 6¢ per pound?

8. How many pieces of cloth containing 45 yards each should be received for 5 baskets of eggs, each basket containing 21 dozen at 18¢ per dozen, if the cloth be valued at 8¢ per yard?

9. If 320 acres of land produce 25,600 bushels of wheat, how many bushels of wheat will 110 acres produce at the same rate?

10. If a horse trots 5 miles in 30 minutes, how far can he trot in 21 minutes at the same rate?

11. How many sections of Texas prairie land, each containing 640 acres, at $ 5 per acre, should be given for an Ohio farm of 400 acres, at $ 40 per acre?

12. A farmer exchanged 196 loads of oats, each load containing 30 sacks of 2 bushels each, worth 30¢ per bushel, for flour at 5¢ per pound. At 196 pounds per barrel, how many barrels should he have received?

13. A farmer exchanged 250 bushels of wheat at 80¢ per bushel for cloth at 40¢ per yard. How many yards of cloth should he have received?

14. If 9 men earn $108 in 6 days, how much will 15 men earn in 4 days at the same rate?

15. 30 half chests of Japan tea containing 75 pounds each at 30¢ per pound were exchanged for brown sugar at $2\frac{1}{4}$¢ per pound. If a barrel of brown sugar weighs 300 pounds, how many barrels should have been received?

UNITED STATES MONEY

153. **Money** is a standard measure of value used as a medium of exchange.

154. **Currency** is the term applied to money or its equivalent.

155. A **decimal currency** is a currency whose denominations increase and decrease on a scale of ten.

156. **United States money**, commonly called Federal money, is the legal currency of the United States. It is a decimal currency and consists of *coin* and *paper money.*

157. The denominations and scale of United States money are shown in the following

Table

10 mills (m.) = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes or 100 cents = 1 dollar ($)
10 dollars = 1 eagle (E.)

The *dollar sign*, $, is always written before the number. The mill is not a coin. It is used only as a decimal of a cent, which is the smallest money of the mint and the smallest recognized in business. The eagle and the dime are used only as names of coins and never in reading United States money.

158. **Bullion** is pure gold or silver in bars or ingots.

159. **Coins** are pieces of metal converted into money by being stamped by the authority of the government in such a way as to indicate the rate at which they shall pass in trade.

160. The coins of the United States are of two kinds, namely:

1. Those made by the authority of the government, in unlimited quantities, for private persons from metal deposited by them.

The government provides that private persons may deposit metal with the United States mints or assay offices in unlimited quantities for the purpose of having it weighed, refined, assayed, and returned to them in the form of standard coins or in ingots of standard fineness.

2. Those subsidiary coins made from silver, nickel, and copper by the authority of the government.

The government has the power to buy metal for the purpose of making it into subsidiary coins for itself. Since subsidiary coins may be sold to private individuals at more than cost, their quantity is restricted.

161. The coins of the United States are *gold, silver, bronze,* and *nickel.*

Coins of the United States

Coins	Composition	Weight	Value
Gold			
Quarter-eagle	$\frac{9}{10}$ pure gold and $\frac{1}{10}$ alloy	64.5 Troy grains	$2.50
Half-eagle	$\frac{9}{10}$ pure gold and $\frac{1}{10}$ alloy	129 Troy grains	5.00
Eagle	$\frac{9}{10}$ pure gold and $\frac{1}{10}$ alloy	258 Troy grains	10.00
Double-eagle	$\frac{9}{10}$ pure gold and $\frac{1}{10}$ alloy	516 Troy grains	20.00
Silver			
Dime	$\frac{9}{10}$ pure silver and $\frac{1}{10}$ alloy	38.58 Troy grains	$0.10
Quarter-dollar	$\frac{9}{10}$ pure silver and $\frac{1}{10}$ alloy	96.45 Troy grains	.25
Half-dollar	$\frac{9}{10}$ pure silver and $\frac{1}{10}$ alloy	192.9 Troy grains	.50
Dollar	$\frac{9}{10}$ pure silver and $\frac{1}{10}$ alloy	412.5 Troy grains	1.00
Bronze and Nickel			
1-cent piece	$\frac{19}{20}$ copper and $\frac{1}{20}$ tin and zinc	48 Troy grains	$0.01
5-cent piece	$\frac{3}{4}$ copper and $\frac{1}{4}$ nickel	77.16 Troy grains	.05

162. The **standard unit of value** in the United States is the gold dollar, which contains 23.22 Troy grains of pure gold and weighs 25.8 Troy grains.

The gold dollar being so inconveniently small is not coined now.

163. The **paper money** of the United States at present consists of *gold certificates, silver certificates, United States notes, treasury notes,* and *national bank bills.*

164. **Gold certificates** are issued for gold deposited with the Treasurer of the United States. They represent values of $20 and upward to $20,000.

165. **Silver certificates** are issued for silver deposited with the Treasurer of the United States in amounts not less than $10. They represent values of $1 and upward to $100.

166. **United States notes** (greenbacks) represent values of $10 and upward to $1000.

167. **National bank bills** are notes issued by national banks under the supervision of the national government. They are now issued in amounts of $5 and upward to $100.

168. **Treasury notes** of 1890 are now in the course of retirement. They were formerly issued in amounts of $1 and upward to $20. They cannot be reissued.

169. **Legal tender** is the term applied to such money as may be legally offered in payment of debts.

170. The following are legal tender in the United States as noted:

1. Gold coins, except when below the standard weight because of abrasion.
2. Silver dollars and treasury notes of 1890 in all cases where the contract does not expressly stipulate otherwise.
3. United States notes (greenbacks), except for interest on the public debts and for duties on imports.
4. National bank notes for any debt to a national bank, and for taxes and other dues to the United States, except duties on imports.
5. Silver coins less than one dollar in all cases where the amount does not exceed ten dollars in one payment.
6. Nickel and copper coins in all cases where the amount does not exceed twenty-five cents in one payment.

NOTATION OF UNITED STATES MONEY

171. The dollars form the integral part of the number, and are written to the left of the dot, called the *decimal point*. The cents and mills form the fractional part of the number and are written to the right of the decimal point.

172. Since *ten* dimes make one dollar, the figures written in the *first place* to the right of the decimal point are *tenths* of a dollar, or dimes. Since *one hundred* cents make one dollar, the figures written in the *second place* to the right of the decimal point are *hundredths* of a dollar, or cents. Since *one thousand* mills make a dollar, the figures written in the *third place* to the right of the decimal point are *thousandths* of a dollar, or mills.

Thus: Six dollars, forty-eight cents, two mills, is written $6.482.

In writing cents less than ten, a cipher should occupy the first place to the right of the decimal point.

In the final results all mills less than five are rejected, and all five or more are counted as a whole cent.

173. Whenever it is desirable to express United States money in written words, the cents should be written as hundredths of a dollar, and in fractional form.

Thus: Twenty-five and $\frac{42}{100}$ dollars.

REDUCTION OF UNITED STATES MONEY

174. **Reduction** is the process of changing the *unit* without changing the *value* of a number.

175. To reduce dollars to cents.

176. Since there are one hundred cents in a dollar, to reduce dollars to cents, *multiply by 100 by annexing two ciphers to the numbers expressing a whole number of dollars, or by moving the decimal point two places to the right in numbers expressing cents, or dollars and cents.*

177. To reduce cents to dollars,

Divide by 100 by removing the decimal point two places to the left.

ORAL EXERCISE

By inspection, reduce:

1. $7.25 to cents.

2. $119 to cents.

3. 241¢ to dollars.

4. 92,798¢ to dollars.

5. $157.32 to cents.

6. 72,572¢ to dollars.

7. Which is the heavier, a gold eagle or a silver dollar? A gold eagle or a silver half-dollar?

8. Which is the heavier, a five-dollar gold piece or a five-cent nickel piece? a gold dollar or a bronze cent?

9. Why are the gold and silver coins of the United States never made more than $\frac{9}{10}$ pure?

10. Are national bank notes legal tender? silver dollars? pennies? gold certificates? silver certificates?

11. Under what circumstances would gold coin not have full legal tender value?

ADDITION AND SUBTRACTION OF UNITED STATES MONEY

178. To add or subtract United States money,

Write dollars under dollars and cents under cents.

Add or subtract as in simple numbers, placing the decimal point in the result directly under the points in the numbers added or subtracted.

WRITTEN EXERCISE

Copy or write from dictation and find the sum of each of the following problems:*

1.	*2.*	*3.*	*4.*
$157,926.04	$47,198.76	$919,010.01	$876,311.40
52.19	2.91	1,889.76	40.32
9,261,549.62	1,487.59	33.44	21.56
1,694,247.57	101.11	221.34	2,197.77
5,216.90	321,876.34	2,345.66	140.40
425.86	8,198.99	1.06	999.88
52.95	2,345.98	66.87	278,811.33
1,076.87	1.11	221,198.32	9,435.23
27,214.95	231.59	44,859.83	1.06
276,421.87	22.81	321.54	6.05
932.17	45,728.67	2,378.95	200,400.58
4.26	3,221.55	34.40	3,211.59
259,426.74	19.88	70.87	20.87
9,275.18	987,111.23	1,100.58	12,549.15

Copy or write from dictation the following numbers and find the sums by horizontal addition:†

5. 932^{10}, $22{,}118^{08}$, 81^{21}, 967^{88}, 221^{99}

6. 346^{50}, 291^{75}, 100^{31}, 269^{11}, 80^{98}.

7. 2165^{84}, 72^{48}, 90^{20}, 117^{65}, 500^{50}, 1127^{14}.

8. 15^{16}, 2^{97}, 11^{46}, 107^{90}, 9^{28}, 81^{45}, 123^{98}, 6^{01}, 15^{30}, 11^{11}.

9. A merchant bought cottons, for 3467^{25}; linens, for 1326^{15}; woolens, for 4215^{75}; delaines, for 1025^{45}; brocades, for 1127^{50}. If all were sold for 13256^{25}, how much was gained?

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* Addition is so interwoven with all arithmetical processes that proficiency in the subject should be insisted upon. If the principles of grouping have not been mastered, simple addition should be reviewed before any advance steps are taken. *Accurate* answers for problems similar to 1, 2, 3, and 4 should be obtained in from twenty to twenty-five seconds.

† Under some circumstances it is desirable to write United States money expressed in dollars and cents without the dollar sign and the decimal point, with the decimal part placed slightly above that expressing the integers or dollars.

Thus, $5.25 may be written 5^{25}. $13.08 may be written 13^{08}. This is advisable only where the sum of several items is to be found by horizontal addition.

MULTIPLICATION OF UNITED STATES MONEY

179. **To multiply United States money,**

Multiply as in simple numbers, and from the right in the product point off as many places as there are places to the right of the decimal point in the multiplicand.

Money is a concrete expression. In a critical analysis of its multiplication, therefore, the money cost or price of the article is a concrete multiplicand. The number of things bought or sold is an abstract multiplier and their product is concrete and of the same name or denomination as the multiplicand. However, since the United States money scale is decimal, these terms may be interchanged for convenience.

WRITTEN EXERCISE

1. Find the amount of the following bill. Make all extensions mentally as explained in **85–88**.

28 lb. lard at 11¢.
46 bu. salt at 22¢.
117 bu. apples at 33¢.
112 lb. butter at 22¢.
132 bu. onions at 44¢.
113 bu. potatoes at 66¢.

2. Find the total cost of the following farm produce. Make all extensions mentally as explained in **95–97**.

24 bu. wheat at $1.02.
215 bu. barley at $1.04.
108 bu. oats at 42¢.
103 bu. rye at 83¢.
105 bu. peas at 72¢.
204 lb. butter at 34¢.

3. Find the amount of the following bill. Make all extensions mentally as explained in **91–93**.

54 yd. jeans at 21¢.
27 yd. delaine at 32¢.
64 yd. gingham at 23¢.
42 yd. drilling at 21¢.
48 yd. print at 24¢.
121 yd. ticking at 15¢.
61 yd. sheeting at 13¢.
217 yd. cashmere at $1.13.

4. Find the total cost of the following items. Make the extensions mentally as explained in **95–97**.

116 yd. moquette carpet at $3.02.
131 yd. Brussels carpet at $2.05.
103 yd. ingrain carpet at $1.04.
115 yd. border No. 1 at $3.06.
64 yd. border No. 2 at $2.07.
43 yd. border No. 3 at $1.08.

5. Find the amount of the following bill. Make all extensions as explained in **90**.

325 yd. tapestry Brussels at $2.17.
547 yd. 3-ply ingrain at $1.26.
427 yd. moquette at $2.79.
648 yd. Axminster at $2.55.
328 yd. body Brussels at $2.48.
255 yd. velvet at $2.79.

DIVISION OF UNITED STATES MONEY

180. **To divide United States money.**

181. **Examples.** *1.* **If 5 hats are worth $25.65, what is 1 hat worth?**

$$5)\overline{25.65}$$ with quotient 5.13

SOLUTION. Since 5 hats are worth 2565 cents ($25.65), 1 hat is worth $\frac{1}{5}$ of 2565 cents, or 513 cents. 513 cents equals $5.13.

2. If 4 boxes of oranges are worth $17, what is 1 box worth?

$$4)\overline{17.00}$$ with quotient 4.25

SOLUTION. Since 4 boxes of oranges cost 1700 cents ($17.00), one box will cost $\frac{1}{4}$ of 1700 cents, or 425 cents. 425 cents equals $4.25.

3. How many boxes of oranges at $4.25 per box can be bought for $17?

$$425)\overline{1700}$$ with quotient 4

SOLUTION. Since 1 box of oranges cost 425 cents ($4.25), as many times 1 box can be bought for 1700 cents as 425 is contained times in 1700, or 4 *times.*

182. Therefore the following rule:

Divide as in simple numbers.

From the right of the quotient point off as many decimal places as the number of places in the dividend exceed those in the divisor.

When the dividend and divisor each contain the same number of decimal places, the numbers may be regarded as integers in performing the operation of division.

When the divisor alone contains cents, both dividend and divisor may be reduced to cents and the division performed exactly as in whole numbers.

WRITTEN EXERCISE

1. A dealer bought wheat at 95¢, oats at 45¢, and corn at 65¢ per bushel. He paid $332.50 for the wheat, $191.25 for the oats, and $113.75 for the corn. How many bushels of each did he buy in all?

2. Having sold my mill for $17,250, and 316 barrels of flour in stock at $5.15 per barrel, I invested out of the proceeds $1185.85 in furnishing a house, $1260 in farming utensils, $1582.25 in live stock, and with the remainder paid in full for a farm of 163 acres. What was the cost of the farm per acre?

METHODS FOR PROVING WORK

183. Addition is generally verified as explained in **44**.

184. A good way to test the accuracy of subtraction is to add the remainder and the subtrahend. If the work is correct, the result obtained should equal the minuend.

185. The work of multiplication may be verified by interchanging the multiplier and the multiplicand, and remultiplying. If the results obtained by both operations are the same, the work is assumed to be correct.

186. The operation of division may be verified by multiplying together the quotient and divisor and adding to the product the remainder, if any.

CASTING OUT NINES AND ELEVENS

187. The basis of our numerical system being 10, every power* of 10 is 1 more than some multiple of 9; and 10 or any power of 10 multiplied by a single digit is some multiple of 9, plus that digit.

Thus, $10 = 9 + 1$; $100 = 11 \times 9 + 1$; and $60 = 6 \times 9 + 6$; $500 = 55 \times 9 + 5$.

188. $4582 = 4000 + 500 + 80 + 2$. Since the excess of nines (the remainder after the nines are cast out) in 4000 is 4, in 500, 5, in 80, 8, in 2, 2, it follows that *the excess of nines in any number is the same as the excess of nines in the sum of the digits of that number.*

Thus, the excess of nines in $4582 = 4 + 5 + 8 + 2$, or 19. $19 = 10$. $10 = 1$.

189. In finding the excess of nines, it is always best to omit all nines and also to reject them as soon as they occur in the addition.

Thus, in casting out the nines in 954,727, begin at the left and drop the nines as soon as they occur. The excess of nines in 954,727 is then found to be 7.

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* A power of a number is the product arising from multiplying a number by itself one or more times.

190. Since 11 is just 1 more than the numerical basis 10, even powers of 10 are multiples of 11, plus 1, and odd powers of 10 are multiples of 11, minus 1.

Thus, 10 × 10 or 100, an even power of 10, equals 11 × 9 + 1. 10 × 10 × 10 × 10 × 10, an odd power of 10, equals 9091 × 11 − 1.

191. Any even power of 10 multiplied by a single digit is some multiple of 11 plus that digit. Any odd power of 10 multiplied by a single digit is some multiple of 11 minus that digit.

Thus, 600 = 54 × 11 + 6; and 6000 = 546 × 11 − 6.

192. 10 multiplied by any single digit is some multiple of 11 minus the difference between 11 and that digit.

Thus, 30 = 2 × 11 + 8; 50 = 4 × 11 + 6; 80 = 7 × 11 + 3; 90 = 8 × 11 + 2; etc.

193. It therefore follows that:

The digit in the odd place of any number of two figures, with eleven added whenever necessary, minus the digit in the even place, equals the excess of elevens in that number.

Applying this principle to *all* numbers, *the sum of the digits in the odd places, increased by eleven or a multiple of eleven whenever necessary, minus the sum of the digits in the even places, is equal to the excess of elevens in the entire number.*

194. The elevens may be dropped from the partial additions in the same manner as explained for nines.

195. The properties of nine and eleven, as explained above, may be used in proving addition, subtraction, multiplication, and division.

196. To prove addition by casting out the nines and elevens.

197. Example. Add 375, 425, 623, and 412. Prove the work by casting out (*a*) the nines and (*b*) the elevens.

(*a*) Excess of nines.

$$\left.\begin{array}{r} 375 = 6 \\ 425 = 2 \\ 623 = 2 \\ \underline{412 = 7} \end{array}\right\} 8$$

$$1835 = 17 = 8$$

Solution. The excess of nines in 375 is 6; in 425 is 2; in 623 is 2; in 412 is 7. The excess of nines in the sum of 6, 2, 2, and 7 is 8. The excess of nines in 1835 is also 8. Since the excess of nines in all the numbers is equal to the excess of nines in the sum of the numbers, the work is assumed to be correct.

(*b*) Excess of elevens.

$$\begin{array}{r}
375 = 1 \\
425 = 7 \\
623 = 7 \\
\underline{412} = \underline{5}
\end{array}\left.\vphantom{\begin{array}{r}1\\1\\1\\1\end{array}}\right\} 20 = 9$$

$$1835 = 20 = 9$$

Solution. 16 (11 + 5) − 7 + 3 = 12. 12 = 1, or the excess of elevens in 375. 5 − 2 + 4 = 7, or the excess of elevens in 425. 3 − 2 + 6 = 7, or the excess of elevens in 623. 2 − 1 + 4 = 5, or the excess of elevens in 412. The sum of 1, 7, 7, and 5 equals 20, which, divided by 11, gives a remainder 9. (5 − 3) + (8 − 1) = 9, the excess of elevens in the sum. Since the excess of elevens in all the numbers is equal to the excess of elevens in the sum, the work is assumed to be correct.

198. To prove subtraction by casting out the nines and elevens.

199. The excess of nines or elevens in the minuend minus the excess of nines or elevens in the subtrahend should equal the excess of nines or elevens in the remainder; or, the excess of nines or elevens in the subtrahend plus the excess of nines or elevens in the remainder should equal the excess of nines or elevens in the minuend.

200. To prove multiplication by casting out the nines and elevens.

201. Example. Find the product of 512 × 324 and verify the result by casting out (*a*) the nines and (*b*) the elevens.

(*a*) Excess of nines.

$$\begin{array}{r}
512 = 8 \\
\underline{324} = 0
\end{array}\left.\vphantom{\begin{array}{r}1\\1\end{array}}\right\} 0$$

$$\begin{array}{r}
2048 \\
\underline{16384} \\
165888 = 0
\end{array}$$

Solution. Use the contracted method of multiplying explained in **90**. The excess of nines in 512 is 8; in 324, 0. 8 × 0 = 0. The excess of nines in the completed product is 0. Since the excess of nines in the multiplicand multiplied by the excess of nines in the multiplier is equal to the excess of nines in the product, the work is assumed to be correct.

(*b*) Excess of elevens.

$$\begin{array}{r}
512 = 6 \\
\underline{324} = 5
\end{array}\left.\vphantom{\begin{array}{r}1\\1\end{array}}\right\} 30 = 8$$

$$\begin{array}{r}
2048 \\
\underline{16384} \\
165888 = 8
\end{array}$$

Solution. 2 − 1 + 5 = 6, or the excess of elevens in 512. 4 − 2 + 3 = 5, or the excess of elevens in 324. 6 × 5 = 30, which, divided by 11, gives a remainder 8. 8 − 8 + 8 − 5 + 6 − 1 = 8, or the excess of elevens in the completed product. Since the excess of elevens in the multiplicand multiplied by the excess of elevens in the multiplier is equal to the excess of elevens in the completed product, the work is assumed to be correct.

202. To prove division by casting out the nines and elevens.

203. Examples in division may be proved by multiplying the excess of nines or elevens in the divisor by the excess of nines or elevens in the quotient. If the work is correct, the result should equal the excess of nines or elevens in the dividend, or the dividend minus the remainder when there is a remainder.

On the whole, the proof of casting out the elevens is more reliable than the proof of casting out the nines, but neither proof is practiced very generally by accountants except in proving long multiplications and divisions.

WRITTEN EXERCISE

1. Multiply 125,426 by 567 in two lines of partial products. Verify the work by casting out the elevens.

2. Multiply 112,121 by 12,816 in two lines of partial products. Verify the work by casting out the nines.

3. Multiply 121,214 by 112,568 in three lines of partial products. Verify the work by casting out the elevens.

4. Find the amount of the following bill, making the extensions mentally as explained in **90**. Verify the work by casting out the elevens.

248 yd. black dress silk, $1.24.
576 yd. Amazon cloth, $2.17.
357 yd. cashmere, $1.55.
248 yd. black wool crepon, $2.79.
568 yd. English camel's hair, $2.17.
124 doz. cotton hose, $1.86.

5. Find the amount of the following bill, making the extensions mentally as explained in **85–88**. Verify the work by casting out the nines.

116 gr. bone buttons, 22¢.
112 pc. black Chantilly lace, 88¢.
53 yd. cotton surah lining, 66¢.
124 yd. wash silk, 44¢.
141 yd. feather ticking, 11¢.
118 yd. fancy gingham, 11¢.
54 yd. gunner's duck, 22¢.
83 yd. Scotch cheviot, 55¢.

WRITTEN REVIEW

1. Find the total of the products called for in the oral exercise, page 40.

2. Find the total cost of the items in the oral review, pages 45 and 46.

3. A man earned $312.50 during February. During March he earned $49.50 more than in February. In April he earned as much as he did during February and May. If he earned $200 in May, how much did he earn in the four months?

4. A finds that in five months he spends as much as he earns in four. At that rate, how long will it take him to save $2400 if he earns $1200 a year?

5. Multiply 12,501 by 486, making only two lines of partial products. Verify the work by casting out the nines.

6. A man bought an equal number of barrels of flour and oatmeal for $260. If the flour cost $6.20 and the oatmeal $6.80 per barrel, how many barrels of each did he buy?

7. A merchant failed in business, and the excess of his liabilities over resources was found to be $3000. If he could pay his creditors but 60¢ on a dollar, what were his total liabilities, and how much did A, whom he owed $1500, receive?

8. A and B had $9245 divided between them. The difference between their shares was $245. What had each?

9. A merchant's cash receipts for a week were as follows: Monday, $921.40; Tuesday, $525.44; Wednesday, $321.50; Thursday, $425.60; Friday, $926.80; Saturday, $120.40. What were his average daily sales?

10. How many barrels of apples at $2.50 must be given for 1200 bushels of potatoes at 50¢?

11. A merchant's gains for February amounted to $1600, or $1100 less than his gains for January. If his gains for January were $60 more than four times his gain for March, what were his total gains for the three months?

12. Multiply 113,214 by 12,816 in two lines of partial products.

13. Multiply 21,213 by 96,486 in three lines of partial products. Verify the work by casting out the elevens.

FRACTIONS

COMMON FRACTIONS

204. *Quantity* is anything which may be measured or which may be regarded as being made up of parts like the whole. *Numbers* are the expressions by which we measure quantities. The basis of all numbers is the *unit.*

There are *integral units* (3) or whole things, and *fractional units* (5) or parts of things obtained by dividing integral units into any number of equal parts.

205. A **fraction** is one or more fractional units.

206. A **common fraction** is a fraction expressed by two numbers, one written above and the other below a horizontal line.

207. The **terms of the fraction** are the two integers, called *numerator* and *denominator,* used to express one or more fractional units.

208. The **denominator** is the term which indicates the number of parts into which a unit has been divided; it is written below the line, and *denominates* or names the size or value of each part of the fraction.

209. The **numerator** is the term which indicates the number of equal parts taken to form the fraction; it is written above the line.

210. **To read fractions,**

Pronounce the numerator first and then the denominator.

Thus, $\frac{1}{7}$, $\frac{2}{3}$, and $\frac{7}{8}$, are read, *one seventh*, *two thirds*, and *seven eighths.*

211. All fractions express unperformed division. The *denominator* is the *divisor,* the *numerator* the *dividend,* and the *value of the number* expressed by the fraction the *quotient.* Hence,

When the numerator and denominator are equal, the value expressed by the fraction is 1; when the numerator is less than the denominator, the value expressed by the fraction is *less than 1; when the numerator is more than the denominator, the value expressed by the fraction is greater than 1;* and

Of two fractions having the same denominator, the one having the larger numerator expresses the greater value.

Of two fractions having the same numerator, the one having the smaller denominator expresses the greater value.

DRILL EXERCISE

1. What is the unit of 5 tons of coal? of 5 thousand feet of lumber? of 7 dozen of eggs? of $\frac{5}{8}$ of a week? of $2\frac{1}{2}$ acres of land?

2. What is the fractional unit of a number divided into 3 parts? into 31 parts? into 13 parts?

3. What is the fractional unit of $\frac{3}{4}$? of $\frac{9}{10}$? of $\frac{7}{15}$? of $\frac{8}{9}$?

4. A unit contains how many sevenths? fifteenths? elevenths? twenty-fifths? eighths? sixths?

5. Which is the greater, $\frac{1}{3}$ of a number or $\frac{1}{6}$ of a number? Why?

6. How many times $\frac{1}{24}$ of a number is 1? Why?

7. A receives $\frac{1}{3}$ of the profits of a business, and B $\frac{1}{6}$. If B's profits in one month are $400, what are A's profits for the same time?

Solution. When a number is divided into 3 equal parts, each part is twice as large as the parts of the same unit divided into 6 equal parts. Hence, $\frac{1}{3}$ of a number is twice $\frac{1}{6}$ of the same number; and if $\frac{1}{6}$ of a number is $400, $\frac{1}{3}$ of the same number is twice $400, or $800. Therefore, A's profits are $800.

8. A invested $\frac{1}{24}$ in the capital stock of a certain business, and B $\frac{1}{8}$. If A's investment was $9000, what was B's? Why?

CLASSIFICATION OF FRACTIONS

212. For convenience fractions may be classified as *simple, complex,* and *compound.*

213. A **simple fraction** is a fraction which has one numerator and one denominator, each of which is an integer.

Simple fractions are either *proper* or *improper.*

214. A **proper fraction** is a fraction whose numerator is less than its denominator, and whose value is less than 1; as, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{6}$.

215. **An improper fraction** is a fraction whose numerator is equal to, or greater than, its denominator, and whose value is 1, or more than 1; as, $\frac{14}{9}$, $\frac{23}{8}$, $\frac{15}{7}$.

216. A **mixed number** is a whole number and a fraction united; as, $2\frac{1}{2}$, $7\frac{1}{9}$, $250\frac{7}{8}$.

217. A **complex fraction** is a fraction having one or both of its terms fractional; as, $\frac{\frac{3}{4}}{9}$, $\frac{7}{\frac{3}{4}}$, $\frac{\frac{3}{4}}{\frac{7}{8}}$, $\frac{2\frac{1}{2}}{\frac{9}{10}}$.

218. A **compound fraction** is a fractional part of a whole number or mixed number, or another fraction; as, $\frac{1}{5} \times \frac{1}{9}$, $\frac{1}{6} \times 2\frac{1}{2}$, $\frac{2}{5}$ of $\frac{9}{10}$.

219. **General Principles.** 1. Multiplying the numerator or dividing the denominator multiplies the fraction.

2. Dividing the numerator or multiplying the denominator divides the fraction.

3. Multiplying or dividing both the numerator and denominator by the same number does not alter the value of the fraction.

REDUCTION OF FRACTIONS

220. The process of changing the form without changing the value of fractions is called **reduction of fractions**.

221. **To reduce a whole number to a fraction.**

222. **Example.** Reduce 7 to a fraction whose denominator is 7.

SOLUTION. Since in 1 unit there are 7 sevenths, in 7 units there must be 7 times 7 sevenths, or 49 sevenths. Hence, 7 equals $\frac{49}{7}$.

223. The value of a mixed number may be represented by an improper fraction.

224. **To reduce a mixed number to an improper fraction.**

225. **Example.** Reduce $17\frac{1}{2}$ to halves.

$$\begin{array}{r}\underline{17\frac{1}{2}}\\ \frac{35}{2}\end{array}$$

SOLUTION. Since in 1 there are 2 halves, in 17 there must be 17 times 2 halves, or 34 halves. Thirty-four halves plus 1 half equals 35 halves. Hence, $17\frac{1}{2}$ equals $\frac{35}{2}$.

ORAL EXERCISE

1. How many fourths in 6? in 25? in 16? in 71? in $12\frac{1}{4}$?

2. How many thirds in 15? fourths? fifths? sixths? ninths?

3. How many thirds in $12\frac{1}{3}$? in $8\frac{1}{3}$? in $17\frac{1}{3}$? in $42\frac{2}{3}$? in $27\frac{2}{3}$?

4. How many sixths in $2\frac{1}{3}$? in $1\frac{1}{3}$? in $4\frac{1}{2}$? in $5\frac{1}{6}$? in $12\frac{1}{2}$?

5. Nine equals how many times $\frac{1}{2}$? how many times $\frac{1}{4}$?

6. How many more pieces in 5 melons cut into fourths than in 3 melons cut into fifths?

7. A has $2\frac{7}{8}$ tons of coal which he proposes to distribute among some poor families. How many families will he aid if he gives each one $\frac{1}{8}$ of a ton?

WRITTEN EXERCISE

Reduce the following mixed numbers to equivalent improper fractions:

1. $13\frac{3}{5}$. *3.* $1040\frac{1}{20}$. *5.* $17\frac{3}{5}$. *7.* $14\frac{4}{9}$. *9.* $121\frac{4}{5}$. *11.* $43\frac{5}{7}$.
2. $27\frac{1}{4}$. *4.* $186\frac{7}{15}$. *6.* $78\frac{3}{4}$. *8.* $26\frac{5}{38}$. *10.* $170\frac{1}{10}$. *12.* $425\frac{7}{16}$.

226. The value of an improper fraction represents a quantity which may be expressed by a whole or a mixed number.

227. To reduce an improper fraction to a whole or a mixed number.

228. Example. Reduce $\frac{134}{5}$ to a mixed number.

$$5\overline{)134}\quad 26\frac{4}{5}$$

SOLUTION. Since 5 fifths equals 1, 134 fifths must equal as many times 1 as there are times 5 in 134, which are $26\frac{4}{5}$ times. Hence, $\frac{134}{5}$ equals $26\frac{4}{5}$.

ORAL EXERCISE

Convert into whole or mixed numbers:

1. $\frac{24}{3}$. *3.* $\frac{44}{5}$. *5.* $\frac{25}{13}$. *7.* $\frac{79}{13}$. *9.* $\frac{47}{8}$. *11.* $\frac{63}{7}$. *13.* $\frac{72}{8}$.
2. $\frac{17}{3}$. *4.* $\frac{12}{5}$. *6.* $\frac{49}{16}$. *8.* $\frac{9}{2}$. *10.* $\frac{21}{9}$. *12.* $\frac{60}{10}$. *14.* $\frac{42}{7}$.

WRITTEN EXERCISE

Convert into whole or mixed numbers:

1. $\frac{1429}{10}$. *3.* $\frac{2000}{73}$. *5.* $\frac{5000}{181}$. *7.* $\frac{411}{17}$. *9.* $\frac{1006}{47}$.
2. $\frac{400}{23}$. *4.* $\frac{1100}{79}$. *6.* $\frac{288}{29}$. *8.* $\frac{1241}{28}$. *10.* $\frac{1476}{91}$

229. To reduce a fraction to its lowest terms.

230. Examples. *1.* Reduce $\frac{63}{105}$ to its lowest terms.

$\frac{63}{105} = \frac{21}{35} = \frac{3}{5}$. Solution. Observing that the terms of the fraction $\frac{63}{105}$ are divisible by 3, first divide by 3 and obtain as a result $\frac{21}{35}$. Observing that the terms of the fraction $\frac{21}{35}$ are divisible by 7, divide by 7 and obtain as a result $\frac{3}{5}$. Since 3 and 5, the terms of the last fraction, are relatively prime, the reduction cannot be carried further. Hence, $\frac{63}{105}$ reduced to its lowest terms equals $\frac{3}{5}$.

Since both numerator and denominator have been divided by the same numbers, the value of the fraction remains unchanged (**219**).

2. Reduce $\frac{697}{779}$ to its lowest terms.

$$\text{G. C. D.} = 41.$$
$$\frac{697 \div 41 = 17}{779 \div 41 = 19}$$

Solution. Being unable to determine a common divisor of 697 and 779 by inspection, find their greatest common divisor (**144**) and obtain 41. Dividing the terms of the fraction by 41 the result is $\frac{17}{19}$. 17 and 19 being relatively prime, the reduction cannot be carried further

WRITTEN EXERCISE

Reduce to their lowest terms:

1. $\frac{18}{24}$. *3.* $\frac{78}{102}$. *5.* $\frac{840}{7285}$. *7.* $\frac{2160}{4080}$. *9.* $\frac{1280}{1920}$.
2. $\frac{25}{60}$. *4.* $\frac{50}{625}$. *6.* $\frac{1287}{1521}$. *8.* $\frac{5184}{6912}$. *10.* $\frac{528}{1760}$.

231. In obtaining final results, *all* proper fractions should be reduced to their lowest terms and *all* improper fractions to whole or mixed numbers.

232. To reduce a fraction to higher terms.

233. Example. Reduce $\frac{5}{9}$ to a fraction whose denominator is 63.

$63 \div 9 = 7$.
$\frac{5}{9} = \frac{35}{63}$.

Solution. Since 9 is contained 7 times in 63, the given fraction may be reduced to a fraction whose denominator is 63, by multiplying both of its terms by 7. Multiplying both terms of $\frac{5}{9}$ by 7, the result is $\frac{35}{63}$. $\frac{35}{63}$ has the same value as $\frac{5}{9}$ (**219**).

In practice think only of results. Thus, for the above work: $63 \div 9 = 7$. $5 \times 7 = 35$. $\frac{35}{63}$.

234. Hence the following rule:

Divide the required denominator by the denominator of the given fraction.

Multiply the numerator of the given fraction by the quotient thus obtained and write the product over the required denominator.

The result is the fraction in higher terms.

ORAL EXERCISE

1. How many ninths in $\frac{2}{3}$? fifteenths? thirtieths? sixtieths?

2. Change $\frac{3}{5}$ to an equivalent fraction having 45 for its denominator.

3. Reduce $\frac{7}{8}$ to a fraction whose denominator is 32; 64; 128.

4. Reduce $\frac{7}{9}$ to sixty-thirds; to one-hundred-eighths; to ninety-ninths.

5. If $\frac{1}{9}$ of a number is 9, what is $\frac{1}{3}$ of the same number?

6. If $\frac{1}{3}$ of a number is 3, what is $\frac{1}{9}$ of the same number?

7. How many thirty-seconds in $\frac{1}{2}$? in $\frac{1}{4}$? in $\frac{1}{8}$? in $2\frac{1}{2}$? in $4\frac{1}{4}$?

235. A **common denominator** of two or more fractions is any number which will contain each of the given denominators an exact number of times.

236. The **least common denominator** of two or more fractions is the least number that will contain each of the given denominators an exact number of times.

237. The least common multiple (**134**) of all the denominators of the given fractions is the **least common denominator.**

238. To replace two or more fractions by two or more equivalent fractions having the least common denominator.

239. Example. Reduce $\frac{3}{4}$, $\frac{1}{6}$, and $\frac{1}{18}$ to equivalent fractions having a common denominator.

$$\begin{array}{r|l}
2 & 4-6-18 \\ \hline
3 & 2-3-\ \ 9 \\ \hline
 & 2-1-\ \ 3
\end{array}
\qquad
\begin{array}{l}
\frac{3}{4} = \frac{27}{36}. \\
\frac{1}{6} = \frac{6}{36}. \\
\frac{1}{18} = \frac{2}{36}.
\end{array}$$

$$2 \times 3 \times 2 \times 3 = 36.$$

$$\text{L. C. M.} = 36.$$

Solution. The least common multiple of the given denominators is found to be 36. Using 36 as the least common denominator reduce each of the given fractions to thirty-sixths, as explained in **233**. Thus, $\frac{3}{4} = \frac{27}{36}$, $\frac{1}{6} = \frac{6}{36}$, $\frac{1}{18} = \frac{2}{36}$.

240. Hence the following rule:

*Find the least common multiple of each of the given denominators for the least common denominator, and proceed as in **234**.*

ORAL EXERCISE

By inspection, reduce to equivalent fractions having the least common denominator:

1. $\frac{3}{4}, \frac{4}{5}$. *3.* $\frac{1}{2}, \frac{2}{3}$. *5.* $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{9}$. *7.* $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{18}$.
2. $\frac{5}{7}, \frac{3}{8}$. *4.* $\frac{2}{5}, \frac{3}{7}$. *6.* $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$. *8.* $\frac{1}{3}, \frac{3}{4}, \frac{1}{12}, \frac{1}{6}$.

WRITTEN EXERCISE

Reduce to equivalent fractions having the least common denominator:

1. $\frac{1}{9}, \frac{4}{7}, \frac{5}{18}$. *3.* $\frac{2}{13}, \frac{4}{5}, \frac{5}{6}$. *5.* $\frac{12}{17}, \frac{25}{51}, \frac{19}{102}$. *7.* $\frac{4}{9}, \frac{2}{27}, \frac{16}{81}$.
2. $\frac{2}{11}, \frac{4}{9}, \frac{7}{15}$. *4.* $\frac{4}{15}, \frac{4}{9}, \frac{5}{6}$. *6.* $\frac{5}{17}, \frac{2}{5}, \frac{3}{85}$. *8.* $\frac{4}{15}, \frac{2}{25}, \frac{3}{5}$.

ADDITION OF FRACTIONS

241. **Similar fractions** are fractions having the same denominator or unit value.

242. In order that fractions may be added, they must be similar and parts of like units.

243. The denominator is the *namer* of the fraction; hence, similar fractions are analogous to like numbers.

244. **To add similar fractions,**

Add the numerators.

Place the sum over the common denominator, and reduce the result to its simplest form.

ORAL EXERCISE

By inspection, find the sums of:

1. $\frac{1}{5}+\frac{4}{5}+\frac{3}{5}+\frac{2}{5}+1+\frac{3}{5}+\frac{4}{5}$. *4.* $\frac{2}{17}+\frac{3}{17}+\frac{5}{17}+\frac{1}{17}$.
2. $\frac{2}{7}+\frac{3}{7}+\frac{5}{7}+\frac{4}{7}+\frac{6}{7}+1\frac{1}{7}+\frac{2}{7}$. *5.* $\frac{3}{19}+\frac{7}{19}+\frac{8}{19}+\frac{1}{19}$.
3. $\frac{4}{9}+\frac{2}{9}+\frac{7}{9}+\frac{5}{9}+4\frac{1}{9}+\frac{7}{9}+\frac{8}{9}$. *6.* $\frac{3}{15}+\frac{2}{15}+\frac{5}{15}+\frac{8}{15}$.

By horizontal addition, find the sums of:

7. 3 pieces of print containing 41^2, 24^1, 40^3 yards, respectively.

NOTE. In the dry goods business fourths (quarters) are very common fractions, and they are usually written without denominators by placing the numerators slightly above the integers. Thus, $41^1 = 41\frac{1}{4}$, $41^2 = 41\frac{2}{4}$ $(41\frac{1}{2})$, and $41^3 = 41\frac{3}{4}$.

8. 3 pieces wash silk containing 42^1, 45^1, 43^1 yards, respectively.

9. 3 pieces duck containing 37^2, 41^2, 45 yards, respectively.

10. 4 pieces cashmere containing 42^1, 45^1, 46^1, 42^1 yards, respectively.

245. **To add fractions not having a common denominator.**

246. **Examples.** *1.* Find the sum of $\frac{5}{7}$ and $\frac{3}{8}$.

$$\frac{5}{7}+\frac{3}{8}=\frac{40+21}{56}$$
$$=\frac{61}{56}=1\tfrac{5}{56}.$$

SOLUTION. Since the given fractions are not similar, reduce them to equivalent fractions having the least common denominator as in **240**. Then add the numerators and place the sum, 61, over the least common denominator, 56. $\frac{61}{56} = 1\frac{5}{56}$.

2. Find the sum of $47\frac{1}{2}$, $16\frac{3}{4}$, and $17\frac{5}{8}$.

$$\begin{array}{ll} & \text{Eighths} \\ 47\frac{1}{2} & 4 \\ 16\frac{3}{4} & 6 \\ \underline{17\frac{5}{8}} & \underline{5} \\ 81\frac{7}{8} & \frac{15}{8} = 1\frac{7}{8}. \end{array}$$

SOLUTION. By inspection, the common denominator of the fractions is found to be 8. The sum of the fractions is then $1\frac{7}{8}$, which, added to the sum of the whole numbers, gives $81\frac{7}{8}$, the required result.

247. Hence the following rule:

Reduce the given fractions to equivalent fractions having the least common denominator, and add as in **244**.

WRITTEN EXERCISE

Find the sums of the following fractions:

1. $\frac{3}{4}, \frac{7}{8}, \frac{3}{8}$. *4.* $\frac{5}{7}, \frac{8}{15}, \frac{9}{20}$. *7.* $\frac{3}{10}, \frac{2}{5}, \frac{9}{62}$. *10.* $\frac{2}{3}, \frac{4}{9}, \frac{3}{8}$.

2. $\frac{6}{7}, \frac{4}{5}, \frac{3}{17}$. *5.* $\frac{3}{17}, \frac{2}{3}, \frac{5}{51}$. *8.* $\frac{3}{5}, \frac{5}{6}, \frac{7}{8}$. *11.* $\frac{9}{10}, \frac{7}{8}, \frac{3}{19}$

3. $\frac{4}{7}, \frac{5}{14}, \frac{9}{42}$. *6.* $\frac{7}{48}, \frac{5}{6}, \frac{2}{3}$. *9.* $\frac{1}{2}, \frac{9}{16}, \frac{7}{25}$. *12.* $\frac{2}{3}, \frac{6}{7}, \frac{1}{9}$.

SHORT METHODS

248. When the numerators of any two fractions are alike, the work of addition may be performed as shown in the following examples.

249. Examples. *1.* Find the sum of $\frac{1}{2}$ and $\frac{1}{17}$.

$\frac{1}{2}+\frac{1}{17}=\frac{19}{34}$.

SOLUTION. The common denominator is 2 × 17, or 34. Since the numerator of each of the fractions is 1, the numerator of the first equivalent fraction having a common denominator 34 is equal to the denominator of the second fraction; and the numerator of the second equivalent fraction having a common denominator 34 is equal to the denominator of the first fraction. $2+17=19$; hence the sum of the given fractions is found to be $\frac{19}{34}$.

2. Find the sum of $\frac{2}{7}$ and $\frac{2}{9}$.

$\frac{2}{7}+\frac{2}{9}=\frac{32}{63}$.

SOLUTION. The common denominator is 7 × 9, or 63. If the numerator of each of the given fractions were 1, the sum of the numerators in the equivalent fractions having a common denominator 63 would be 16 (7 + 9). But the numerator of the given fractions is 2; hence the sum of the numerators of the equivalent fractions having a common denominator 63, is 32 (2 × 16). Therefore, $\frac{2}{7}+\frac{2}{9}=\frac{32}{63}$.

In practice think only of results. Thus, $\overline{7+9}\times 2=32\cdot\frac{32}{63}$.

ORAL EXERCISE

By inspection, find the sum of:

1. $\frac{1}{5}+\frac{1}{7}$.
2. $\frac{1}{7}+\frac{1}{8}$.
3. $\frac{1}{9}+\frac{1}{4}$.
4. $\frac{1}{18}+\frac{1}{3}$.
5. $\frac{1}{15}+\frac{1}{4}$.
6. $\frac{1}{14}+\frac{1}{8}$.
7. $\frac{1}{21}+\frac{1}{8}$.
8. $\frac{1}{19}+\frac{1}{2}$.
9. $\frac{1}{8}+\frac{1}{4}$.
10. $\frac{1}{2}+\frac{1}{8}$.
11. $\frac{1}{6}+\frac{1}{5}$.
12. $\frac{1}{2}+\frac{1}{4}$.
13. $\frac{2}{7}+\frac{2}{5}$.
14. $\frac{2}{9}+\frac{2}{7}$.
15. $\frac{2}{7}+\frac{2}{8}$.
16. $\frac{2}{9}+\frac{2}{5}$.
17. $\frac{2}{18}+\frac{2}{8}$.
18. $\frac{2}{15}+\frac{2}{7}$.
19. $\frac{2}{5}+\frac{2}{8}$.
20. $\frac{2}{11}+\frac{2}{8}$.
21. $\frac{2}{17}+\frac{2}{8}$.
22. $\frac{4}{5}+\frac{4}{9}$.
23. $\frac{2}{9}+\frac{2}{15}$.
24. $\frac{2}{5}+\frac{2}{11}$.
25. $\frac{3}{8}+\frac{3}{5}$.
26. $\frac{3}{7}+\frac{3}{4}$.
27. $\frac{3}{5}+\frac{3}{7}$.
28. $\frac{3}{11}+\frac{3}{5}$.
29. $\frac{5}{6}+\frac{5}{9}$.
30. $\frac{6}{7}+\frac{6}{10}$.
31. $\frac{5}{7}+\frac{5}{11}$.
32. $\frac{3}{8}+\frac{3}{7}$.
33. $\frac{3}{10}+\frac{3}{7}$.
34. $\frac{3}{5}+\frac{3}{9}$.
35. $\frac{3}{4}+\frac{3}{9}$.

In the following problems add the first two fractions, and to the sum add the other fraction.

36. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$.
37. $\frac{1}{4}+\frac{1}{3}+\frac{5}{12}$.
38. $\frac{1}{5}+\frac{1}{3}+\frac{1}{15}$.
39. $\frac{1}{4}+\frac{1}{5}+\frac{7}{20}$.
40. $\frac{2}{5}+\frac{2}{7}+\frac{11}{35}$.
41. $\frac{2}{7}+\frac{2}{9}+\frac{11}{63}$.

250. The business man's fractions are usually of the simplest sort, and ability to add them rapidly is of the utmost importance. In a great many cases the least common denominator can be determined by inspection and the fractions added as rapidly as whole numbers.

251. Examples. *1.* Find the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}=\frac{15}{16}$.

SOLUTION. By inspection, find the least common denominator to be 16. Reducing each fraction to sixteenths at sight, and adding, say or think, 1, 3, 7, 15, $\frac{15}{16}$.

2. Find the sum of $\frac{3}{4}$, $\frac{1}{16}$, $\frac{1}{8}$, and $\frac{1}{4}$.

$\frac{3}{4}+\frac{1}{16}+\frac{1}{8}+\frac{1}{4}=\frac{19}{16}=1\frac{3}{16}$.

SOLUTION. By inspection, find the least common denominator to be 16. 4, 6, 7, 19, $\frac{19}{16}$, or $1\frac{3}{16}$.

ORAL EXERCISE

By inspection, find the sums of the following problems:

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*	*9.*	*10.*	*11.*	*12.*
$\frac{1}{8}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{3}{7}$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{3}{4}$	$\frac{1}{7}$
$\frac{1}{16}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{15}$	$\frac{1}{14}$	$\frac{1}{6}$	$\frac{7}{8}$	$\frac{1}{9}$
$\frac{1}{32}$	$\frac{1}{18}$	$\frac{1}{20}$	$\frac{5}{28}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{28}$	$\frac{1}{3}$	$\frac{5}{16}$	$\frac{5}{7}$
$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{1}{7}$	$\frac{7}{8}$	$\frac{1}{15}$	$\frac{1}{6}$	$\frac{3}{5}$	$\frac{2}{7}$	$\frac{1}{18}$	$\frac{1}{2}$	$\frac{4}{9}$

The above may be used as an "open-book" exercise, different students being required to announce at sight the least common denominator, and then the successive steps necessary to arrive at a total. Students should be drilled on exercises similar to the above until they can add the fractions given as rapidly as they can whole numbers.

WRITTEN EXERCISE

Copy or write from dictation and add the following problems. Add the fractions as explained in **251**.

1.	*2.*	*3.*	*4.*
$2746\frac{1}{2}$	$1017\frac{3}{16}$	$1142\frac{1}{20}$	$1410\frac{1}{3}$
$4292\frac{1}{8}$	$1492\frac{3}{4}$	$1460\frac{3}{40}$	$1040\frac{1}{3}$
$3041\frac{1}{4}$	$1932\frac{7}{8}$	$1932\frac{1}{20}$	$1620\frac{1}{9}$
$3742\frac{3}{4}$	$1042\frac{1}{4}$	$1735\frac{1}{5}$	$1342\frac{2}{9}$
$1692\frac{3}{16}$	$1114\frac{1}{2}$	$2518\frac{3}{5}$	$1647\frac{5}{9}$
$9241\frac{5}{16}$	$7522\frac{3}{4}$	$1420\frac{3}{5}$	$1842\frac{1}{3}$
$1832\frac{3}{8}$	$3742\frac{1}{8}$	$1310\frac{1}{10}$	$1621\frac{1}{6}$
$7531\frac{1}{2}$	$4714\frac{3}{16}$	$1416\frac{3}{10}$	$1831\frac{5}{6}$

SUBTRACTION OF FRACTIONS

252. In order that fractions may be subtracted they must be similar and parts of like units.

DRILL EXERCISE

1. $127 − $62 = ? *4.* $\frac{18}{20}-\frac{7}{20}=$? *7.* $\frac{10}{11}-\frac{6}{11}=$?

2. 478 yards − 28 yards = ? *5.* $\frac{4}{5}-\frac{2}{5}=$? *8.* $\frac{31}{35}-\frac{18}{35}=$?

3. 195 acres − 88 acres = ? *6.* $\frac{7}{8}-\frac{3}{8}=$? *9.* $\frac{119}{241}-\frac{71}{241}=$?

253. **To subtract similar fractions,**

Find the difference between the numerators of the given fractions and write the result over the denominator.

254. **To subtract fractions not having a common denominator.**

255. **Examples.** *1.* Subtract $\frac{2}{3}$ from $\frac{7}{8}$.

Twenty-fourths

$$\begin{aligned}\tfrac{7}{8} &= 21\\ \tfrac{2}{3} &= \underline{16}\\ &\ \ \tfrac{5}{24}\end{aligned}$$

SOLUTION. Since only similar fractions and parts of like units may be subtracted, reduce the given fractions to equivalent fractions having a common denominator. $\frac{7}{8}=\frac{21}{24}$; $\frac{2}{3}=\frac{16}{24}$. $\frac{21}{24}-\frac{16}{24}=\frac{5}{24}$.

Hence, $\frac{7}{8}-\frac{2}{3}=\frac{5}{24}$.

2. From $16\frac{1}{3}$ take $12\frac{3}{4}$.

$$\begin{aligned}16\tfrac{1}{3} &= 15\tfrac{16}{12}\\ 12\tfrac{3}{4} &= \underline{12\tfrac{9}{12}}\\ &\quad 3\tfrac{7}{12}\end{aligned}$$

SOLUTION. Reduce the fractions to a common denominator. Since we cannot take $\frac{9}{12}$ from $\frac{4}{12}$, we take 1 from 16, change it to $\frac{12}{12}$, and add it to $\frac{4}{12}$, making $\frac{16}{12}$. Then $\frac{16}{12}-\frac{9}{12}=\frac{7}{12}$; and 15 − 12 = 3.

Hence, $16\frac{1}{3}-12\frac{3}{4}=3\frac{7}{12}$.

256. Therefore the following rule:

Reduce the given fractions to equivalent fractions having the least common denominator and subtract as in 253.

ORAL EXERCISE

Find the values of the following:

1. $1\frac{5}{8}-\frac{1}{7}$. *3.* $7\frac{5}{8}-1\frac{5}{9}$. *5.* $12\frac{3}{5}-3\frac{3}{8}$. *7.* $11\frac{7}{10}-8\frac{2}{5}$.

2. $3\frac{7}{8}-\frac{4}{9}$. *4.* $25\frac{3}{4}-14\frac{1}{2}$. *6.* $17\frac{11}{12}-\frac{5}{6}$. *8.* $120-56\frac{41}{88}$.

WRITTEN EXERCISE

Find the difference between:

1. $240\frac{2}{5}$ and $89\frac{1}{6}$.

2. $\frac{11}{12}$ and $\frac{12}{13}$.

3. $14\frac{5}{9}$ and $21\frac{7}{8}$.

4. $11\frac{149}{204}$ and $21\frac{7}{8}$.

5. $104\frac{8}{10}$ and $84\frac{4}{5}$.

6. $9\frac{19}{40}$ and 21.

7. $1050\frac{20}{21}$ and $2020\frac{2}{3}$

8. $79\frac{1}{3}$ and $49\frac{7}{8}$.

9. $54\frac{11}{13}$ and $29\frac{1}{6}$.

10. From $216\frac{7}{8}$ acres of land, lots of 21 acres, $16\frac{3}{4}$ acres, $26\frac{15}{16}$ acres, $41\frac{1}{2}$ acres, and $63\frac{3}{8}$ acres were sold. How many acres remained unsold?

SHORT METHODS

257. When the numerators of any two fractions are alike, the work of subtraction may be shortened, as shown in the following examples.

258. Examples. *1.* From $\frac{1}{9}$ take $\frac{1}{17}$.

$\frac{1}{9}-\frac{1}{17}=\frac{8}{153}$.

SOLUTION. The common denominator is 17 × 9, or 153. The numerator of the first equivalent fraction having a common denominator 153 is 1 × 17, or 17. The numerator of the second equivalent fraction having a common denominator 153 is 1 × 9, or 9. Hence 17, the denominator of the subtrahend, minus 9, the denominator of the minuend, written over the common denominator, 153, equals the required result, or $\frac{8}{153}$.

2. From $\frac{2}{7}$ take $\frac{2}{13}$.

$\frac{2}{7}-\frac{2}{13}=\frac{12}{91}$.

SOLUTION. The common denominator is 91. $\overline{13-7}\times 2=12$, which write over the common denominator 91.

ORAL EXERCISE

By inspection, find the value of:

1. $\frac{1}{9}-\frac{1}{12}$.

2. $\frac{1}{7}-\frac{1}{8}$.

3. $\frac{1}{5}-\frac{1}{6}$.

4. $\frac{1}{7}-\frac{1}{10}$.

5. $\frac{1}{4}-\frac{1}{5}$.

6. $\frac{1}{2}-\frac{1}{3}$.

7. $\frac{2}{5}-\frac{2}{9}$.

8. $\frac{2}{7}-\frac{2}{9}$.

9. $\frac{2}{7}-\frac{2}{11}$.

10. $\frac{2}{3}-\frac{2}{7}$.

11. $\frac{2}{3}-\frac{2}{11}$.

12. $\frac{2}{9}-\frac{2}{11}$.

13. $\frac{2}{5}-\frac{2}{17}$.

14. $\frac{2}{3}-\frac{2}{5}$.

15. $\frac{2}{11}-\frac{2}{13}$.

16. $\frac{2}{9}-\frac{2}{10}$.

17. $\frac{3}{4}-\frac{3}{5}$.

18. $\frac{3}{4}-\frac{3}{7}$.

19. $\frac{3}{4}-\frac{3}{9}$.

20. $\frac{3}{4}-\frac{3}{10}$.

21. $\frac{3}{4}-\frac{3}{11}$.

22. $\frac{3}{5}-\frac{3}{7}$.

23. $\frac{3}{5}-\frac{3}{8}$.

24. $\frac{3}{4}-\frac{3}{8}$.

25. $27\frac{4}{5}-19\frac{4}{11}$.

26. $121\frac{6}{7}-17\frac{6}{11}$.

27. $124\frac{5}{6}-13\frac{5}{7}$.

28. $64\frac{1}{7}-52\frac{1}{9}$.

29. $83\frac{1}{4}-72\frac{1}{5}$.

30. $89\frac{3}{5}-72\frac{3}{8}$.

MULTIPLICATION OF FRACTIONS

DRILL EXERCISE

1. If 1 pound of sugar is worth 5¢, how much are 5 pounds worth?

2. If 1 pound of tea is worth $\$\frac{1}{5}$, how much are 3 pounds worth?

3. If 12 men earn $48, how many dollars does one man earn?

4. What is $\frac{1}{2}$ of 48?

5. $48 \times \frac{1}{12} = ?$

6. What is $\frac{2}{5}$ of 15?

7. $15 \times \frac{2}{5} = ?$

259. To multiply a fraction by a whole number, a whole number by a fraction, or a fraction by a fraction.

260. Examples. *1.* $\frac{3}{8} \times 4 = ?$

(*a*)

$$\frac{3}{8} \times 4 = \frac{3 \times \cancel{4}}{\underset{2}{\cancel{8}}} = \frac{3}{2} = 1\tfrac{1}{2}.$$

(*b*)

$$\frac{3}{8} \times 4 = \frac{3}{8 \div 4} = \frac{3}{2} = 1\tfrac{1}{2}.$$

(*c*)

$$\frac{3}{8} \times 4 = \frac{3 \times \cancel{4}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} = 1\tfrac{1}{2}.$$

SOLUTIONS. (*a*) Multiplying the numerator of a fraction multiplies that fraction (**219**). Hence, to multiply $\frac{3}{8}$ by 4, multiply the numerator 3 by 4 and divide the result by the denominator 8 as shown in (*a*). Or,

(*b*) Dividing the denominator of any fraction multiplies that fraction (**219**). Hence, to multiply $\frac{3}{8}$ by 4 divide the denominator 8 by 4 and write the numerator over the quotient as shown in (*b*). Or,

(*c*) Arranging (*a*) in another convenient form for cancellation we have the process (*c*).

Multiplying the numerator and leaving the denominator unchanged as in (*a*), the number of parts taken has been multiplied, and the value of each part left the same. Hence, the whole fraction has been multiplied.

Dividing the denominator and leaving the numerator unchanged as in (*b*), the size of each part has been multiplied and the number of parts left the same. Hence, the whole fraction has been multiplied.

A whole number may be expressed in fractional form by writing 1 for its denominator. Hence, $\frac{3}{8} \times 4 = \frac{3 \times 4}{8 \times 1}$, and the process is indicated in convenient form for cancellation.

2. What will 5 dozen oranges cost at $\$\frac{3}{5}$ per dozen?

$$\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{1} = 3.$$

SOLUTION. Since 1 dozen oranges cost $\$\frac{3}{5}$, 5 dozen will cost 5 times $\$\frac{3}{5}$, which, by cancellation, as shown in the margin, is equal to $3.

3. If 1 mat of coffee cost \$24, what will $\frac{3}{7}$ of a mat cost?

(*a*)

$$24 \times \frac{3}{7} = 10\frac{2}{7}.$$

(*b*)

$$24 \div 7 = 3\frac{3}{7}.$$

$$3\frac{3}{7} \times 3 = 10\frac{2}{7}.$$

Solutions. (*a*) $\frac{3}{7}$ of \$24 = $\frac{1}{7}$ of 3 × \$24. 3 × \$24 = \$72. $\frac{1}{7}$ of \$72 (\$72 ÷ 7) = \$$10\frac{2}{7}$. Hence, $\frac{3}{7}$ of a mat of coffee at \$24 a mat will cost \$$10\frac{2}{7}$. Or,

(*b*) Since 1 mat of coffee cost \$24, $\frac{3}{7}$ of a mat will cost $\frac{3}{7}$ of \$24. $\frac{3}{7}$ of \$24 is equal to 3 times $\frac{1}{7}$ of \$24. $\frac{1}{7}$ of \$24 (\$24 ÷ 7) = \$$3\frac{3}{7}$. 3 times \$$3\frac{3}{7}$ = $10\frac{2}{7}$.

Solution (*b*) is shorter than solution (*a*) when the denominator of the divisor is a factor of the whole number.

4. If 1 pound of tea cost \$$\frac{5}{8}$, what will $\frac{2}{3}$ of a pound cost?

(*a*)

$$\frac{5}{8} \times \frac{2}{3} = \frac{10}{24} = \frac{5}{12}.$$

(*b*)

$$\frac{5}{\underset{4}{\cancel{8}}} \times \frac{\cancel{2}}{3} = \frac{5}{12}.$$

Solutions. (*a*) If 1 pound of tea cost \$$\frac{5}{8}$, $\frac{2}{3}$ of a pound will cost $\frac{2}{3}$ of \$$\frac{5}{8}$. $\frac{2}{3}$ of \$$\frac{5}{8}$ is equal to 2 times $\frac{1}{3}$ of \$$\frac{5}{8}$. $\frac{1}{3}$ of \$$\frac{5}{8}$ equals \$$\frac{5}{24}$. 2 × \$$\frac{5}{24}$ = \$$\frac{10}{24}$ = \$$\frac{5}{12}$. Hence, if 1 pound of tea cost \$$\frac{5}{8}$, $\frac{2}{3}$ of a pound will cost \$$\frac{5}{12}$. Or,

(*b*) By cancellation the operation is a little shorter and the result appears in its lowest form.

5. Find the product of $7\frac{1}{2} \times 3\frac{2}{3}$.

$$\frac{\overset{5}{\cancel{15}}}{2} \times \frac{11}{\cancel{3}} = \frac{55}{2} = 27\frac{1}{2}.$$

Solution. Reduce the mixed numbers to improper fractions and proceed as explained in Example 4.

261. From the foregoing solutions the following general rule may be derived:

Express the whole or mixed numbers as improper fractions. Cancel all equivalent factors from the numerators and denominators.

Find the product of the remaining numerators for the numerator of the resulting fraction, and the product of the remaining denominators for the denominator of the resulting fraction.

Note. The same rule holds good for finding the product of more than two fractions.

ORAL EXERCISE

Find the value of:

1. $\frac{1}{3}$ of 18.

2. $18 \times \frac{2}{3}$.

3. $\frac{2}{3} \times 3$.

4. $\frac{5}{8} \times 4$.

5. $\frac{2}{3}$ of 27.

6. $\frac{2}{3}$ of 6.

7. $\frac{11}{15}$ of 45.

8. $\frac{9}{16}$ of 32.

9. Find the cost of 20 yards of cashmere at $\$\frac{7}{8}$ a yard.

10. If 1 yard of silk is worth $\$\frac{7}{16}$, what are 8 yards worth?

11. Required, the cost of 150 yards of muslin at $\$\frac{1}{8}$ a yard.

12. What will $2\frac{1}{3}$ pounds of sugar cost at $3\frac{1}{3}$¢ per pound?

13. What will $2\frac{1}{2}$ pounds of beef cost at $6\frac{2}{3}$¢ per pound?

14. What will $\frac{3}{8}$ of a pound of tea cost if 1 pound cost $\$\frac{2}{3}$?

15. John was given $\frac{2}{7}$ of a farm and James $\frac{3}{5}$ as much. What part had James?

16. If $\frac{1}{4}$ of a stock were lost by fire and the remainder sold at $\frac{3}{4}$ of its cost, what part of the first cost was received?

17. Divide 21 into 2 parts, one of which shall be $\frac{3}{4}$ of the other.

18. Divide 60 into 2 parts, one of which shall be $\frac{7}{8}$ of the other.

19. So divide $150 between A and B that A's part may be $\frac{3}{4}$ of B's.

20. Tea costing $\$\frac{5}{8}$ per pound is sold for $\frac{4}{5}$ of its cost. For what price per pound is the tea sold? What is the loss per pound?

WRITTEN EXERCISE

Find the product of:

1. $\frac{5}{17} \times 85.$ *3.* $\frac{11}{3} \times 12.$ *5.* $\frac{31}{2} \times 16.$ *7.* $\frac{104}{9} \times 40.$

2. $\frac{13}{16} \times 8.$ *4.* $\frac{20}{6} \times 9.$ *6.* $\frac{17}{7} \times 11.$ *8.* $\frac{27}{5} \times 28.$

9. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{7}{8} \times \frac{8}{9} \times 10 = ?$

10. $\frac{19}{20} \times \frac{18}{19} \times 5 \times \frac{1}{15} \times \frac{11}{12} \times \frac{1}{2} = ?$

11. What will be the cost of $7\frac{1}{2}$ tons of hay at $\frac{3}{4}$ of $15 per ton?

12. Find the cost of $7\frac{1}{2}$ pounds of beef at $8\frac{1}{2}$¢ per pound.

13. At $\$2\frac{1}{2}$ per day, how much will a man earn in $17\frac{1}{2}$ days?

14. Paid $\$\frac{9}{10}$ for some stationery and $\frac{2}{3}$ as much for some pens and ink. How much did I pay for both?

15. A having $750 invested $\frac{1}{5}$ of it in insurance and paid $\frac{3}{8}$ of the remainder for a horse. How much did he have left?

16. A owning $\frac{7}{8}$ of a mill sold $\frac{3}{8}$ of his share to B. What part of the whole mill did he still own?

SHORT METHODS

262. **To multiply together two mixed numbers when the integers are alike and the fractions are ½.**

263. **Example.** What will 8½ yards of lining cost at 8½¢ per yard?

8½¢
8½
72¼¢

SOLUTION. *The sum of ½ of each of two like numbers is equal to either of the numbers.* Since 8 multiplied by ½ added to ½ of 8 is equal to 8, in finding the product of 8½ by 8½, we may say, 9 × 8 + ½ of ½ = 72¼, the required result.

ORAL EXERCISE

By inspection, find the value of:

1. 2½ yards at $2½. *4.* 3½ yards at $3½. *7.* 6½ pounds at 6½¢.
2. 7½ barrels at $7½. *5.* 8½ pounds at 8½¢. *8.* 11½ acres at $11½.
3. 5½ yards at $5½. *6.* 9½ pounds at 9½¢. *9.* 12½ dozen at $12½.

264. **To multiply together *any* two mixed numbers when the fractions are ½.**

265. **Examples.** *1.* Find the cost of 120½ yards of lining at 6½¢ per yard.

120½
.06½
63½
720
7.83¼

SOLUTION. ½ of 120 + 6 times ½ is equal to ½ of $\overline{120 + 6}$. ½ of 126 = 63, which write as shown in the margin. ½ of ½ = ¼, which write as shown in the margin. 6 times 120 = 720. Adding the partial products, the result is 7.83¼. The required result is, therefore, $7.83.

2. Find the cost of 87½ pounds of crackers at 8½¢ per pound.

87½
.08½
47¾
696
7.43¾

SOLUTION. ½ of $\overline{87 + 8}$ = 47½, which, added to ½ of ½ = 47¾. 87 × 8 = 696. Add, and the result is 7.43¾, or $7.44.

Observe that in dividing numbers by 2 there can never be a remainder of more than 1.

Also that in multiplying together any two numbers ending in ½, the fraction in the resulting product must be either ¼ or ¾.

266. Therefore the following rule:

If the sum of the integers is even, write $\frac{1}{4}$ in the resulting product. If not, write $\frac{3}{4}$ in resulting product.

Find one half of the sum of the integers and to the result add the product of the integers.

ORAL EXERCISE

By inspection, find the value of:

1. $9\frac{1}{2}$ yards at $6\frac{1}{2}$¢.
2. $4\frac{1}{2}$ pounds at $9\frac{1}{2}$¢.
3. $12\frac{1}{2}$ pounds at $7\frac{1}{2}$¢.
4. $16\frac{1}{2}$ yards at $10\frac{1}{2}$¢.
5. $20\frac{1}{2}$ yards at $8\frac{1}{2}$¢.
6. $32\frac{1}{2}$ yards at $2\frac{1}{2}$¢.
7. $35\frac{1}{2}$ yards at $3\frac{1}{2}$¢.
8. $12\frac{1}{2}$ yards at $5\frac{1}{2}$¢.
9. $21\frac{1}{2}$ pounds at $4\frac{1}{2}$¢.

WRITTEN EXERCISE

Find the value of:

1. $162\frac{1}{2} \times 16\frac{1}{2}$.
2. $144\frac{1}{2} \times 3\frac{1}{2}$.
3. $75\frac{1}{2} \times 4\frac{1}{2}$.
4. $123\frac{1}{2} \times 5\frac{1}{2}$.
5. $60\frac{1}{2} \times 12\frac{1}{2}$.
6. $90\frac{1}{2} \times 3\frac{1}{2}$.
7. $120\frac{1}{2} \times 4\frac{1}{2}$.
8. $115\frac{1}{2} \times 5\frac{1}{2}$.
9. $16\frac{1}{2} \times 5\frac{1}{2}$.
10. $204\frac{1}{2} \times 7\frac{1}{2}$.
11. $215\frac{1}{2} \times 8\frac{1}{2}$.
12. $110\frac{1}{2} \times 8\frac{1}{2}$.

Mixed numbers ending in $\frac{1}{4}$, $\frac{1}{8}$, etc., may be multiplied together in practically the same manner as shown in the foregoing explanations.

DIVISION OF FRACTIONS

DRILL EXERCISE

1. If 8 hats cost $24, what will 1 hat cost?

2. If $\frac{8}{9}$ of an article cost $24, what will $\frac{1}{9}$ of an article cost?

3. If 5 barrels of apples cost $15, what will 7 barrels cost?

4. If $\frac{5}{6}$ of an acre of land cost $15, what will $\frac{7}{6}$ acre cost?

5. If 3 pounds of tea cost $1.05, what will 5 pounds cost?

6. If $\frac{3}{5}$ of a pound of tea cost 21¢, what will $\frac{5}{5}$ of a pound cost?

7. If 5 dimes will purchase 1 pound of tea, how many pounds will $4 purchase?

8. If $\frac{5}{10}$ of a pound of tea cost 25¢, what will 4 pounds cost?

9. If 4 pounds of sugar cost $$\frac{1}{4}$, what will 1 pound cost?

10. If $\frac{3}{8}$ of a yard of cloth cost $$\frac{9}{4}$, what will 1 yard cost?

267. The **reciprocal of a whole number** is 1 divided by that number.

Thus, the reciprocal of 3 is $\frac{1}{3}$.

268. **The reciprocal of a fraction is 1 divided by that fraction.**

269. Example. Find the reciprocal of $\frac{4}{5}$.

SOLUTION. In 1 there are 5 fifths. $\frac{1}{5}$ is contained in 5 fifths 5 times. $\frac{4}{5}$ is 4 times $\frac{1}{5}$. Therefore, $\frac{4}{5}$ is contained in $\frac{5}{5}$ $\frac{1}{4}$ as many times as $\frac{1}{5}$. $\frac{1}{4}$ of 5 is $\frac{5}{4}$; the reciprocal of $\frac{4}{5}$ equals $\frac{5}{4}$. Hence,

1 divided by any fraction is the fraction inverted.

270. $\frac{1}{4}$ is the reciprocal of 4, and 4 is the reciprocal of $\frac{1}{4}$. Hence,

The dividend, multiplied by the reciprocal of the divisor, is equal to the quotient.

271. **To divide a fraction by a whole number, a whole number by a fraction, or a fraction by a fraction.**

272. Examples. *1.* Divide $\frac{4}{5}$ by 2.

(*a*)

$$\frac{4}{5}\div 2=\frac{4\div 2}{5}=\frac{2}{5}.$$

SOLUTIONS. (*a*) Dividing the numerator (dividend) divides the fraction (**219**). Hence, divide the numerator of the fraction $\frac{4}{5}$ by 2 and the quotient is $\frac{2}{5}$. Or,

(*b*)

$$\frac{4}{5}\div 2=\frac{4}{5\times 2}=\frac{4}{10}=\frac{2}{5}.$$

(*b*) Multiplying the denominator (divisor) divides the fraction (**219**). Hence, multiply the denominator of the fraction $\frac{4}{5}$ and the quotient is $\frac{2}{5}$. Or,

(*c*)

$$\frac{4}{5}\div 2=\frac{\overset{2}{\cancel{4}}}{5}\times\frac{1}{\cancel{2}}=\frac{2}{5}.$$

(*c*) The dividend multiplied by the reciprocal of the divisor is equal to the quotient (**270**). The reciprocal of 2 is $\frac{1}{2}$. Therefore, $\frac{4}{5}$ divided by 2 is equal to $\frac{4}{5}\times\frac{1}{2}$. $\frac{4}{5}\times\frac{1}{2}=\frac{2}{5}$.

2. If 2 pounds of coffee cost $\$\frac{4}{5}$, what will 1 pound cost?

$$\frac{\overset{2}{\cancel{4}}}{5}\times\frac{1}{\cancel{2}}=\frac{2}{5}.$$

SOLUTION. If 2 pounds of coffee cost $\$\frac{4}{5}$, 1 pound will cost $\frac{1}{2}$ of $\frac{4}{5}$, or $\$\frac{2}{5}$.

3. At $\$\frac{7}{8}$ per yard, how many yards of cloth may be purchased for \$14?

(*a*)

$$14\div\frac{7}{8}=\frac{112}{8}\div\frac{7}{8}$$
$$=112\div 7=16.$$

SOLUTIONS. (*a*) Since the denominator names or tells the size or value of the parts taken, similar fractions may be divided as concrete whole numbers. In concrete whole numbers the operation of division is performed without regard to the unit named, so in dividing similar fractions the denominator may be ignored. Reducing 14 to the same value as $\frac{7}{8}$, the result is $\frac{112}{8}$. $\frac{112}{8}\div\frac{7}{8}$ being analogous to $\frac{10}{\text{dollars}}\div\frac{5}{\text{dollars}}$, divide 112 by 7 and obtain 16.

In a critical analysis of the above problem say in general: Since 1 yard cost \$$\frac{7}{8}$, as many yards may be purchased for \$14 as $\frac{7}{8}$ is contained times in 14. $14 = \frac{112}{8}$, and $\frac{7}{8}$ is contained in $\frac{112}{8}$ 16 times. Hence, 16 × 1 yard, or 16 yards, may be purchased for \$14. Or,

(b)

$$14 \div \frac{\overset{2}{\cancel{14}}}{1} \times \frac{8}{\cancel{7}} = 16.$$

(b) Since 1 yard cost \$$\frac{7}{8}$, as many times 1 yard may be purchased for \$1 as \$$\frac{7}{8}$ is contained times in \$1, or $\frac{8}{7}$ times (the reciprocal of $\frac{7}{8}$). $\frac{8}{7} \times 1$ yard $= \frac{8}{7}$ yards. Since for \$1 $\frac{8}{7}$ yards may be purchased, for \$14 14 times $\frac{8}{7}$ yards, or 16 yards, may be purchased.

4. If 1 yard of cloth cost \$$\frac{1}{3}$, how many yards may be purchased for \$$\frac{3}{4}$?

(a)

$$\frac{3}{4} \div \frac{1}{3} = \frac{9}{12} \div \frac{4}{12} = 9 \div 4 = 2\tfrac{1}{4}.$$

Solutions. (a) Since 1 yard cost \$$\frac{1}{3}$, as many yards may be bought for \$$\frac{3}{4}$ as $\frac{1}{3}$ is contained times in $\frac{3}{4}$. $\frac{3}{4} = \frac{9}{12}$; $\frac{1}{3} = \frac{4}{12}$. 4 is contained in 9 $2\frac{1}{4}$ times. Hence, $2\frac{1}{4} \times 1$ yard, or $2\frac{1}{4}$ yards, may be purchased. Or,

(b)

$$\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\tfrac{1}{4}.$$

(b) Since \$$\frac{1}{3}$ is contained in \$1 $\frac{3}{1}$ times (the reciprocal of $\frac{1}{3}$), 3 yards of cloth may be purchased for \$1. If 3 yards may be purchased for \$1, $\frac{3}{4}$ of 3 yards may be purchased for \$$\frac{3}{4}$. Hence the simple process $\frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}$.

273. Therefore the following rule:

Multiply the dividend by the reciprocal of the divisor.

Note. Reduce mixed numbers to improper fractions before applying the rule.

ORAL EXERCISE

Find the value of:

1. $6 \div \frac{2}{3}$.	*3.* $24 \div \frac{2}{3}$.	*5.* $10 \div \frac{2}{5}$.	*7.* $\frac{2}{3} \div 6$.
2. $12 \div \frac{3}{4}$.	*4.* $15 \div \frac{1}{6}$.	*6.* $25 \div \frac{5}{6}$.	*8.* $\frac{3}{5} \div 3$.

9. If 4 pounds of coffee cost \$$\frac{8}{9}$, what will 1 pound cost?

10. If $\frac{21}{26}$ of a farm be grain land and evenly divided into 3 fields, what part of the farm will each field contain?

11. Divide 9 by $\frac{4}{5}$; 12 by $1\frac{1}{2}$; $\frac{1}{3}$ by $\frac{7}{8}$; $2\frac{5}{6}$ by $\frac{5}{17}$.

12. What part of 1 is $\frac{3}{8}$? of 2? of 3? of 4? of 5? of 10?

13. What part of 9 is 4? of 5? of 6? of 7? of 11? of 17?

WRITTEN EXERCISE

1. If $\frac{5}{8}$ of an acre of land sells for $45, what will 1 acre sell for at the same rate?

2. A farm of 471 acres is divided into shares of $94\frac{1}{5}$ acres each. How many shares are there?

3. A church collection of $232 was divided among poor families, to each of which was given $$5\frac{4}{5}$. How many families shared the bounty?

4. When potatoes are worth $$\frac{4}{5}$ per bushel and apples $$\frac{3}{4}$ per bushel, how many bushels of potatoes would pay for a load of apples measuring 30 bushels?

5. A woman buys $\frac{3}{5}$ of a cord of wood worth $$6\frac{2}{3}$ per cord and pays for it in work at $$\frac{4}{5}$ per day. How many days must she work to make full payment?

6. A dealer paid $\frac{4}{7}$ of $$78\frac{3}{4}$ for $\frac{2}{5}$ of 25 cords of wood. What was the cost per cord?

7. If $\frac{3}{16}$ of a farm of $67\frac{1}{5}$ acres be divided into 63 village lots, what part of an acre will each lot contain?

8. 1760 bushels of wheat were put into sacks containing $2\frac{1}{4}$ bushels. How many sacks were there?

9. At $$\frac{7}{8}$ per day, how long would it take to earn $$15\frac{3}{4}$?

10. How many fields of $9\frac{5}{8}$ acres can be made from a farm containing $125\frac{1}{8}$ acres?

WRITTEN REVIEW

1. From the sum of $\frac{3}{7}$ and $5\frac{2}{3}$ take the difference between $17\frac{1}{2}$ and 21.

2. Divide into six equal parts the product of $11\frac{7}{10}$ multiplied by $3\frac{7}{9}$.

3. An estate is so divided among A, B, and C that A gets $\frac{2}{5}$, B $\frac{3}{10}$, and C the remainder, which was $4200. What is the amount of the estate?

4. If 14 bushels of apples can be bought for $$3\frac{1}{2}$, how many bushels can be bought for $$\frac{3}{4}$?

5. A woman having $1 gave $\frac{2}{3}$ of it for coffee at $33\frac{1}{3}$¢ per pound. How many pounds did she buy?

6. Having bought $\frac{2}{9}$ of a ship, I sold $\frac{2}{5}$ of my share for $12,000. What was the value of the ship at that rate?

7. What must be the amount of an estate it, when it is divided into three parts, the first part is double the second, the second double the third, and the difference between the second and the third is $7500?

8. Having paid $119 for a watch and chain, I discover that the cost of the chain was only $\frac{7}{10}$ the cost of the watch. What was the cost of the watch?

9. An estate valued at $120,000 was so distributed that A received $\frac{1}{5}$; B $\frac{1}{10}$ of the estate more than A; C as much as A and B together, less $600; and 2 charities the remainder in equal parts. How much did each charity receive?

10. A painter worked $17\frac{1}{2}$ days, and after expending $\frac{4}{7}$ of his wages for board had $15 left. How much did he earn per day?

11. A mechanic worked $21\frac{2}{5}$ days, and after paying his board with $\frac{3}{8}$ of his earnings had $$66\frac{7}{8}$ left. How much did he earn per day?

12. If $\frac{1}{2}$ of the trees of an orchard are apple, $\frac{1}{4}$ peach, $\frac{1}{8}$ pear, $\frac{1}{16}$ plum, and the remaining 21 trees cherry, how many trees in all?

13. A, B, and C rented a pasture for $37. A put in 3 cows for 4 months, B 5 cows for 6 months, and C 8 cows for 4 months. How much had each to pay?

14. A farmer sold two cows for $75, receiving for one cow only $\frac{7}{8}$ as much as for the other. What was the price of each?

15. After selling 450 horses a dealer had $\frac{4}{7}$ of his stock remaining. How many had he at first?

16. If 8 horses consume $4\frac{1}{2}$ bushels of oats in $3\frac{1}{2}$ days, how many bushels will 12 horses consume in the same time?

17. A and B can do a piece of work in 10 days which A alone can do in 18 days. In what time can B alone do the work?

18. John and Calvin agree to build a wall for $86. If Calvin can work only $\frac{4}{5}$ as fast as John, how should the money be divided?

19. What is the length of a pole that stands $\frac{1}{3}$ in the sand, $\frac{4}{9}$ in the water, and $25\frac{1}{2}$ feet above the water?

20. A colt and cow cost $124. If the colt cost $4 more than 3 times the cost of the cow, what was the cost of each?

21. A tree 84 feet high was so broken in a storm that the part standing was $\frac{5}{7}$ of the length of the part broken. How many feet were standing?

22. A farmer has $\frac{3}{8}$ of his sheep in one pasture, $\frac{2}{5}$ in another, and the remainder of his flock, 72 sheep, in a third pasture. How many sheep has he?

23. For a horse and carriage I paid $540. What was the cost of each, if the cost of the carriage was $1\frac{1}{2}$ times the cost of the horse?

24. Peter can do a piece of work in 12 days, and Charles in 15 days. How many days will it require for its completion if they both join in the work?

25. A can do a piece of work in 21 days, B in 18 days, and C in 15 days. In how many days can the three working together perform the work?

26. John and his father have joint work which they can do, working together, in 25 days. If it requires 60 days for John working alone to complete the work, how many days will it require for the father to complete it?

27. A, B, and C together have $2520. C has twice as much as B, who has $\frac{1}{3}$ as much as A. How much has each?

28. A farmer bought 3 farms of 240 acres each at $\$11\frac{3}{4}$ an acre. He built three barns at a cost of $1245 each, spent $1275 in improving the houses, and put up $752\frac{1}{2}$ rods of fence at $\$2\frac{7}{8}$ per rod. He then sold the farms for $\$35\frac{3}{4}$ per acre. Did he gain or lose, and how much?

29. A and B joined in purchasing a farm costing $4500, A paying $2000 and B the remainder. After owning the farm six months they sold it for $6300. Of this sum how much should each receive?

30. By what number must $\frac{1}{9}$ be multiplied to produce $240\frac{1}{9}$?

31. The difference between $\frac{1}{4}$ and $\frac{1}{8}$ of a number is 90 less than $\frac{1}{5}$ of the same number. What is the number?

32. If a man can dig 20 bushels of potatoes in a day, and can pick up 30 bushels in a day, how many bushels can he dig and pick up in 20 days?

33. D can cut $\frac{3}{4}$ of a cord of wood in $\frac{1}{2}$ day. In $\frac{1}{2}$ day E can cut $\frac{2}{3}$ as much as D can cut in a whole day. If they work together, how long will it take them to cut 70 cords?

34. If $3\frac{1}{4}$ acres of land cost \$ 65, what will $125\frac{1}{13}$ acres cost at the same rate?

35. A works at the rate of \$ $2\frac{3}{8}$ a day, and B at the rate of \$ $3\frac{1}{2}$ a day. How long will it take A to earn as much as B earns in 19 days?

36. A tank has an inlet by which it may be filled in 10 hours, and an outlet by which, when filled, it may be emptied in 6 hours. If both inlet and outlet be opened when the tank is full, in what time will it be emptied?

37. A and B are engaged to perform a certain piece of work for \$ 35.55. It is supposed that A does $\frac{1}{4}$ more work than B, and they are to be paid proportionately. How much should each receive?

38. If you buy 60 lemons at the rate of 6 for 10¢, and twice as many more at the rate of 5 for 8¢, and sell the entire lot at the rate of 3 for 4¢, will you gain or lose, and how much?

39. Henry bought a basket of oranges at the rate of 3 for 2¢, and gained 50¢ by selling them at the rate of 2 for 3¢. How many oranges did he buy?

40. There are 108 bushels of corn in 2 bins. In one of the bins there are 12 bushels less than one half as many bushels as in the other. How many bushels in each?

41. Three brothers join in paying off the mortgage on their father's farm. The eldest pays $\frac{2}{5}$ of it, and the others pay the remainder in equal shares. If the eldest brother pays \$ 90 more than the amount paid by each of his younger brothers, what is the amount of the mortgage?

42. A can dig $7\frac{1}{2}$ bushels of potatoes in $\frac{1}{8}$ of a day, and B can dig $5\frac{1}{4}$ bushels in $\frac{1}{2}$ of a day. How many bushels can they both dig in $7\frac{3}{4}$ days?

43. A dealer bought $250\frac{3}{4}$ bushels of corn at $60\frac{3}{4}$¢ per bushel. If he sold the whole amount of his purchase at $65\frac{1}{2}$¢ per bushel, what was his gain?

44. Having bought $120\frac{3}{4}$ cords of wood at \$ $5\frac{1}{2}$ per cord, I sold $\frac{1}{2}$ of it at \$ 6 per cord and the remainder for \$ 340. Did I gain or lose, and how much?

45. What is the value of 8 pieces of dress silk containing 48^1, 42^2, 45^2, 40^1, 43^3, 42^2, 45^2, and 42^1 yards at \$ $1\frac{1}{4}$ per yard?

46. I bought 240¼ bushels of oats at 30¼¢ per bushel, 190½ bushels of corn at 60½¢ per bushel, and 30 bushels of wheat at $1.12½, and sold the whole for $320. Did I gain or lose, and how much?

47. Find the total cost of the items in the oral exercise, page 85; of the items in the oral exercise, page 86.

48. A man bought 5 bags of wheat, weighing respectively 120, 124¾, 128¼, 132⅝, and 131⅞ pounds, at $1⅛ per bushel. If each bag, independent of the wheat it contained, weighed 1 pound, and there are 60 pounds in a bushel of wheat, did he gain or lose by selling the whole purchase for $15?

49. A produce dealer's sales for a day are as follows: 341½ bushels of wheat at $1⅛ per bushel, 410¼ bushels of barley at 80¢ per bushel, 1120½ bushels of oats at 30½¢ per bushel, 310¼ bushels of buckwheat at $¾ per bushel, 250 bushels of beans at $2⅛ per bushel, 1386½ bushels of potatoes at 50½¢ per bushel, 1050¼ bushels apples at $¼ per bushel, and 630½ bushels of turnips at 70½¢ per bushel. Find the total sales for the day.

50. The six fields of a farm measure, respectively, 10, 12½, 19¾, $26\frac{5}{16}$, 30⅛, and $2\frac{5}{16}$ acres, and are valued at $250 per acre. How much is the farm worth?

DECIMAL FRACTIONS

274. A **decimal fraction**, or a **decimal**, is a fraction having for its denominator 10 or some power of 10; as, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{9}{1000}$.

If a unit be divided into ten equal parts, the parts are called *tenths*; if a tenth of a unit be divided into ten equal parts, the parts are called *hundredths*; if a hundredth of a unit be divided into ten equal parts, the parts are called *thousandths*; and so on.

To obviate the trouble of writing the denominators of decimal fractions, an abbreviated method of notation is used as shown in the following examples:

$$\tfrac{3}{10} = .3;\ \tfrac{5}{100} = .05;\ \tfrac{9}{1000} = .009.$$

275. Compared with Common Fractions. Decimal fractions are in most respects quite similar to common fractions. The points of difference may be stated as follows:

1. The denominator of a common fraction is always written, while that of a decimal fraction is only indicated.

2. The denominator of a common fraction may be any number, while that of a decimal fraction must be 10 or some power of 10.

276. The **decimal point** is a period (.). It is always placed at the left of tenths, and by its position indicates the denominator and determines the value of the decimal fraction; as, .4, .47, .315.

When the decimal point is used to separate the integral from the fractional part in a mixed decimal, or dollars and cents in a decimal currency, it is called a *separatrix*. The *figures* written at the right of the decimal point constitute the *numerator* of the fraction, and the *number of figures* written at the right of the decimal point indicates the *power of 10* which constitutes the *denominator* of the fraction.

277. A **pure decimal** corresponds to a proper fraction, the value being less than 1; as .5, .27, .207, .3241.

278. A **mixed decimal** corresponds to an improper fraction, the value being more than 1; as, 8.17, 17.8, 24.113.

279. A **complex decimal** corresponds to a complex fraction, and always has a common fraction in its right-hand place; as, $.33\frac{1}{3}\left(\frac{33\frac{1}{3}}{100}\right)$, $.16\frac{2}{3}\left(\frac{16\frac{2}{3}}{100}\right)$.

280. **General Principles.** 1. Decimals increase in value from right to left, and decrease in value from left to right in a tenfold ratio.

2. A decimal should contain as many places as there would be ciphers in its denominator if written, the decimal point representing the unit 1 of such denominator.

3. The value of any decimal fraction depends upon its distance from the decimal point.

4. Prefixing a cipher to a decimal is equivalent to dividing it by 10.

5. Annexing one or more ciphers to a decimal does not alter its value.

NUMERATION OF DECIMAL FRACTIONS

281. The abbreviated method used to indicate decimal fractions is nothing more than an extension of the method by which whole numbers are represented.

The relation of orders in a mixed decimal fraction is clearly shown by the following

NUMERATION TABLE

Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths
9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.
1	2	7	3	9	2	4	1	6	·	7	2	5	1	9	2	7	5

THE INTEGRAL PART THE FRACTIONAL PART

The above number is read, *one hundred twenty-seven million, three hundred ninety-two thousand, four hundred sixteen and seventy-two million, five hundred nineteen thousand, two hundred seventy-five hundred-millionths.*

282. The order of a decimal fraction may be found by numerating either from right to left or from left to right, but it should be remembered that the decimal point stands in the position of the unit 1 in the decimal denominator. The order of a decimal may usually be determined by inspection if the fact to be drawn from the following illustration be observed.

If .29 be numerated from the right as in integers, the point is in the hundreds' place, hence read *twenty-nine hundredths.* In .1137 the point is in the ten-thousands' place, hence read *eleven hundred thirty-seven ten-thousandths;* in .031631 the point is in the millions' place, hence read *thirty-one thousand, six hundred thirty-one millionths.*

283. Hence the following rule:

Numerate from the decimal point to determine the denominator.

Read the decimal as a whole number, and give to it the denomination of the right-hand figure.

In reading whole numbers never read *and* between periods or between hundreds and tens and units. Thus, in reading 615, say *six hundred fifteen,* and not *six hundred and fifteen.* In reading mixed decimals always connect the integral and fractional parts by *and;* as 2.5, read *two and five tenths;* 17.016, read *seventeen and sixteen thousandths.*

ORAL EXERCISE

Read the following decimals:

1. .297.
2. .1471.
3. .2442.
4. .105.
5. .963.
6. .56007.
7. .2.
8. .20.
9. .200.
10. .2000.
11. .214698.
12. 4003755.
13. .02.
14. .002.
15. .0002.
16. .00002.
17. .000002.
18. 136.251.
19. 638.6$\frac{3}{8}$.
20. 341.131$\frac{1}{8}$.
21. 801.00801.
22. 6000.58302.
23. 9001.00901.
24. 3000.00030003.

NOTATION OF DECIMAL FRACTIONS

284. Example. Write as a decimal thirty-four hundredths.

Solution. Observe that in writing thirty-four hundredths as a common fraction the mental operation is as follows: After writing 34, the numerator, the question is, "34 what?" The answer is "34 hundredths," and 100 is written below as a denominator. The result is $\frac{34}{100}$. In writing the decimal form of the same fraction reason in practically the same way. Write 34, the numerator, and make it hundredths by placing before 34 a decimal point, which represents 1 of the decimal denominator. The result is .34. Notice that 34 occupies two places corresponding to the two ciphers in the denominator.

285. Hence the following rule:

Write the decimal the same as a whole number, prefixing ciphers when necessary to give each figure its true local value.

Place the decimal point before the left-hand figure of the decimal.

WRITTEN EXERCISE

Express by figures the following decimals:

1. Twenty-six hundredths.
2. Twenty-seven hundredths.
3. Six ten-thousandths.
4. Four hundredths.
5. Five and seven tenths.
6. Five hundred and five hundredths.
7. Twenty-two hundred-thousandths.
8. Five thousand and five thousandths.
9. One million and one millionth.
10. Five hundred thousandths.
11. Five hundred-thousandths.

12. Seventy-seven tenths.

13. Two thousand two thousandths.

14. Two thousand and two thousandths.

15. Eleven and one hundred seven millionths.

16. Eighty-three and five hundred four ten-thousandths.

17. Seven hundred ten and two hundred forty-three hundred-thousandths.

18. Fifty-four million fifty-four thousand fifty-four and fifty-four million fifty-four thousand fifty-four ten-billionths.

19. Write the following as decimal fractions:

$\frac{87}{100}$, $\frac{4}{10000}$, $1\frac{97}{1000}$, $3\frac{898}{10000}$, $9\frac{17}{100}$.

REDUCTION OF DECIMAL FRACTIONS

286. **To reduce a decimal to a common fraction.**

287. **Examples.** *1.* Reduce .035 to a common fraction.

$.035 = \frac{35}{1000} = \frac{7}{200}$.

Solution. The decimal .035 is read thirty-five thousandths, which as a common fraction is written $\frac{35}{1000}$. $\frac{35}{1000} = \frac{7}{200}$.
Hence, .035 as a common fraction in its simplest form is equal to $\frac{7}{200}$.

2. Reduce $.66\frac{2}{3}$ to a common fraction.

$$.66\frac{2}{3} = \frac{.66\frac{2}{3}}{100} = \frac{\frac{200}{3}}{100} = \frac{200}{300} = \frac{2}{3}.$$

Solution. $.66\frac{2}{3}$ is a complex decimal. In the form of a complex fraction it is equal to $\frac{66\frac{2}{3}}{100}$. Divide the numerator of the complex fraction by the denominator and the result is $\frac{2}{3}$.

288. Hence the rule:

Omitting the decimal point write the proper denominator and reduce the fraction to its simplest form.

ORAL EXERCISE

Reduce to equivalent common fractions in their lowest terms:

1. .25. *3.* .72. *5.* .125. *7.* $.33\frac{1}{3}$. *9.* .025. *11.* $.12\frac{1}{2}$.
2. .24. *4.* .75. *6.* .016. *8.* $.44\frac{4}{9}$. *10.* .250. *12.* $.16\frac{2}{3}$.

WRITTEN EXERCISE

Reduce to equivalent common fractions in their lowest terms:

1. .62. *4.* .4025. *7.* .42504. *10.* .24042. *13.* $.008\frac{1}{3}$.
2. .105. *5.* .2244. *8.* .28828. *11.* .08004. *14.* $.326\frac{2}{3}$.
3. .372. *6.* .1878. *9.* .425. *12.* .1146. *15.* $.313\frac{1}{3}$.

Reduce to ordinary mixed numbers:

16. 5.16. *18.* 11.75. *20.* 16.162. *22.* 65.132. *24.* 15.016.
17. 13.205. *19.* 31.135. *21.* 81.1888. *23.* 35.0012. *25.* $28.44\frac{4}{9}$.

289. **To reduce a common fraction to a decimal.**

290. **Example.** Reduce $\frac{4}{5}$ to an equivalent decimal.

$$5\overline{)4.0}\quad .8$$

SOLUTION. $\frac{4}{5}$ is equal to $\frac{1}{5}$ of 4 units. 4 units is equal to 40 tenths; $\frac{1}{5}$ of 40 tenths (4.0) = .8. Hence the rule:

291. *Divide the numerator by the denominator.*

If the denominator of a common fraction reduced to its lowest terms contains any other factor than 2 or 5 (the prime factors of 10), the quotient of the numerator by the denominator cannot be expressed as a complete decimal; thus $\frac{1}{6} = .166\frac{2}{3}$ or .166+.

WRITTEN EXERCISE

Reduce to equivalent decimals:

1. $\frac{1}{16}$. *4.* $\frac{3}{32}$. *7.* $\frac{21}{400}$. *10.* $\frac{15}{16}$. *13.* $\frac{7}{250}$.
2. $\frac{3}{20}$. *5.* $\frac{11}{80}$. *8.* $\frac{15}{32}$. *11.* $\frac{1}{64}$. *14.* $\frac{19}{20}$.
3. $\frac{11}{40}$. *6.* $\frac{13}{25}$. *9.* $\frac{3}{125}$. *12.* $\frac{31}{32}$. *15.* $\frac{47}{50}$.

ADDITION OF DECIMAL FRACTIONS

292. **Example.** Add .7, 2.43, .865, 11.5, 113.2675, and 200.00165.

$$\begin{array}{r@{}l} & .7 \\ 2 & .43 \\ & .865 \\ 11 & .5 \\ 113 & .2675 \\ \underline{200} & \underline{.00165} \\ 328 & .76415 \end{array}$$

SOLUTION. By the decimal system numbers increase in value from right to left in a tenfold ratio, and the decimal point separates the integral from the fractional part. Hence, to add decimals the points should be placed so that they fall in the same vertical line. The units of the same order will then fall in the same column. The result of the addition may then be obtained in the same manner as in simple numbers.

293. Therefore the following rule:

Write the decimals so that the points will fall in the same vertical line.

Add as in whole numbers and place the point in the sum directly below the points in the numbers added.

WRITTEN EXERCISE

1. Add 3.04, 25.001, .67, .2146, and 819.256.

2. Add 30.1257, 605.2146, 1000.864532, and 16.25694.

3. Add 896.111, 9530.216753, 1111.230094, and 1100.960005.

4. Find the sum of seventeen and forty-six ten-thousandths, eighty-three and one thousand four millionths, five hundred two and seventy-five hundred-thousandths, one million six and six million one billionths.

5. Add fifty-six thousand one hundred twelve and one thousand twenty millionths, six and ninety-seven million five billionths, one thousand five hundred seventy-nine and twenty-six thousand twenty-one hundred-thousandths.

6. I buy 10 bales of cloth containing $32\frac{1}{4}$, $41\frac{11}{16}$, $39\frac{1}{32}$, $46\frac{3}{4}$, 29.875, $38\frac{1}{16}$, $43\frac{5}{8}$, $31\frac{3}{32}$, $42\frac{13}{16}$, and 40.635 yards, respectively. How many yards in my purchase?

7. A man cut 16.5 cords of wood the first week, 15.76 cords the second week, and $9\frac{5}{8}$ cords the third week. How many cords did he cut in all?

8. How many thousand feet of lumber in 4 piles measuring as follows: 16,815, 28,185, 16,189, 75,141 feet, respectively?

9. How many tons of coal in 5 car loads weighing respectively 22.815 tons, 21.86 tons, $1\frac{5}{8}$ tons, 19.998 tons, and 18.125 tons?

10. A lumber dealer bought 6 car loads of lumber as follows: 1 car of 16.185 thousand feet at a cost of $178.12, 1 car of 15.998 thousand feet at a cost of $$198.37\frac{1}{2}$, 1 car of 14.17 thousand feet at a cost of $132.78, 1 car of 19.175 thousand feet at a cost of $$300.96\frac{1}{2}$, 1 car of 20.156 thousand feet at a cost of $418.50, and 1 car of 15.500 thousand feet at a cost of $175. Find the total number of thousand feet bought and the total cost of same.

11. Add 21^1, 54^2, 17^1, 30^3, 46^1, 61^2, 80^1, 39^3, and 24^2

12. Add 121^1, 97^3, 46^2, 111^3, 43, 71^2, 86^3, 50^1, 103^3, 72^1, 71^3, and 50.

SUBTRACTION OF DECIMAL FRACTIONS

294. Examples. *1.* Find the difference between .127 and .102.

$$\begin{array}{r} .127 \\ .102 \\ \hline .025 \end{array}$$

SOLUTION. Both decimals are of the same order; hence, write the subtrahend under the minuend and subtract as in whole numbers keeping the points in the minuend, subtrahend, and remainder, respectively, in the same vertical line.

2. From .7 take .37.

$$\begin{array}{r} .7 \\ .37 \\ \hline .33 \end{array}$$

SOLUTION. Annexing ciphers to a decimal does not alter its value; hence, consider .7 as .70 and the decimals are of the same order and may be subtracted as whole numbers. The result is then .33.

3. Find the difference between .73 and 2.

$$\begin{array}{r} 2. \\ .73 \\ \hline 1.27 \end{array}$$

SOLUTION. 2 being a whole number must be greater than the decimal fraction .73. Consider two decimal ciphers as annexed to 2 and subtract as in whole numbers. The result is then 1.27.

295. Hence the following rule:

Write the terms so that the points fall in the same vertical line and subtract as in whole numbers, placing the point in the remainder below the points in the other terms.

WRITTEN EXERCISE

Find the difference between:

1. .13823 and .668.

2. 2 and .72152.

3. 6.7584 and 1.232.

4. 2.3 and .753452.

5. .900 and .09.

6. .002 and .200.

7. 3491.5 and 4246.1005.

8. 24.6852 and 25.

9. 250.98754 and 386245.

10. .0001 and 1000.01.

11. 3 and .00015.

12. 259.00702 and 156.07.

MULTIPLICATION OF DECIMAL FRACTIONS

296. Examples. *1.* Find the product of .09 × .5.

$$\begin{array}{r} .09 \\ .5 \\ \hline .045 \end{array}$$

SOLUTION. $.09 = \frac{9}{100}$ and $.5 = \frac{5}{10}$. Applying the rule for the multiplication of common fractions $\frac{9}{100} \times \frac{5}{10} = \frac{45}{1000} = .045$. The *figures* to the right of the decimal point always form the numerator of the decimal fraction and the *number of figures* to the right of the

decimal point indicate the power of 10 which forms the denominator; hence, in performing the above multiplication, arrange the work as shown in the margin. $9 \times 5 = 45$, or the numerator of the product. The denominator of the multiplicand is the second power of ten, hence, contains two ciphers. The denominator of the multiplier is the first power of 10, hence, contains one cipher. The denominator of the product will thus contain three ciphers $(2 + 1)$ equal to three decimal places; hence, $.09 \times .5 = .045$.

2. Find the product of $2.05 \times .007$.

$$\begin{array}{r} 2.05 \\ \underline{.007} \\ .01435 \end{array}$$

SOLUTION. Multiply as in Example 1. Since the number of figures in the product is one less than the number of decimal places in the multiplicand and multiplier, supply the deficiency by prefixing a cipher, as shown in the margin. The result is then .01435.

297. From the foregoing solutions the following **rule is derived:**

Multiply as in whole numbers.

From the right in the product point off a number of decimal places equal to the sum of the number of decimal places in the multiplicand and multiplier.

WRITTEN EXERCISE

1. $.52 \times .25 \times 0 \times 95 = ?$

2. $1625.426 \times .0725 = ?$

3. $2.5 \times .095 = ?$

4. $25000 \times .000024 = ?$

5. $.009 \times .00001 \times 10000 = ?$

6. $716.0025 \times 10.1005 = ?$

7. Multiply three hundred forty-seven ten-thousandths by fifty-two thousandths.

8. I sold 14.4 bales of cloth, each bale containing 61.625 yards at 62½¢ per yard. How much did I receive?

9. From 10.85 acres of wheat a farmer harvested 31.875 bushels per acre and sold his crop at 97¼¢ per bushel. How much was received for the crop?

10. A man having 650 acres of land sold .625 of it. What is the remainder worth at $ 60.95 per acre?

11. Sold Mohawk Valley Lumber Co. for cash, 185.998 thousand feet of pine lumber at $ 18.87½ per thousand. Find the amount of the invoice.

12. A man's income is $ 3500 a year. If his average daily expenses are $ 8.23, how much can he save in a year?

13. Bought of Field Bros. & Co. three car loads of lumber at $ 17.87½ per thousand feet. The cars contained 21.255 thousand feet, 22.275 thousand feet, and 16.250 thousand feet, respectively. Find the amount of the invoice.

14. For constructing a house and barn, I bought 46.21 thousand feet matched pine at $ 21 per thousand, 13.516 thousand feet of siding at $ 28.50 per thousand, 11.260 thousand feet chestnut at $ 32 per thousand, 4.68 thousand feet black walnut at $ 45 per thousand, 58.66 thousand feet hemlock at $ 6.25 per thousand, 13.7 thousand brick at $ 5.50 per thousand, 9.28 thousand feet cherry at $ 86 per thousand, and 33.725 thousand feet hemlock timber at $ 11 per thousand. What was the total cost?

SHORT METHODS

298. Changing the position of the decimal point in any number affects the local value of each figure in that number.

Thus, if .721 be written 7.21, 7 tenths are made 7 units; 2 hundredths, 2 tenths; and 1 thousandth, 1 hundredth. Hence,

299. To multiply a decimal by 1 followed by any number of ciphers,

Move the decimal point to the right as many places as there are ciphers in the multiplier.

300. To multiply a number by .1, .01, .001, etc., is equivalent to dividing by 10, 100, 1000, etc. Hence,

301. To multiply a number by .1, .01, .001, etc.,

Move the decimal point as many places to the left as there are places in the multiplier.

ORAL EXERCISE

By inspection, find the value of:

1. 75.42 × .01.
2. 15.216 × 100.
3. 952.1003 × 10,000.
4. 10,000 × .72142.
5. 321.49 × 100.
6. .151 × 3000.
7. 32 × .00001.
8. .0001 × 75.
9. 16.295 × .0001.
10. .15 × 400.
11. 500 × .025.
12. 1201 × .001.
13. 17.02 × .001.
14. 157.271 × .0001.
15. 40.002 × .001.

302. $8.5 \times 8.5 = 8\frac{1}{2} \times 8\frac{1}{2}$. Hence, to multiply together numbers ending in .5, *proceed as explained in* **261–265**.

303. Examples. *1.* Find the value of 8.5 acres of prairie land at $ 8.50 per acre.

$$\begin{array}{r} 8.5 \\ 8.5 \\ \hline 72.25 \end{array}$$

Solution. $.5 \times .5 = .25$. $9 \times 8 = 72$. If 1 acre cost $ 8.50, 8.5 acres will cost 8.5 times $ 8.50, or $ 72.25.

2. Find the value of 17.5 acres of prairie land at $ 4.50 per acre.

$$\begin{array}{r} 17.5 \\ 4.5 \\ \hline 78.75 \end{array}$$

Solution. The sum of the numbers to the left of the decimals is odd; hence, the decimal fraction in the product is .75($\frac{3}{4}$). $\frac{1}{2}$(.5) of $\overline{17+4}+\overline{17 \times 4} = 78.5$. Rejecting the .5, which was included in the .75 first written, the result is $ 78.75.

3. Find the cost of 15.5 dozen men's kid gloves at $ 5.50 per dozen.

$$\begin{array}{r} 15.5 \\ 5.5 \\ \hline 85.25 \end{array}$$

Solution. The sum of the numbers to the left of .5 is even; hence, the decimal fraction in the product is .25($\frac{1}{4}$). $\frac{1}{2}$(.5) of $\overline{15+5}+\overline{15 \times 5} = 85$. Hence, the required result is $ 85.25.

ORAL EXERCISE

Find the value of:

1. 21.5 A. at $ 5.50.
2. 17.5 doz. at $ 5.50.
3. 16.5 doz. at $ 4.50.
4. 25.5 yd. at $ 4.50.
5. 19.5 yd. at $ 2.50.
6. 15.5 yd. at $ 2.50.
7. 125 lb. at 35 ¢.[*]
8. 16.5 doz. at $ 3.50.
9. 15.5 lb. at $8\frac{1}{2}$ ¢.
10. 12.5 yd. at 7.5 ¢.
11. 115 lb. at 5.5 ¢.
12. 135 lb. at 35 ¢.
13. 105 lb. at 45 ¢.
14. 255 lb. at 45 ¢.
15. 115 lb. at 55 ¢.
16. 115 gal. at 35 ¢.
17. 205 gal. at 45 ¢.
18. 215 gal. at 55 ¢.

DIVISION OF DECIMAL FRACTIONS

304. Division is the reverse of multiplication. Since in multiplication the decimal places of the two factors are added to determine the number of decimal places in the product, in division the number of decimal places in the quotient is found by subtracting the decimal places, if any, in the divisor from those in the dividend.

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[*] In this example and all similar cases, perform the multiplication just as if the decimal point were to the left of the 5's; then, in the product point off as many places as there are decimal places in the numbers multiplied. Thus, in this problem say, or think: *odd number, therefore write 75.* $\frac{1}{2}$ *of* $\overline{12+3}+\overline{12 \times 3} = 43\frac{1}{2}$. *Reject* $\frac{1}{2}$ *which has been included in the 75 just written and the product is 4375. Point off two places and the result is $ 43.75.*

305. Example. Divide .085 by .17.

.17).085(.5
85

Solution. The divisor has two decimal places, and the dividend has three decimal places, therefore point off one (3 − 1) place from the right in the quotient.

306. Hence the following rule:

When needed, annex ciphers to the dividend to make its places equal in number to those of the divisor.

Divide as in whole numbers, and from the right of the quotient point off as many places as the number of places in the dividend exceeds those in the divisor.

Do not commence the division until the number of decimal places in the dividend is at least equal to the number of decimal places in the divisor. Supply any deficiency in the dividend by annexing ciphers.

If the divisor and dividend have the same number of decimal places, the quotient obtained to the limit of the dividend as given will be a whole number.

If the number of decimal places in the dividend be greater than the number of decimal places in the divisor, point off from the right of the quotient the number of places equal to such excess, prefixing ciphers to the quotient if necessary.

If after division there be a remainder, ciphers may be annexed to it and the division continued to exactness or to two or three places ordinarily demanded in business computations. Such ciphers should be considered as parts of the dividend.

ORAL EXERCISE

By inspection, find the value of:

1. 1÷1.
2. 1÷.1.
3. 1÷.01.
4. 10÷.1.
5. 10÷.01.
6. .1÷1.
7. .1÷.1.
8. .1÷.01.
9. .1÷.001.
10. .1÷10.
11. .1÷100.
12. 1÷10.
13. 1÷100.
14. 1÷1000.
15. .001÷100.
16. .0001÷.1.
17. 100÷.01.
18. 10÷10000.
19. .22÷11.
20. 2.2÷.011.
21. 220÷11000.
22. .022÷110.
23. 2.2÷.00011.
24. 2200÷.00011.
25. .022÷11000.
26. 2200÷.000022
27. .00001÷10000.
28. 10000÷.0001.

WRITTEN EXERCISE

Find the sums of the quotients in the following problems:

1.

9 ÷ 9.
9 ÷ .9.
9 ÷ .09
9 ÷ .009.
.009 ÷ 9.
900 ÷ 900.
900 ÷ .09.
9000 ÷ .0009.
900 ÷ 9000.
.009 ÷ 90000.

2.

75 ÷ 250.
7.5 ÷ 2500.
75 ÷ .25.
7500 ÷ .25.
.75 ÷ 2500.
750 ÷ 25000.
7500 ÷ .0025.
.075 ÷ .025.
750 ÷ .0025.
75 ÷ .000025.

3.

64 ÷ 16.
.64 ÷ 16.
64 ÷ .16.
640 ÷ .16.
640 ÷ 1600.
64 ÷ .016.
64 ÷ .00016.
6400 ÷ .16.
6400 ÷ .00016.
640 ÷ 16000.

4.

11 ÷ 22.
110 ÷ .22.
11 ÷ .022.
1100 ÷ .22.
11000 ÷ .022.
110 ÷ 2200.
1100 ÷ 22000.
11 ÷ .000022.
11000 ÷ .22.
11 ÷ 2200.

5.

150 ÷ .3.
150 ÷ .03.
150 ÷ 3000.
150 ÷ .003.
1500 ÷ .03.
15 ÷ 30000.
15 ÷ .0003.
1500 ÷ .003.
150 ÷ .00003.
15 ÷ 300.

6.

39 ÷ 130.
3900 ÷ .13.
390 ÷ 13000.
3900 ÷ 130000.
.039 ÷ 13.
.0039 ÷ 130.
.00039 ÷ 13000.
.000039 ÷ 13.
.0000039 ÷ 1.3.
.039 ÷ .013.

SHORT METHODS

307. To divide a decimal by 1 followed by any number of ciphers,

Move the decimal point in the dividend to the left as many places as there are ciphers in the divisor.

308. To divide a number by .01, .001, or 1 preceded by any number of decimal ciphers,

Move the decimal point in the dividend as many places to the right as there are places in the divisor.

ORAL EXERCISE

1. 397 ÷ 10.
2. 1.37 ÷ 1000.
3. 17.3 ÷ 10.
4. 2.47 ÷ 1000.
5. 357.16 ÷ 1000.
6. 14.27 ÷ 100.
7. .82 ÷ 100.
8. .075 ÷ .01.
9. .113 ÷ .001.
10. .171 ÷ .0001.
11. .75 ÷ .0001.
12. 13.54 ÷ .001.

ORAL REVIEW

1. The sum of two numbers is .3. The smaller number is .05. What is the product of the two numbers?

2. If .75 of a mill is worth $ 7500, what is .5 of it worth?

3. If .75 of a stock of goods is worth $ 225, what is three times the stock worth?

4. Five times a certain decimal is .4. What is the decimal?

5. Three times a certain decimal is .15. What is twelve times the same decimal?

6. How many thousandths in seven units?

7. $\frac{1}{2} - .5 \times \frac{1}{2} = ?$

8. The product of two numbers is .0006. If one of the numbers is .03, what is the other?

9. The sum of two numbers is 15. If one of them is 6.5, what is the product of the numbers?

10. $2.50 is how many hundredths times $75?

11. What will 7.5 thousand envelopes cost at $2.50 per thousand?

12. Find the cost of 12.5 thousand feet of plank at $8.50 per thousand.

WRITTEN REVIEW

1. The sum of three numbers is 4.5. If the smaller is .95 and the larger 2.05, what is the product of the three numbers?

2. Multiply the sum of sixty-five hundred and sixty-five and one hundred seven millionths by the product of nine hundred millionths and one hundred twenty-seven and seventeen hundredths.

3. What is the cost of 6 barrels of sugar weighing 301, 314, 297, 309, 313, and 315 pounds, respectively, at $6\frac{7}{8}$¢ per pound?

4. If a wheelman travels 10.3 hours per day, how many days will be required for him to travel 558.0025 miles at the rate of 7.88 per hour?

5. I sold a lumber man 381.25 pounds of butter at 28¾¢ per pound, 2468.375 pounds of cheese at 11.4¢ per pound, and 2356.5 pounds of dressed beef at 7⅞¢ per pound, and received pay in lumber at $23.12½ per thousand feet. How many thousand feet of lumber should I have received?

6. A man's salary is $2500 per year. If he spends $650.25 for board, $119.25 for books, $31.85 for other literature, $63.40 for charity, $209.75 for clothes, $109.90 for traveling expenses, $115.60 for incidental expenses, and saves the remainder, how long will it take him to pay for a piece of property valued at $8400?

7. A merchant had on hand Jan. 1, 1904, a stock of merchandise aggregating $11750.90. During the year he bought goods amounting to $7315.90 and sold goods amounting to $15364.85. If on Dec. 31, 1904, he has stock on hand valued at $9215.75, has he gained or lost for the year and how much?

8. Having bought 25 gross of steel pens at $1.25 per gross, I sold them at 12¢ each. If there are 144 pens in a gross, did I gain or lose, and how much?

9. Find the total cost of the items in the oral exercise, page 103.

10. If a boy receives $1.25 a day, and a man $3.75 a day, how long will it take the boy to earn as much as the man can earn in 16 days?

11. In a certain business school .5 of the students study bookkeeping, .75 of the remainder study shorthand and typewriting, and the remainder, 125 pupils, study the English branches. How many students in each department, and in the entire school?

12. C. W. Allen bought of J. E. Seel & Co., dealers in flour and feed, 135 barrels roller process flour at $6.75 per barrel, 135 barrels searchlight pastry flour at $5.75 per barrel, 375 sacks puritan pancake flour at 23¢ per sack, 195 sacks chef pastry flour at 25¢ per sack, 250 bags bran at $1.50 per bag, 1500 pounds corn meal at 2¾¢ per pound. Find the amount of the bill.

13. A and B are in partnership. A is to receive .75 of the profits and B the remainder. At the end of one year B draws $1250 as his share of the profits. If the total losses for the year were $950, what was the total gain for the year? the net gain?

14. January 1, A and B join in the purchase of some real estate, A paying .4 of the purchase price, and B the remainder. They share the profits arising from the sale in proportion to their investments. The property is sold at a profit of $2500 and B receives $6000 as his share. How much did A and B pay for the real estate?

QUANTITY, PRICE, AND COST

309. The essential elements of every business transaction involving the money value of property or labor are **quantity, price,** and **cost.**

310. The fixed unit used in estimating the money value of commodities is termed a **commercial unit.**

A yard, a dozen, a bushel, and an acre are commercial units.

311. **Quantity** is the number of commercial units in any given commodity.

312. **Price** is the value put upon a commercial unit.

313. **Cost** is the value of a quantity.

314. An **aliquot part** of a number is one of the even parts of that number.

20, 25, 33⅓, 50, etc., are aliquot parts of 100.

315. The **unit of an aliquot part** is the number which must be divided to obtain the part.

$1 is the unit of the aliquot parts 20¢, 25¢, 33⅓¢, etc.

DRILL EXERCISE

1. Name three commodities of which a dozen is the commercial unit; a yard; a pound; a ton.

2. Name three quantities of which the commercial unit is 1 rod; 1 acre; 1 barrel; 1 bag; 1 bale.

3. Name three aliquot parts of 1 yard; of 1 bushel; of 1 day; of $1.

4. Name three aliquot parts of 50; of 250; of ½.

5. Name four aliquot parts of 1 ton; of 2 yards; of 30.

6. What aliquot part of $1 is 50¢? 33⅓¢? 25¢? 20¢? 16⅔¢? 12½¢? 10¢? 8⅓?

7. What aliquot part of 25¢ is 12½¢? 6¼¢? 5¢? 8⅓¢? 2½¢? 1⅔¢?

8. What aliquot part of 50¢ is 2½¢? 3⅓¢? 6¼¢? 8⅓¢? 10¢? 12½¢?

316. The aliquot parts of $1 are especially useful in computations where the quantity and price are given to find the cost.

TABLE OF ALIQUOT PARTS

PARTS OF $1.00	PARTS OF 50¢	PARTS OF 25¢	PARTS MORE OR LESS THAN $1.00
50¢ = ½			112½¢ = ⅛ more
33⅓¢ = ⅓			125¢ = ¼ more
25¢ = ¼	25¢ = ½		133⅓¢ = ⅓ more
20¢ = ⅕			110¢ = $\frac{1}{10}$ more
16⅔¢ = ⅙	16⅔¢ = ⅓		90¢ = $\frac{1}{10}$ less
12½¢ = ⅛	12½¢ = ¼	12½¢ = ½	87½¢ = ⅛ less
10¢ = $\frac{1}{10}$	10¢ = ⅕		80¢ = ⅕ less
8⅓¢ = $\frac{1}{12}$	8⅓¢ = ⅙	8⅓¢ = ⅓	83⅓¢ = ⅙ less
6¼¢ = $\frac{1}{16}$	6¼¢ = ⅛	6¼¢ = ¼	75¢ = ¼ less
5¢ = $\frac{1}{20}$	5¢ = $\frac{1}{10}$	5¢ = ⅕	66⅔¢ = ⅓ less
3⅓¢ = $\frac{1}{30}$	3⅓¢ = $\frac{1}{15}$		62½¢ = ⅜ less
2½¢ = $\frac{1}{40}$	2½¢ = $\frac{1}{20}$	2½¢ = $\frac{1}{10}$	40¢ = ⅗ less
1⅔¢ = $\frac{1}{60}$	1⅔¢ = $\frac{1}{30}$	1⅔¢ = $\frac{1}{15}$	37½¢ = ⅝ less

317. Many of the ordinary business computations may be materially shortened by the use of aliquot parts.

DRILL EXERCISE

1. Formulate a short method for finding the cost of a quantity when the price is 33⅓¢.

SOLUTION. Since 33⅓¢ is ⅓ of $1, to find the cost of a quantity when the price is 33⅓¢ *consider the quantity as dollars and divide by 3.*

2. Formulate a short method for finding the cost of a quantity when the price is 50¢; 25¢; 20¢; 16⅔¢; 12½¢; 6¼¢; 8⅓¢.

3. Formulate a short method for finding the cost of a quantity (*a*) when the price is 10¢; (*b*) when the price is 5¢.

Solutions. (*a*) Since 10¢ is $\frac{1}{10}$ of $1, to find the cost of a quantity when the price is 10¢, *point off from the right one place in the quantity considered as dollars.*

(*b*) Since 5¢ is ½ of 10¢, to find the cost of a quantity when the price is 5¢, *point off one place in the quantity considered as dollars and divide by 2.*

4. Formulate a short method for finding the cost of a quantity when the price is 3⅓¢; 2½¢; 1⅔¢; 1¼¢; 1⅑¢.

318. General Rule. *Find the cost of the total quantity by multiplying $1 by the given quantity; then take such part of the product thus obtained as the given price is a part of* $1.

319. An **abbreviation is a part of a word used to indicate an** entire word.

Many abbreviations are used in computations involving quantity, price, and cost. The most important of these are shown in the following list.

BUSINESS ABBREVIATIONS

A 1.	first quality	doz.	dozen	No.	number
Apr.	April	Dr.	debtor	Nov.	November
acct.	account	ea.	each	Oct.	October
amt.	amount	E. & O. E.	errors and omissions excepted	oz.	ounce
Aug.	August			p.	page
bal.	balance	Feb.	February	pp.	pages
bbl.	barrel	ft.	foot *or* feet	pt.	pint
B/L	bill of lading	frt.	freight	payt.	payment
bot.	bought	gal.	gallon	Pd.	paid
bu.	bushel	gro.	gross	pkg.	package
bx.	box	hhd.	hogshead	pc.	piece
cd.	cord	hr.	hour	pr.	pair
ctg.	cartage	in.	inch	qt.	quart
Co.	Company	Jan.	January	recd.	received
C. O. D.	collect on delivery	Jr.	Junior	R.R.	railroad
Cr.	creditor	lb.	pound	sec.	second
cwt.	hundredweight	Mar.	March	*s.*	shilling
da.	day	mem.	memorandum	Sept.	September
d.	pence	mo.	month	Sr.	Senior
Dec.	December	Messrs.	Gentlemen *or* Sirs	wk.	week
disc.	discount	Mr.	Mister	yd.	yard
do.	ditto, *or* the same	Mrs.	Mistress	yr.	year

BUSINESS CHARACTERS

#	number	¢	cents	C	by the hundred
@	at	1^1	1¼	✓	check mark
a/c	account	1^2	1 2/4 *or* 1½	"	ditto
c/o	care of	1^3	1¾	′	feet
×	by (in surface measures)	%	per cent	″	inches
$	dollars	M	by the thousand	£	pounds sterling

ORAL EXERCISE

By inspection, find the cost of:

1. 350 lb. tea at 50¢.
2. 870 lb. coffee at 33⅓¢.
3. 124 lb. raisins at 25¢.
4. 24 lb. raisins at 16⅔¢.
5. 190 lb. rice at 10¢.
6. 160 lb. seed at 6¼¢.
7. 123 lb. meal at 3⅓¢.
8. 855 yd. prints at 20¢.
9. 144 yd. gingham at 6¼¢.
10. 180 yd. silesia at 16⅔¢.
11. 192 yd. lining at 8⅓¢.
12. 1140 yd. prints at 33⅓¢.
13. 284 yd. lining at 25¢.
14. 960 yd. ticking at 6¼¢.
15. 168 lb. ham at 16⅔¢.
16. 368 yd. plaids at 33⅓¢.
17. 88 yd. lace at 87½¢.
18. 340 yd. mohair at 75¢.
19. 390 yd. alpaca at 66⅔¢.
20. 484 lb. lard at 12½¢.
21. 1680 doz. eggs at 16⅔¢.
22. 240 lb. pork at 6⅔¢.
23. 1152 yd. linen at 37½¢.
24. 728 gal. cider at 8⅓¢.
25. 111 qt. berries at 3⅓¢.
26. 880 lb. salt at 1¼¢.
27. 164 yd. cotton at 6¼¢.
28. 86 lb. meal at 1½¢.

DRILL EXERCISE

1. Formulate a short method for finding the quantity when the cost is given and the price is 33⅓¢.

SOLUTION. Since the price is contained 3 times in $1, the quantity bought will be 3 times the number of dollars invested; hence, ***multiply the cost by 3 and consider the result as quantity.***

2. Formulate a short method for finding the quantity when the cost is given and the price is 50¢; 25¢; 20¢; 16⅔¢; 8⅓¢; 6¼¢; 5¢.

3. Formulate a short method for finding the quantity (*a*) when the cost is given and the price is 10¢; (*b*) when the price is 3⅓¢.

Solutions. (*a*) Since 10¢ is contained 10 times in $1, when the cost is given and the price is 10¢, ***annex a cipher to the dollars and consider the result as quantity.***

(*b*) 10¢ is 3 times 3⅓¢. If, to find the quantity at 10¢, we annex a cipher to the cost and consider the result as quantity, to find the cost at 3⅓¢, we should ***annex one cipher to the cost, multiply by 3, and consider the result as quantity.***

4. Formulate a short method for finding the quantity when the cost is given and the price is 2½¢; 1⅔¢; 1¼¢; 1⅓¢; 5¢.

ORAL EXERCISE

1. How many pounds of tea worth 33⅓¢ can be bought for $123?

2. How many pounds of tea worth 50¢ can be bought for $419.50?

3. How many yards of cloth at 16⅔¢ can be bought for $25?

4. At 25¢, how many yards of prints can be bought for $25?

5. At 33⅓¢, how many gallons of molasses can be bought for $15?

6. At 3⅓¢, how many pounds of dried apples can be bought for $32?

7. At 2½¢, how many pounds of sugar can be bought for $12?

8. At 3⅓¢, how many pounds of sugar can be bought for $18?

9. At 20¢, how many yards of cotton can be bought for $125?

10. How many yards of gingham at 8⅓¢ can be bought for $11?

11. How many yards of ticking at 6⅔¢ can be bought for $8?

12. How many yards of lining worth 6¼¢ can be bought for $15?

WRITTEN EXERCISE

1. A farmer sold 26½ bu. buckwheat, at 87½¢ per bu., and took his pay in sugar at 6¼¢ per lb. How many pounds should he have received?

2. A gardener exchanged 132 qt. of berries, at 8⅓¢ per qt., and 75 doz. corn, at 12½¢ per doz., for cloth at 25¢ per yd. How many yards did he receive?

3. If I exchange 1920 acres of wild land, at $7.50 per acre, for an improved farm at $125 per acre, what should be the number of acres in my farm?

4. A farmer gave 8¾ cwt. of pork, at $7.50 per cwt., 15 bu. of beans, at $3.25 per bu., and 46⅔ bu. of oats, at 33⅓¢ per bu., for 28 yd. of dress silk, at $1.25 per yd., and 52½ yd. of delaine, at 16⅔¢ per yd., receiving for the remainder, cotton goods at 12½¢ per yd. How many yards of cotton goods should be delivered to him?

5. When potatoes are worth 66⅔¢ per bu., and turnips 25¢ per bu., how many pounds of coffee, at 16⅔¢ per lb., will pay for 24 bu. of potatoes and 18 bu. of turnips?

6. Having bought 1487 lb. A. sugar, at 6¼¢ per lb.; 872 lb. C. sugar, at 5¢ per lb.; 628⅓ lb. Y.H. tea, at 33⅓¢ per lb.; 522 lb. J. tea, at 25¢ per lb.; 650 lb. Rio coffee, at 12½¢ per lb.; and 81 sacks of flour, at $1.25 per sack, I give in payment seven one-hundred dollar bills. How much should be returned to me?

GENERAL APPLICATIONS OF ALIQUOT PARTS

320. The principles of aliquot parts may be applied to a great many business exercises, as will be shown in the following examples.

321. Examples. *1.* What will be the cost of 15 bbl. of pork at $16⅔ per barrel?

Solution. Since $16⅔ is ⅙ of $100, 16⅔ times any number is ⅙ of 100 times that number. Hence,

To multiply by 16⅔, *annex two ciphers to the multiplicand and divide the result by 6*, obtaining as a result $250.

2. What will 25 acres of land cost at $164 per acre?

Solution. 25 acres at $164 is equal to 164 acres at $25 per acre. Hence, *Annex two ciphers to 164 and divide by 4*, obtaining as a result $4100.

3. Find the cost of 25 yd. of cloth at 44¢ per yard.

Solution. *Interchange 25 and 44 and apply the principles of aliquot parts.* Then, 44 ÷ 4 = 11. Therefore, 25 yd. of cloth at 44¢ will cost $11.

4. Find the cost of 2500 lb. of coffee at 32¢ per pound.

Solution. 2500 lb. at 32¢ per pound is equal to 3200 lb. at 25¢ per pound. Hence,

Interchange the significant figures (in this case 32 and 25) *and apply the principles of aliquot parts.* Then, 3200 ÷ 4 = 800. Therefore, 2500 lb. of coffee at 32¢ will cost $800.

5. Find the cost of 250 lb. of tea at 64¢ per pound.

Solution. 250 lb. at 64¢ per pound is equal to 640 lb. at 25¢ per pound. Hence,

Interchange the significant figures and apply the principles of aliquot parts. Then, 640 ÷ 4 = 160. Therefore, 250 lb. of tea at 64¢ will cost $160.

6. Find the cost of 250 acres of land at $44 per acre.

Solution. 250 acres at $44 per acre is equal to 440 acres at $25 per acre. Annexing two ciphers to 440 the result is 44,000. Then, 44,000 ÷ 4 = 11,000. Therefore, 250 acres of land at $44 per acre will cost $11,000.

DRILL EXERCISE

1. Formulate a short method for multiplying any number by 12½.

Solution. Since 12½ is ⅛ of 100, 12½ times any number is ⅛ of 100 times the same number. Hence,

To multiply any number by 12½, ***annex two ciphers to the multiplicand and divide by 8.***

2. Formulate a short method for multiplying any number by 25; by 2.5; by 333⅓; by 1⅓; by 125; by 250; by 66⅔; by 6⅔; by 33⅓; by 1250.

3. Demonstrate that 2500 lb. of coffee at 44¢ per pound is equal to 4400 lb. at 25¢ per pound.

ORAL EXERCISE

By inspection, find the value of:

1. 72 head cattle at $25.
2. 75 acres land at $33⅓.
3. 48 bbl. beef at $16⅔.
4. 162 bbl. pork at $12½.
5. 25 lb. tea at 64¢.
6. 250 acres land at $44.
7. 2500 acres land at $16.
8. 45 bbl. beef at $16⅔.
9. 125 yd. cashmere at 64¢.
10. 333⅓ yd. silk at 96¢.
11. 166⅔ yd. cotton at 36¢.
12. 64 acres land at $25.
13. 250 yd. wool crepon at $2.40
14. 16⅔ gro. buttons at 24¢.
15. 125 yd. cheviot at $1.05.
16. 250 yd. taffeta silk at 88¢.
17. 45 gro. buttons at 33⅓¢.
18. 250 acres land at $88.
19. 33⅓ lb. tea at 54¢.
20. 96 yd. cloth at 33⅓¢.
21. 24 tons coal at $12½.
22. 36 tons coal at $8⅓.
23. 32 tons coal at $6¼.
24. 18 tons coal at $3⅓.

WRITTEN EXERCISE

1. Find the total cost of:

236 lb. tea at 50¢.
1152 yd. linen at 33⅓¢.
528 lb. lard at 12½¢.
488 gal. molasses at 25¢.
1848 doz. eggs at 16⅔¢.
844 yd. cambric at 25¢.
250 yd. alpaca at 72¢.
112 yd. silk at $2.50.
3608 yd. gingham at 12½¢.
25 yd. brilliantine at $1.84.

2. Find the total cost of:

350 yd. plaids at 33⅓¢.
248 yd. ticking at 10¢.
1140 yd. prints at 33⅓¢.
950 yd. lining at 25¢.
720 yd. drilling at 8⅓¢.
2500 yd. lining at 41¢.
250 yd. gingham at 14¢.
2500 yd. mohair at $1.23.
250 yd. sateen at 32¢.
125 yd. diagonals at $1.04.

3. Find the total cost of:

188 lb. of ham at 16⅔¢.
250 lb. Japan tea at 44¢.
125 gal. molasses at 91¢.
25 sacks flour at $1.24.
125 sacks flour at $1.80.
25 doz. cans of corn at $1.25.
166⅔ lb. tea at 42¢.
125 doz. cans peaches at $1.28.
250 cans tomatoes at 88¢.
125 doz. cans pears at $1.20.

DRILL EXERCISE

1. Formulate a short method for finding the cost when the quantity is given and the price is $1.66⅔.

SOLUTION. $1.66⅔ is 10 times 16⅔¢; hence, to find the cost when the price is $1.66⅔, *annex a cipher to the quantity considered as dollars and divide by 6.*

2. Formulate a short method for finding the cost when the quantity is given and the price is $3.33⅓; $1.25.

3. How may the cost be found when the quantity is given and the price is 66⅔¢?

SOLUTION. 66⅔ is 10 times 6⅔. 6⅔ is $\frac{1}{15}$ of a dollar; hence, to find the cost when the price is 66⅔¢, *add a cipher to the quantity considered as dollars and divide by 15.*

4. **Formulate a short method for finding the cost when the quantity is given and the price is $6.66⅔.**

5. How may the cost be found (*a*) when the price is 75¢? (*b*) when the price is $1.33⅓?

Solutions. (*a*) 75¢ is ¼ less than $1; hence, to find the cost when the price is 75¢, *consider the quantity as dollars and subtract ¼ of itself.*

(*b*) $1.33⅓ is ⅓ more than $1; hence, to find the cost when the price is $1.33⅓, *to the quotient considered as dollars add ⅓ of itself.*

6. Formulate a short method for finding the cost when the quantity is given and the price is $7.50; $75; $1.25; $1.08⅓.

7. Formulate a short method for finding the quantity when the cost is given and the price is $1.66⅔.

Solution. When the price is 16⅔¢, the cost is ⅙ of the quantity. When the price is $1.66⅔, the cost is 10 times ⅙ of the quantity; hence, to find the quantity when the price is $1.66⅔, *point off one place in the dollars considered as quantity and multiply by 6.*

8. Formulate a short method for finding the quantity when the cost is given and the price is $6.66⅔; $2.50.

9. How may the quantity be found when the cost is given and the price is $7.50?

Solution. 75¢ plus ⅓ of itself is equal to $1; hence, when the price is 75¢, ⅓ of the cost added to itself is equal to the quantity. $7.50 is 10 times 75¢; hence, to find the quantity when the price is $7.50, *point off one place in the cost considered as quantity and add ⅓*

10. Formulate a short method for finding the cost when the quantity is given and the price is $25; $75.

ORAL EXERCISE

Find the cost of:

1. 240 lb. at 75¢.
2. 165 lb. at $3.33⅓.
3. 366 lb. at $1.66⅔.
4. 333⅓ lb. at 24¢.
5. 166⅔ lb. at 66¢.
6. 750 lb. at 84¢.
7. 1000 lb. at 7½¢.
8. 125 lb. at 88¢.
9. 484 lb. at $2.50.
10. 33⅓ lb. at 99¢.
11. 300 lb. at 42¢.
12. 3000 lb. at 16¢.
13. 2500 lb. at $.444.
14. 250 lb. at 16¢.
15. 125 lb. at 64¢.

WRITTEN EXERCISE

In the following problems make all extensions mentally.

1. Find the total cost of:

316 lb. at 10¢.
484 lb. at 25¢.
1000 lb. at 7½¢.
2500 lb. at 16¢.
3000 lb. at 11¢.
216 lb. at 12½¢.
1124 lb. at 50¢.
320 lb. at 6¼¢.
816 lb. at 25¢.
381 lb. at 33⅓¢.
1095 lb. at 33⅓¢.
125 lb. at 64¢.
95 lb. at 8⅓¢.
64 lb. at 6¼¢.
1445 lb. at 20¢.

2. Find the total cost of:

835 yd. at 10¢.
450 yd. at 33⅓¢.
288 yd. at 26¢.
250 yd. at 44¢.
1200 yd. at $2.50.
298 yd. at 50¢.
333 yd. at 33⅓¢.
240 yd. at 8⅓¢.
966 yd. at 16⅔¢.
750 yd. at 6⅔¢.
1000 yd. at 19½¢.
2000 yd. at 21¢.
648 yd. at 12½¢.
125 yd. at 88¢.
2500 yd. at 64¢.

3. Find the total cost of:

1400 lb. at 4¢.
2163 lb. at 3⅓¢.
7200 lb. at 5¢.
1250 lb. at 8.4¢.
125 lb. at 6.4¢.
3980 lb. at 6¼¢.
128 lb. at 16⅔¢.
291 lb. at 50¢.
1437 lb. at 25¢.
840 lb. at 75¢.
880 lb. at 1¼¢.
83^2 lb. at 50¢.
107^2 lb. at 50¢.
450 lb. at 6⅔¢.
240 lb. at $1.33⅓.

4. Find the total cost of:

525 yd. at 20¢.
2853^3 yd. at 10¢.
1400 yd. at 6½¢.
450 yd. at 6⅔¢.
364 yd. at $1.25.
360 yd. at 75¢.
280 yd. at $2.50.
320 yd. at 75¢.
146 yd. at 25¢.
2500 yd. at 88¢.
500 yd. at 37¢.
1000 yd. at $1.87½.
2000 yd. at $1.12½.
296 yd. at 12½¢.
180 yd. at 16⅔¢.

5. Find the total cost of:

1095 yd. at 3⅓¢.
484 yd. at 6¼¢.
366 yd. at 8⅓¢.
1291 yd. at 11¢.
250^2 yd. at 24¢.
400 yd. at 15½¢.
756 yd. at 16⅔¢.
320 yd. at 12½¢.
1515 yd. at 66⅔¢.
906 yd. at $1.33⅓.
1250 yd. at 32¢.
64 yd. at $25.
1644 yd. at 25¢.
964 yd. at $2.50.
3000 yd. at 23¢.

322. **To find the cost of articles sold by the hundred.**

323. **Example.** What is the cost of 444 lb. of phosphate at $2.50 per hundred pounds?

Solution. 444 lb. equals 4.44 hundred pounds. If 1 hundred lb. cost $2.50, 4.44 hundred pounds will cost 4.44 times $2.50, or $11.10.

324. Hence the following rule:

Reduce the quantity to hundreds and decimals of a hundred by pointing off 2 places from the right. Multiply the number of hundreds by the price per hundred.

WRITTEN EXERCISE

Find the cost of:

1. 1600 lb. salt at $1.25 per C; 1700 lb. at $1.12½ per C.

2. 378 fence posts at $7.50 per C; 420 posts at $6.66⅔ per C.

3. 905 lb. lead at $3.33⅓ per C; 250 lb. at $3.20 per C.

4. 1125 lb. castings at $2.50 per C; 2500 lb. at $2.40 per C.

5. 1620 handles at $7.50 per C; 250 at $6.40 per C.

6. 5045 lb. beef at $12.50 per C; 1250 lb. at $14.40 per C.

7. 24828 lb. nails at 12½¢ per C; 37500 lb. at 16⅔¢ per C.

8. 840 lb. scrap iron at $1.33 per C; 750 lb. at $1.60 per C.

9. 3295 lb. guano at $4.50 per C; 4500 lb. at $5.50 per C.

10. 2456 fence rails at $2.50 per C; 3750 rails at $3.33⅓ per C.

325. **To find the cost of articles sold by the thousand.**

326. **Example.** At $7 per M, what will be the cost of 1544 bricks?

Solution. 1544 bricks equals 1.544 thousand bricks; if 1 thousand bricks cost $7, 1.544 thousand will cost 1.544 times $7, or $10.808 = $10.81.

327. Therefore the following rule:

Reduce the quantity to thousands and decimals of a thousand by pointing off 3 places from the right. Multiply the number of thousands by the cost per thousand.

WRITTEN EXERCISE

Find the cost of:

1. 1650 ft. pine lumber at $5 per M.
2. 611 ft. oak lumber at $24 per M.
3. 21168 ft. hemlock lumber at $7.50 per M.
4. 9475 ft. elm lumber at $13 per M.
5. 2120 ft. ash lumber at $25 per M.
6. 2768 ft. maple lumber at $14 per M.
7. 1100 ft. chestnut lumber at $18 per M.
8. 4560 ft. oak lumber at $22 per M.
9. 11265 ft. spruce lumber at $12.50 per M.
10. 76000 shingles at $5.25 per M.

328. To find the cost of articles sold by the ton of 2000 lb.

329. Example. What will be the cost of 3108 lb. of coal at $6 per ton?

SOLUTION. 3108 lb. equals 3.108 half tons. Since 1 ton, or 2000 lb., costs $6, ½ of a ton, or 1000 lb., will cost ½ of $6, or $3. If ½ of a ton costs $3, 3.108 half tons will cost 3.108 times $3, or $9.32.

330. Hence the following rule:

Divide the price of 1 ton by 2 and the result will be the price of 1000 lb.

From the right of the quantity point off 3 places and multiply by the price of 1000 lb.

WRITTEN EXERCISE

Find the cost of:

1. 2680 lb. at $2 per ton.
2. 1345 lb. at $7 per ton.
3. 4372 lb. at $36 per ton.
4. 1135 lb. at $2.50 per ton.
5. 116780 lb. at $34 per ton.
6. 84,725 lb. at $38 per ton.
7. 15,066 lb. at $120 per ton.
8. 9362 lb. at $4.50 per ton
9. 2040 lb. at $12.40 per ton.
10. 1115 lb. at $35 per ton.

331. To find the cost of products of varying weights per bushel.

332. Examples. *1.* Find the cost of 600 lb. clover seed at $7.50 per bushel of 60 lb.

SOLUTION. At $7.50 per pound, the cost would be 600 times $7.50, or $4500, but since the price was not $7.50 per pound, but $7.50 per bushel of 60 lb., the cost will be $\frac{1}{60}$ of $4500, or $75.

2. Find the cost of 400 lb. seed at $1.25 per bushel of 14 lb.

SOLUTION. At $1.25 per pound the cost would be $500; but since the price was not $1.25 per pound, but $1.25 per bushel of 14 lb., the cost would be $\frac{1}{14}$ of $500, or $35.71.

3. Find the cost of 6400 lb. barley at 75¢ per bushel of 48 lb.

SOLUTION. At 75¢ per pound the cost would be 6400 times 75¢, or $4800; but since the price was not 75¢ per pound, but 75¢ per bushel of 48 lb., the cost will be $\frac{1}{48}$ of $4800. Dividing by 48 the result is $100.

333. Hence the following rule:

Multiply the number of pounds weight by the price per bushel and divide the product by the number of pounds in one bushel.

WRITTEN EXERCISE

Find the cost of:

1. 2400 lb. wheat at 80¢ per bushel of 60 lb.

2. 2560 lb. corn at 65¢ per bushel of 56 lb.

3. 3361 lb. barley at 75¢ per bushel of 48 lb.

4. 1768 lb. millet at $1 per bushel of 45 lb.

5. 2255 lb. oats at 35¢ per bushel of 32 lb.

6. 2172 lb. buckwheat at 50¢ per bushel of 48 lb.

7. 2761 lb. beans at $1.25 per bushel of 62 lb.

8. 2500 lb. peas at $1.40 per bushel of 60 lb.

9. 3140 lb. Hungarian grass seed at $2.50 per bushel of 45 lb.

10. 2059 lb. red top grass seed at 90¢ per bushel of 14 lb.

BILLS AND ACCOUNTS

334. **Merchandise** is any goods or commodities held for the purpose of exchange.

335. A **bill** or **invoice** is a written statement in detail of merchandise sold, or of services rendered.

336. **Outline of a Bill.** The following is an outline of what a bill should show:

1. The place and date of sale.
2. The names of the buyer and seller.
3. The terms of sale.
4. The distinguishing marks and numbers, if any, placed on the goods.
5. The quantity, name, and price of each item.
6. The entire amount of the separate items.
7. All extra charges.
8. Any discounts allowed.

Formerly the term "invoice" was applied only to a written statement of merchandise sold at wholesale or shipped to an agent to be sold on commission. Now, however, the terms "invoice" and "bill" are used interchangeably when applied to a detailed statement of goods bought or sold in the course of trade. The term "invoice" is never applied to a list of expense items or a statement of services rendered. Thus we say, "an expense bill," "a physician's bill," "a freight bill," "a bill of lading," etc.

337. A bill is *receipted* when the words "Received payment" are written at the bottom, and the name of the seller is signed either by himself or by some authorized person.

Any authorized person may receipt a bill. When any person other than the seller receipts a bill, he should first sign the name of the seller, and on the next line below his own name or initials, preceded by the word "by" or "per."

338. A **debit** is that which costs value; a **credit** is that which produces value.

339. An **account** is a collection, under an appropriate title, of related debits and credits.

340. The debits of an account are written on the left side; the credits on the right side.

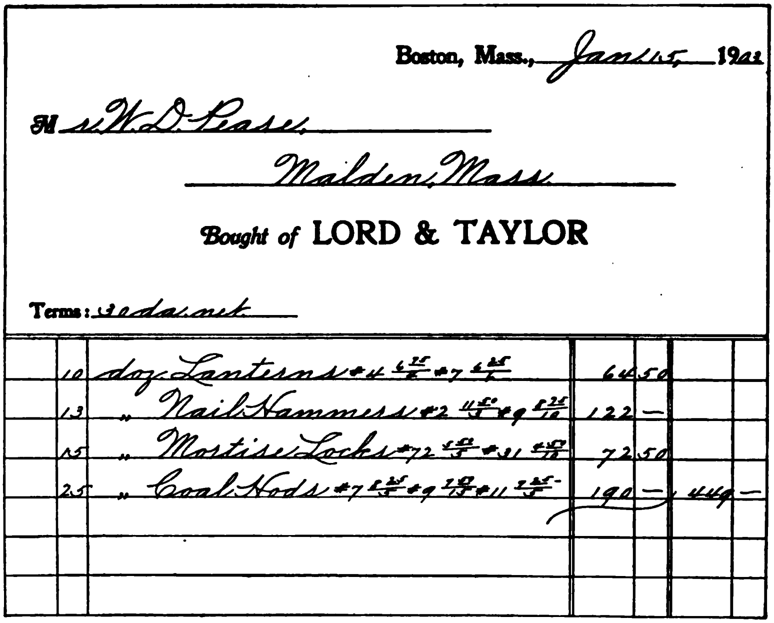
341. The **ledger** is the principal book of accounts. In it are entered in classified form the debits and credits of all business transactions.

342. An **account current** is an itemized statement of all the debits and credits between two persons.

343. A **statement** is a summary of the debits and credits of a personal account.

344. An **inventory** is usually an itemized schedule of the property of an individual, firm, or corporation not shown on the regular books of account.

An inventory is usually made upon the event of taking off a balance sheet, of a change in the business, of the admission of a partner, of the issue of stock, or, in case of embarrassment or insolvency, for examination by creditors, who wish to know the exact resources and liabilities of the business.



Boston, Mass., Jan. 15, 1921

Mr. W. D. Pease,

Malden, Mass.

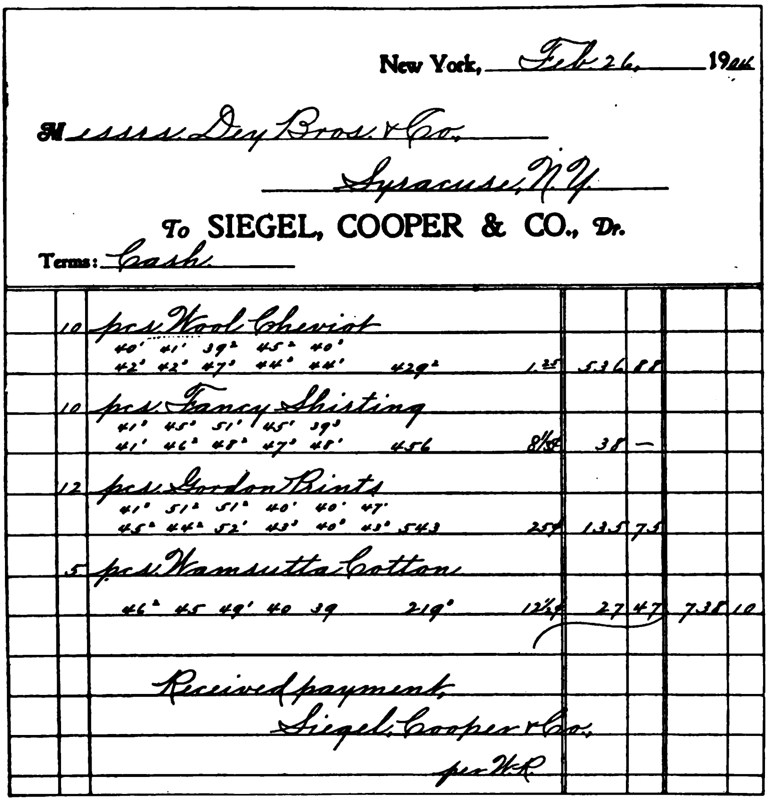
Bought of LORD & TAYLOR

Terms: 30 da. net

10	doz. Lanterns #4 $\frac{6^{25}}{6}$ #7 $\frac{6^{75}}{4}$	64	50		
13	" Nail Hammers #2 $\frac{11^{50}}{3}$ #9 $\frac{8^{75}}{10}$	122	—		
15	" Mortise Locks #72 $\frac{5^{50}}{5}$ #31 $\frac{4^{50}}{10}$	72	50		
25	" Coal Hods #7 $\frac{8^{25}}{5}$ #9 $\frac{7^{50}}{15}$ #11 $\frac{7^{25}}{5}$	190	—	449	—

Model Bill (Hardware)

In the model given above, the first figure shows the list number of the article, the figures above the horizontal line show the price, and the figures below the line the number of dozens of that special number which were sold. Thus, the first item in the model means that ten dozen lanterns in all were sold, of which 4 dozen were #4, at $6.75 per dozen, and 6 dozen were #7, at $6.25 per dozen.



New York, Feb 26 19ou

Messrs. Dey Bros. & Co.

Syracuse, N.Y.

To SIEGEL, COOPER & CO., Dr.

Terms: Cash

10	pcs. Wool Cheviot			
	40^1 41^1 39^2 45^2 40^3			
	42^3 42^3 47^3 44^3 44^1 429^2	1.25	536 88	
10	pcs. Fancy Shirting			
	41^3 45^3 51^1 45^1 39^3			
	41^1 46^2 48^2 47^3 48^1 456	8⅓	38 —	
12	pcs. Gordon Prints			
	41^3 51^2 51^2 40^1 40^1 47^1			
	45^2 44^2 52^1 43^3 40^3 43^3 543	25	135 75	
5	pcs. Wamsutta Cotton			
	46^2 45 49^1 40 39 219^3	12½	27 47	738 10

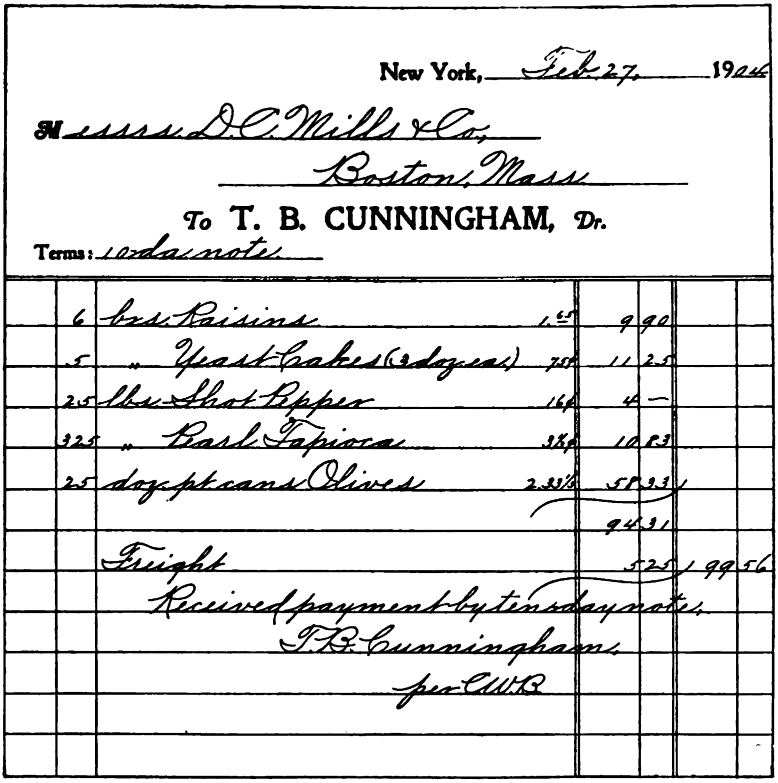
Received payment,
Siegel, Cooper & Co.
per W.R.

Model Receipted Bill (Dry Goods)

In the model given above the number of yards in the different pieces of cloth is not uniform. Since the price is so much per yard, it is necessary to list the number of yards in each piece as shown.

40^1, 39^2, 40^3, etc., in the first item on the bill are understood to mean 40¼, 39½(½), 30¾, etc. 429^2 equals the total number of yards in the 10 pieces.

In finding the total number of yards in any number of pieces the various items should not be copied to another sheet, but should be added horizontally as they stand. The fractions should be added first, and then the integers.



New York, Feb. 27, 1904

Messrs. D. C. Mills & Co.,
Boston, Mass.

To T. B. CUNNINGHAM, Dr.

Terms: 10 da. note.

	6	bxs. Raisins	1.65	9	90		
	5	" Yeast Cakes (3 doz. ea.)	75¢	11	25		
	25	lbs. Shot Pepper	16¢	4	—		
	325	" Pearl Tapioca	3⅓¢	10	83		
	25	doz. pt. cans Olives	2.33⅓	58	33		
				94	31		
		Freight		5	25	99	56
		Received payment by ten day note,					
		T. B. Cunningham.					
		per C. W. B.					

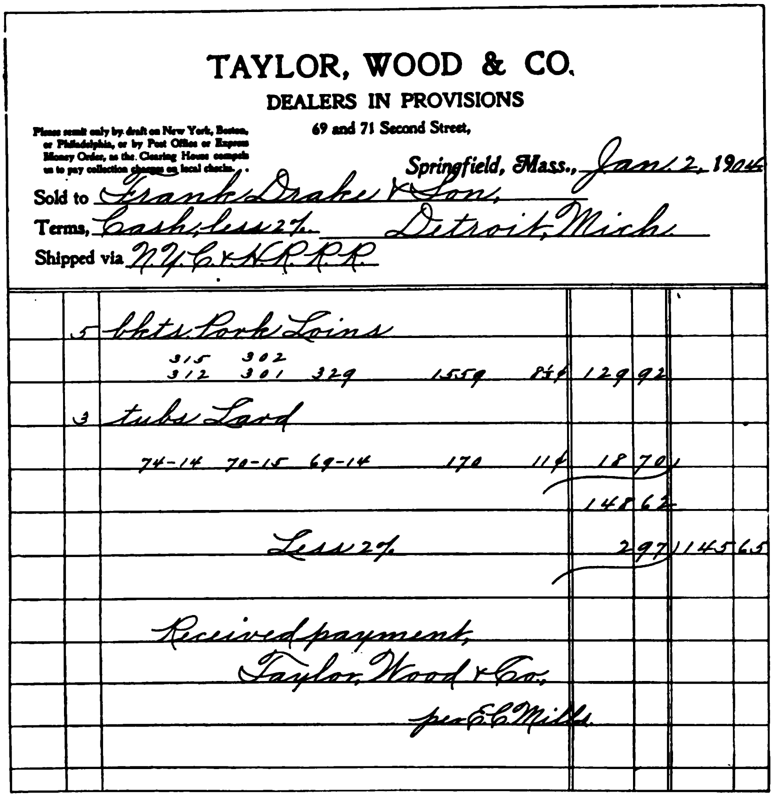
Model Receipted Bill (Groceries)

In the above model the prices given are free on board cars New York city, and the shippers prepaid the freight charges to Boston, Mass. In all such cases the freight is part of the selling price, and is usually added to the bill as shown in the model.

When goods are sold so that all transportation charges fall upon the buyers, the cost of cartage is also added to the amount of the bill. In certain lines of business a charge is also made for the crates used in packing. The above model shows the proper arrangement for all such additions.

Had the above bill of goods been sold free on board cars Boston, Mass., and had the shippers not prepaid the freight charges, these charges would be deducted by the consignees from the amount of the bill, on the arrival of the goods. The freight bill would then be sent to the shippers for credit.

Any conditions as to time of credit, manner of payment, or discount for prepayment should always be recorded on a bill.



TAYLOR, WOOD & CO.

DEALERS IN PROVISIONS

69 and 71 Second Street,

Please remit only by draft on New York, Boston, or Philadelphia, or by Post Office or Express Money Order, as the Clearing House compels us to pay collection charges on local checks.

Springfield, Mass., Jan. 2, 1904

Sold to Frank Drake & Son,

Terms, Cash, less 2% Detroit, Mich.

Shipped via N. Y. C. & H. R. R. R.

5	bbls. Pork Loins				
	315 302 312 301 329 1559 8⅓	129	92		
3	tubs Lard				
	74-14 70-15 69-14 170 11	18	70		
		148	62		
	Less 2%	2	97	145	65

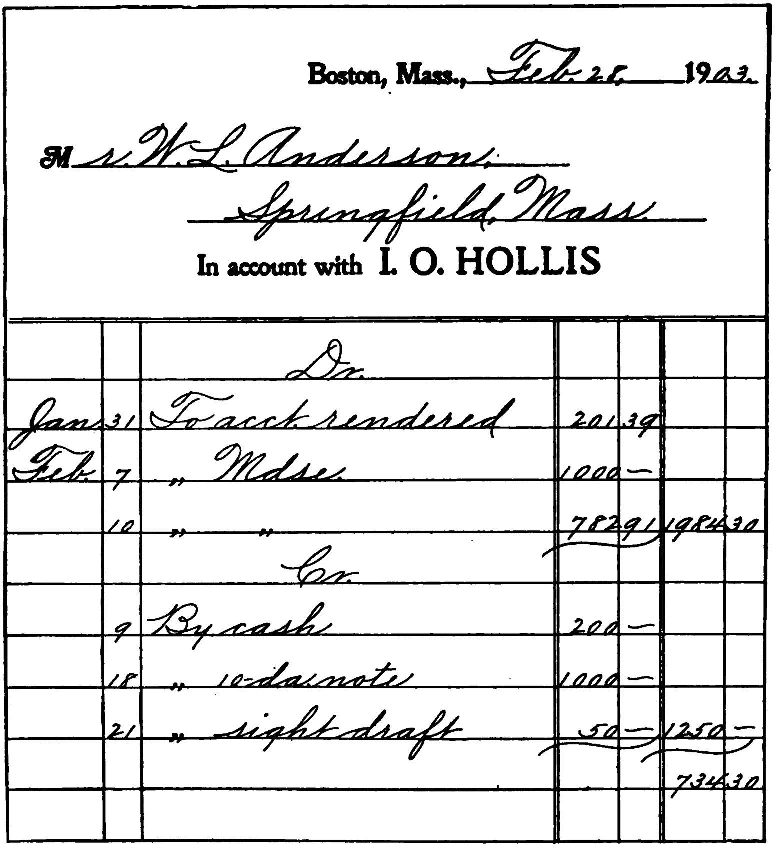
Received payment,
Taylor, Wood & Co.
per E. C. Mills

Model Receipted Bill (Provisions)

The second item in the above model shows how gross weights and tares are recorded in billing. The numbers to the left of the hyphen are the gross weights, and the numbers to the right of the hyphen, the tares. Thus, 74-14 is understood to mean that the gross weight of a tub is 74 lb. and the tare 14 lb. The 170 is the net weight of the three tubs. In finding the net weight the various items should not be copied to another sheet, but should be added horizontally as explained in **62**.

Bills on which commercial discounts are allowed should always be arranged as shown in the above model. Commercial discounts are fully explained on pages 198–206.

In retail business, where running accounts are kept with customers, a transcript of the charges, or of the charges and credits, is made, giving items and dates of purchases and payments. This transcript partakes of the nature of both statement and bill and is called an *account current.*



Boston, Mass., Feb. 28 1902.

Mr. W. L. Anderson,
Springfield, Mass.

In account with I. O. HOLLIS

		Dr.				
Jan.	31	To acct. rendered	201	39		
Feb.	7	" Mdse.	1000	—		
	10	" "	782	91	1984	30
		Cr.				
	9	By cash	200	—		
	18	" 10-da. note	1000	—		
	21	" sight draft	50	—	1250	—
					734	30

Model Statement

The above statement is an abstract of W. L. Anderson's account. On Jan. 31 a statement was rendered showing a debit balance of $201.39. This amount is taken as a basis for the February statement, and to it are added the debit items of the ledger since Jan. 31, making a total of $1984.30. From this total is deducted the sum of the credits in the ledger since Jan. 31, or $1250, leaving a debit balance of $734.30. This $734.30 will be the first item entered upon the March statement.

345. Statements are usually rendered the end of each month. By an exchange of statements errors are less likely to occur, and when made, are more readily detected.

346. Wages are usually calculated on the basis of 8 or 10 hours to a day. In finding the amount due, in order to avoid fractions, it is best to find the total time in hours, multiply by the rate per day and then divide by 8 or 10, carrying decimals to three places.

PAY ROLL. Week ending Aug. 6, 1904

No.	Names of Employees	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Totals	Rate	Amount	Remarks
	Allen, Wm. L.	8½	10	10	10	11	8	57½	2.50	17.97	
	Brown, Martin	10	10	7½	8	5	10	50½	2.75	17.36	
	Eaton, J. D.	10	4	10	8	12½	8	52½	2.60	17.06	
	Parker, Chas.	10	10½	9	11	12	12	64½	2.25	18.14	
	Reade, Chas.	9½	8	10	10	12	10	59½	1.75	13.02	
	Rogers, John	10	10	4½	9	10	10	53½	2.25	15.05	
	Watson, Jas.	10	12½	10	12½	10	12½	67½	2.75	23.20	
										121.80	

Model Pay Roll

Checks are sometimes used in paying off employees, but more generally the envelope system is used and each employee is paid in currency the amount due him. To pay off the employees in this manner the bookkeeper usually draws from the bank the *exact* amount of money and just the denominations wanted. To do this with absolute accuracy it is generally necessary for him to classify and record the denominations required for the payment of the amount due each name on the pay roll in a manner similar to the following :

BILLS					FRACTIONAL CURRENCY				
$20	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
	1	1	1		1	1	2		2
	1	1	1			1	1		1
	1	1	1					1	1
	1	1	1	1			1		4
	1		1	1					2
	1	1						1	
1			1	1			2		
1	6	5	6	3	1	2	6	2	10

Second National Bank
CHELSEA, MASS.

Pay Roll Memorandum

E. W. FOWLE & CO.
require the following:

Pennies	10		10
Nickels	2		10
Dimes.	6		60
Quarters	2		50
Halves	1		50
Dollars	3	3	
2's	6	12	
5's	5	25	
10's	6	60	
20's	1	20	
		121	80

The foregoing model shows what should be done to ascertain the denominations required to pay off the amount shown in the model pay roll, page 127. Many times columns are provided for this memoranda on the right-hand side of the pay roll, immediately after the "Remarks" column.

After the amount of the pay roll and the denominations required have been ascertained, they are written on the pay roll memorandum as shown in the accompanying model. This memorandum is attached to a check, payable to "Pay Roll," which is taken to the bank and cashed.

WRITTEN EXERCISE

Find the amount of each of the following bills:

1

SAGINAW, MICH., Sept. 1, 1904.

Messrs. SAGE BROS. & Co.,
Tonawanda, N.Y.

Bought of WESTON & BROWN

TERMS: Sight draft without notice after 30 da.

		26416 ft. Clear Pine	25^{00} per M				
		146250 ft. Pine Plank	12^{50} per M				
		11670 Cedar Posts	10^{00} per C				
		81275 ft. Clapboards	25^{00} per M				
		71000 Shingles	4^{10} per M				
		56200 ft. Pine Lumber	25^{00} per M				
		111224 Cedar R. R. Ties	$33^{33\frac{1}{3}}$ per C				
		31000 ft. Pine Boards	$16^{66\frac{2}{3}}$ per M				

2

WORCESTER, MASS., May 15, 1904.

Messrs. R. E. BARNES & Co.,
Detroit, Mich.

Bought of OSGOOD, TOWER & Co.

TERMS : Cash

Case			No. of Yd.	Price	Items	Amount
#19	15	pcs. Bleached Cotton				
		41^2 46^3 41^1 45^2 44 44^1				
		47^1 45^3 42 42^3 43^1 43^3				
		47 44 44^2		6¼¢		
#5	12	pcs. Muslin				
		37^1 32^3 33 35^3 34^1 32				
		35^2 33^3 37 38^1 38^1 36		10¢		
#31	9	pcs. Delaine				
		39 40^2 41^1 39^3 38^2 40				
		42^3 44^1 42		16⅔¢		
#7	24	pcs. Windsor Prints				
		21^3 27^3 25^3 28 26 22^2				
		24 25 32 31^2 28 24^1				
		25 27^2 22 28^1 24^1 22				
		26 24 31^2 32 22 21^2		6¼¢		
#21	21	pcs. Merrimac Prints				
		28^1 32 34^3 28^2 26 24^1				
		22^2 24^2 26^2 24 26^1 33				
		28^2 34 27^1 30 32^3 24				
		30^2 31 30^2		6⅔¢		
#169	20	pcs. Simpson Mourning Prints				
		40 38 34^2 40^1 32 40				
		41^2 40 16^2 40 29^3 30				
		27^2 19^2 41^1 38^2 30 43				
		42 41^3		6⅔¢		
#173	15	pcs. Striped Denim				
		40 42 41 40 38 41				
		43^1 44^2 45^1 40^3 46 38				
		40^1 38^3 40^1		8⅓¢		

3

BOSTON, MASS., Jan. 3, 1904.

Messrs. MARTIN & WARREN,
Milwaukee, Wis.

Bought of HARRIS BROS. & Co.

TERMS: Interest after 60 da.

	8	pcs. F. A. Cambric					
		56 52^1 45^3 50^2 52 40^1 50 51^1	22¢				
	5	gro. Jet Buttons	$1^{12\frac{1}{2}}$				
	8	pcs. P. D. Goods					
		35 45^3 55^2 50^3 51 52 46^1 50	25¢				
	4	pcs. G. Flannel					
		35^3 40 40^2 40^3	$33\frac{1}{3}$¢				
	8	pcs. V. Barege					
		20^1 25 24^2 27 26^3 22 24^2 22	$16\frac{2}{3}$¢				
	6	pcs. E. Lining					
		40 52^2 54 55^1 45^2 50^2	$8\frac{1}{3}$¢				
	3	pcs. B. Silk					
		58 58 56	98¢				

4

CLEVELAND, OHIO, Oct. 15, 1904.

Messrs. BROWN, HORTON & Co.,
Springfield, Mass.

Bought of ROBINSON, CAREY & Co.

TERMS: Net cash.

	10	bbl. Pork	$16^{66\frac{2}{3}}$				
	5	bbl. Mess Beef	11^{25}				
	3	bbl. Hams					
		275–56 281–60 287–62	$12\frac{1}{2}$¢				
	3	bbl. Shoulders					
		248–37 252–42 371–40	$8\frac{1}{3}$¢				
	3	tubs Lard					
		71–14 70–15 69–14	11¢				
	6	bkt. Pork Loins					
		314 301 294 312 302 315	$8\frac{1}{3}$¢				

WRITTEN REVIEW

Find the amount of each of the following invoices:

1. Thurston & Denton, Buffalo, N.Y., bought of Brown Bros. & Co., Boston, Mass., Jan. 27, the following: 8 pcs. M. shirting, 40^2 41^1 46^2 51^2 45^1 50^3 43 34^1, at 6⅜¢; 15 pcs. crash, 613 yd., at 6¼¢; 6 pcs. C. jeans, 50^2 45 50 55^1 51^1 46^2, at 5¢; 25 doz. M. L. thread at 59¢; 10 pcs. R. print, 41 55^2 45^1 51 46 50^3 40 56^2 42^1 52^2, at 6¼¢; 4 pcs. N. sateen, 55^1 55^2 60^3 50^2, at 6⅜¢; 5 gro. F. braid at $7.62½; 16 doz. L. shirts at $7.25; 6 pcs. T. R. prints, 25^1 35^3 30^2 31 21^1 25^1, at 4¾¢; 25 cases E. batts at $6; 20 gro. S. P. buttons at 49¢.

2. I. F. Hoyt, Milwaukee, Wis., bought of Mann & Co., of the same city, Sept. 4: 10 pcs. N. sateen, 55^2 51 50^3 54^1 56 55 52^2 53 51^3 50, at 5½¢; 15 pcs. T. A flannel, 62^3 65^1 61 58^2 55 63^1 65^3 62 60^2 63 56^3 60^1 58 62^2 65^1, at 33⅓¢; 20 pcs. R. gingham, 50 52^1 51 51^2 55 60^3 62^1 61^2 58 55^2 56^1 53^3 51 55^3 61^2 61 58^1 56 54^2 51^1, at 6¼¢; 10 pcs. B. checks, 45 52^1 41^2 40 55^3 50^2 45 51^1 42 50^3, at 25¢.

3. Brown Bros. & Co., Malden, Mass., bought of W. D. Adams & Co., Boston, Mass., June 18: 20 pcs. L. gingham, 58^2 46^1 41^3 38^1 46^2 45^3 51^2 55 38^2 35 37^3 49^3 40^2 51^3 44 44^2 40 37^1 33^3 46^2, at 8½¢; 24 pcs. W. print, 44^1 46^3 51^2 39^3 41^2 45 48^3 51 34^3 37^2 35 36^2 41^3 34^3 49^1 37^2 34 36^2 42^3 48 43^2 53^1 38^1 42, at 5½¢; 20 pcs. E. lining, 45 54^1 39^2 48^3 46^2 38^2 47^2 37^2 45^3 46^3 42^3 44^3 45^3 43^1 35^2 54^2 34^3 42^2 53^2 44^1, at 4¼¢.

4. Jan. 21, 1904, J. H. Palmer, Sons & Co., sold Morrison, Price & Long the goods shown below; terms, 2% 10 da., 1% 30 da., net 60 da. 10 doz. knives and forks: 3 doz. #5 at $8.33⅓, 3 doz. #7 at $6.66⅔, 4 doz. #9 at $10; 9 doz. razors: 3 doz. #12 at $9, 3 doz. #13 at $12.50, 3 doz. #18 at $16.66⅔; 12 doz. panel saws: 6 doz. #1 at $15, 4 doz. #4 at $21, 2 doz. #5 at $25; 4 doz. nutmeg graters: 2 doz. #1 at $2.25, 2 doz. #4 at $1.75; 6 doz. pocket knives: 2 doz. #12 at $6, 2 doz. #16 at $7.50, 2 doz. #20 at $3.75; 5 doz. burnished teapots: 2 doz. #1 at $6.75, 3 doz. #2 at $7.25; 35 doz. dippers: 18 doz. #3 at $1.75, 10 doz. #6 at $1.35, 7 doz. #4 at $1.25; 2¾ doz. wash boilers at $37.75; ¾ doz. #2 kettles at $5.87½; 27 tea kettles at 97¢; 8½ doz. padlocks at $8.75; ¾ doz. 3-qt. saucepans at $9.37½; 1½ doz. 2-qt. saucepans at $7.85; 2½ doz. #44 dishpans at $2.47; 3¾ doz. #14 cups at 78¾¢; 59 faucets at $1.47; 2½ doz. carpet stretchers at $2.95; 6¾ doz. wrought wrenches at $12.75; 5½ doz. cast steel axes at $12.50.

5. W. C. Blanchard, Hartford, Conn., bought of M. C. Woods, Utica, N.Y., July 15: 10 pcs. R. gingham, 60 61^{2} 50^{1} 60^{3} 51 61^{3} 61^{1} 50 55 51^{3}, at 8¢; 10 doz. F. E. braid, at 23¢; 10 pcs. B. checks, 45 41 55^{1} 42 52 40^{2} 50 55 51^{3} 45^{2}, at 24¢; 15 gro. G. buttons, at $1.12½; 2 pcs. T. A. flannel, 65 60, at 30¢; 6 pcs. E. lining, 40 55^{1} 45^{2} 52 41 50^{1}, at 5¢; 5 doz. L. L. gloves, at $3.05; 4 pcs. M. sateen, 55^{3} 55 50 60^{3}, at 5½¢; 5 gro. T. braid, at $7.62½; 3 doz. L. shirts, at $7.20; 6 pcs. T. R. print, 25 35 30^{3} 31 21 25^{1}, at 4¾¢; 10 cases E. batts, at $6; 20 gro. S. P. buttons, at 49¢; 4 pcs. V. barege, 20 23 25 25, at 16⅔¢; 7 pcs. W. print, 45^{3} 51 45 50 46^{2} 55 50^{3}, at 5¼¢.

6. Find the amount of Baker, Taylor & Co.'s inventory, Jan. 1, with items as follows: 8 pcs. F. A. cambric, 56 52 45 50 52 54 46 50, at 22¢; 5 gro. J. buttons, at $1.12½; 15 pcs. P. D. goods, 55 45^{3} 55^{2} 50^{3} 51 52 46^{1} 50 52^{1} 54 48^{2} 50^{3} 52 55^{1} 50, at 50¢; 4 pcs. G. flannel, 35^{3} 40 40^{2} 40^{3}, at 25¢; 6 pcs. E. lining, 40 52^{2} 54 55^{1} 45^{2} 50^{2}, at 3½¢; 10 pcs. V. barege, 20^{1} 25 23^{2} 27 26^{3} 22 24^{2} 22 26^{3} 28, at 16⅔¢; 10 pcs. B. H. checks, 45 52 55 41 40^{2} 51^{3} 51^{1} 53 50^{2} 46, at 24¢; 5 pcs. W. prints, 25^{2} 31^{3} 30 28^{2} 27, at 5¼¢; 15 pcs. A. F. cashmere, 62^{1} 65^{3} 60^{1} 63 58^{3} 60^{2} 56^{2} 58^{2} 60 62^{2} 55^{3} 58^{1} 60^{3} 58 55^{1}, at 19¢; 20 pcs. L. gingham, 45 48^{1} 46^{1} 44^{2} 45^{3} 44^{3} 46 44 48 46 42 50^{2} 51^{3} 46^{2} 47^{1} 46^{1} 48 49 45^{1} 48, at 8½¢.

Render the following statements:

7. On Feb. 28, 1904, the debits and credits of Mason & Hamlin's account with Lord & Taylor, Boston, Mass., were as follows: Debits: Jan. 1, To merchandise, $900.62; Jan. 27, To merchandise, $200.56; Feb. 18, To merchandise, $260.93. Credits: Feb. 1, By cash, $175; Feb. 15, By Cash, $200.

8. On May 31 the debits and credits of Burke, Fitzsimmons & Hone's account with C. D. Gray, Rochester, N.Y., were as follows: Debits: April 15, To merchandise, $900.46; April 30, To merchandise, $340.92; May 15, To merchandise, $135.40. Credits: April 30, By merchandise returned, $35.40; May 15, By cash, $300.90; May 20, By cash, $600.

9. The amount of the model pay roll, page 127, was determined on the basis of an 8-hour day. Find the amount on the basis of a 10-hour day.

10. Find the amount of the following pay roll (*a*) on the basis of an 8-hour day; (*b*) on the basis of a 10-hour day:

NAME	MON.	TUES.	WED.	THUR.	FRI.	SAT.	RATE
Adams, Henry	7	8	10	9	8	12	$2.00
Carter, James	8	8	8	8	9	11	3.00
Coxe, Robert	5	8	5	1	10	9	3.50
Drowne, William	9	7	9	8	8	8	3.00
Harris, John	8	8	8	7	8	9	2.50
Keyser, Frederick	8	7	7	8	9	8	1.75
Martin, Charles	9	10	8	7	9	8	3.00
Smith, Martin	7	8	8	9	9	5	3.00
Warren, William	9	9	9	9	9	10	3,50
Weeks, Thomas	8	9	10	9	8	9	2.50

11. Find the amount of the following pay roll (*a*) on the basis of an 8-hour day; (*b*) on the basis of a 10-hour day:

NAME	MON.	TUES.	WED.	THUR.	FRI.	SAT.	RATE
Breen, Mildred	9	9	8	10	11	9	$3.50
Carey, Alice	5	8	8	9	9	8	2.75
Carret, Ellen	7	7	8	9	9	9	3.25
Cutter, James	11	9	8	8	8	9	4.10
Ernst, Harry	9	9	7	6	7	9	4.00
Foley, Maude	9	5	8	8	7	10	3.25
Gordon, Ruth	8	7	8	8	8	8	2.75
Healey, Grace	7	8	6	8	6	5	1.75
Laddy, Mary	8	8	7	9	9	9	4.00
Lang, James	8	8	9	10	8	11	4.50
Penny, George A.	7	6	8	8	8	8	4.50
Pratt, Helen	8	9	9	7	7	8	3.50
Schiller, Helen	7	9	8	8	7	7	6.00
Smith, Frank	8	7	8	8	9	8	4.20
Tuckerman, Leo	8	5	9	8	7	9	5.10
Walker, Florence	8	9	9	9	8	9	5.50

12. Make pay roll memorandums for (*a*) and (*b*) in problem 10; for (*a*) and (*b*) in problem 11. Assume that you are to draw the money from City National Bank.

DENOMINATE NUMBERS

MEASURES

347. Concrete numbers in which the unit has been established by law or custom are called **denominate numbers.** Numbers expressed in units of the same denomination are **simple numbers.** Simple numbers having denominate units are **simple denominate numbers.** Numbers expressed in units of two or more denominations are **compound numbers,** or **compound denominate numbers.**

348. A **measure** is a standard unit by which quantity is estimated.

Quantity may be length, breadth, thickness, area, volume, capacity, weight, value, time, number, or amount.

349. The principal measures are those of *Weight, Extension, Time, Capacity, Value,* and *Angles.*

350. A **standard unit of measure** is a unit which has been established by law or custom as the one by which other units are to be adjusted.

The *Winchester bushel* has been adopted by the United States as the standard unit for dry quantities, such as grain, seeds, etc.; the gold dollar has been established as the standard unit of money value; etc.

351. A quantity is measured by finding how many times it contains any standard unit of measure.

The unit of extent is the *yard;* of weight, the *Troy pound;* etc. The number of yards in a piece of cloth may be ascertained by applying the *yard* measure; the weight of a body, by the use of the *pound;* etc.

MEASURES OF WEIGHT

352. **Weight** is a quantity of matter expressed numerically with reference to some standard unit.

353. The **standard unit of weight** in the United States is the *Troy pound.*

354. There are three kinds of weight in use: *Troy Weight, Avoirdupois Weight,* and *Apothecaries' Weight*

Troy Weight

355. Troy weight is used for weighing gold, silver, and jewels.

TABLE

24 grains (gr.) = 1 pennyweight (pwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

$$\begin{array}{ccccccc} \text{lb.} & & \text{oz.} & & \text{pwt.} & & \text{gr.} \\ 1 & = & 12 & = & 240 & = & 5760. \end{array}$$

The grains of the Troy, Avoirdupois, and Apothecaries' weights are the same.

The Troy pound is equal to 22.7944 cubic inches of pure water at its greatest density, and is identical with the Troy pound of Great Britain.

In weighing diamonds and gems the unit generally employed is the *carat*, which is about 3.2 Troy grains.

The term *carat* is also used to express the fineness of gold, 24 carats being pure. Thus, a carat means $\frac{1}{24}$ part, and gold 18 carats fine contains 18 parts gold, or is $\frac{3}{4}$ pure.

Avoirdupois Weight

356. Avoirdupois weight is used for weighing all heavy articles, such as groceries, coal, provisions, grain, and the metals, except gold and silver.

357. The unit of Avoirdupois weight is the *pound*, which contains 7000 grains.

TABLE

16 ounces (oz.) = 1 pound (lb.).
100 pounds = 1 hundredweight (cwt.).
20 hundredweight, or 2000 pounds = 1 ton (t.).

$$\begin{array}{ccccccc} \text{t.} & & \text{cwt.} & & \text{lb.} & & \text{oz.} \\ 1 & = & 20 & = & 2000 & = & 32{,}000. \end{array}$$

"Hundredweight" and "pounds" may be read together as pounds, or pounds may be read as so many hundredths of a hundredweight. Thus, 17 hundredweight, 29 pounds, may be read "1729 pounds," or "17.29 hundredweight"; and 2 tons, 7 hundredweight, 31 pounds, may be read "2 tons, 7.31 hundredweight."

366. The **standard unit of extension** in the United States is the *yard*.

The *standard yard* prescribed at Washington has been fixed with the greatest precision. It is determined by a brass rod or pendulum, which vibrates seconds in a vacuum at the sea level at 62° Fahrenheit, in the latitude of London, Eng. This pendulum is divided into 391,393 equal parts, and 360,000 of these parts constitute a yard. A copy of the standard, which is identical with the present standard of Great Britain, is kept in each State capitol.

Long Measure

367. **Long measure** is used in measuring lengths and distances.

TABLE

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards or 16½ feet = 1 rod (rd.).
320 rods or 5280 feet = 1 mile (mi.).

mi. rd. yd. ft. in.
1 = 320 = 1760 = 5280 = 63,360.

The terms *pole* and *perch* are sometimes used instead of *rod*.

Formerly the mile was divided into 8 furlongs of 40 rods each. The furlong is now practically obsolete.

The *hand*, used in measuring horses, is equal to four inches.

5280 feet is the legal mile in the United States and England, and hence is sometimes called the *statute mile*.

The *knot*, used in navigation, is equal to 1.152⅔ statute miles, or 6086 feet. It is sometimes called a *geographic mile*.

A *league* is equal to three knots or geographic miles.

A *pace* is equal to three feet, and five paces approximate a rod.

The *fathom*, used in measuring depths at sea, is equal to six feet.

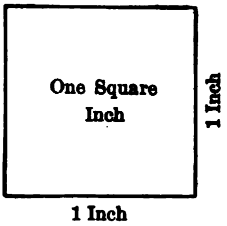
Square Measure

368. **Square measure** is used in computing surfaces such as land, floors, boards, walls, and roofs.

369. A **square** is a flat surface bounded by four equal sides and having four square corners.

370. The **unit** of square measure is a *square*, each side of which is bounded by a unit of length; as, a square inch, a square yard.

A square inch is a square, each side of which is one inch; a square foot is a square, each side of which is one foot; a square yard is a square, each side of which is one yard; a square rod is a square, each side of which is one rod.



371. The **area** of a figure is the number of square units contained in its surface.

TABLE

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards, or 272¼ square feet = 1 square rod (sq. rd.).
160 square rods, or 43,560 square feet = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).

sq. mi. A. sq. rd. sq. yd. sq. ft. sq. in.
1 = 640 = 102,400 = 3,097,600 = 27,878,400 = 4,014,489,600.

All the units of square measure except the acre are derived from the corresponding units of long measure. Thus, 144 (12 × 12) square inches = 1 square foot; 9 (3 × 3) square feet = 1 square yard; 30¼ (5½ × 5½) square yards = 1 square rod; 102,400 (320 × 320) square rods, or 640 acres, which are equal to 1 square mile.

Surveyors' Long Measure

372. **Surveyors' long measure** is used by surveyors in measuring land, laying out roads, establishing boundaries, etc.

373. The **unit** of surveyors' long measure is the *Gunter's chain*, which is 4 rods, or 66 feet, in length.

The chain has 100 links, which may be written as hundredths of a chain. Thus, 4 chains, 27 links = 4.27 chains.

TABLE

7.92 inches = 1 link (l.).
25 links = 1 rod.
4 rods, or 100 links = 1 chain (ch.).
80 chains = 1 mile (mi.).

mi. ch. rd. l. in.
1 = 80 = 320 = 8000 = 63,360.

Surveyors' Square Measure

374. **Surveyors' square measure** is used by surveyors in measuring land by acres and sections. It is sometimes called *land measure.*

375. The **unit of land measure** is the *acre.*

Table

625 square links (sq. li.)	= 1 square rod (sq. rd.).
16 square rods	= 1 square chain (sq. ch.).
10 square chains, or 160 square rods	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi).

In some parts of the country 36 square miles, or 6 miles square, is a *township.* A square mile, or 640 acres, is also called a section in surveying public lands.

sq. mi.		A.		sq. ch.		sq. rd.		sq. l.
1	=	640	=	6400	=	10,2400	=	64,000,000.

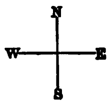
376. **United States public lands** are surveyed by selecting a north and south line as a *principal meridian,* and an east and west line intersecting this as a *base line.* From these, other lines are run at right angles, six miles apart, thus dividing the territory into *townships* six miles square.

377. The rows of townships running north and south are called *ranges.* The townships in each range are numbered north and south of the base line, and the ranges are numbered east and west from the principal meridian.

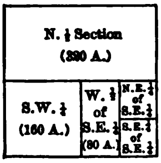
378. Each township is divided into 36 squares of 1 square mile each. These squares are called *sections,* and are divided into halves and quarters; each quarter section is in turn divided into halves and quarters.

A TOWNSHIP

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36



A SECTION



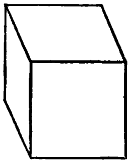
379. The numbering of the sections in every township is as in the accompanying diagram. The corners of all sections are permanently marked by monuments of stone or wood, and a description of the monument and its location is made in the field notes of the surveyor.

Cubic Measure

380. **Cubic measure** is used in determining the contents or volume of solids.

381. A **cube** is a regular solid bounded by six equal square sides or faces. Its length, breadth, and thickness are, therefore, equal.

382. The **unit** of cubic measure is a cube, each side of which is bounded by a unit of length; as, a cubic inch, a cubic yard.



Cubic Foot.

A cubic inch is a cube, each side of which is one inch; a cubic foot is a cube, each side of which is one foot; a cubic yard is a cube, each side of which is one yard.

383. The **contents** or **volume** of a cubical body is the number of cubic units it contains.

Table

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

cu. yd. cu. ft. cu. in.
1 = 27 = 46,656.

The units of cubic measure are derived from the corresponding units of long measure. Thus 1728 (12 × 12 × 12) cubic inches = 1 cubic foot; 27 (3 × 3 × 3) cubic feet = 1 cubic yard.

A cubic foot of water contains 6¼ gallons and weighs 1000 ounces, or 62½ Avoirdupois pounds. Hence, a gallon of water weighs 10 pounds.

Wood Table

16 cubic feet = 1 cord foot (cd. ft.).
8 cord feet, or 128 cubic feet = 1 cord (cd.).

A cord of wood is a pile 8 feet long, 4 feet high, and 4 feet thick.

A perch of masonry is 16½ feet long, 1½ feet wide, and 1 foot high, and contains 24¾ cubic feet.

A cubic yard of earth is called a *load*.

MEASURES OF CAPACITY

384. **Capacity** signifies room for things.

385. There are two measures of capacity in general use — *dry measure* and *liquid measure*.

Dry Measure

386. **Dry measure** is used in measuring grain, fruit, vegetables, coal, etc.

387. The **unit** of dry measure is the *Winchester bushel,* which is 8 inches deep, 18½ inches in diameter, and contains 2150.42 cubic inches.

This is the standard unit in uniform use for measuring shelled grains. The *heaped bushel* of 2747.71 cubic inches is used for measuring apples, roots, corn in the ear, etc.

The *British imperial bushel* contains 2218.19 cubic inches.

TABLE

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

bu.	pk.	qt.	pt.
1 =	4 =	32 =	64.

The *dry gallon,* or half peck, contains 268.8 cubic inches.

Liquid Measure

388. **Liquid measure** is used in measuring liquids and in estimating the capacity of cisterns, reservoirs, etc.

389. The **unit** of liquid measure is the *gallon,* which contains 231 cubic inches.

TABLE

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
31½ gallons = 1 barrel (bbl.).

gal.	qt.	pt.	gi.
1 =	4 =	8 =	32.

Casks, called hogsheads, pipes, butts, etc., are not fixed measures, their capacity varying for commercial purposes.

In the sale of oils and liquors, and in certain other cases, the barrel is also an indefinite quantity.

Apothecaries' Fluid Measure

390. **Apothecaries' fluid measure** is used in measuring the liquids used in compounding medical prescriptions.

TABLE

60 minims (♏) = 1 fluid drachm (f. ʒ).
8 fluid drachms = 1 fluid ounce (f. ℥).
16 fluid ounces = 1 pint (O).

COMPARATIVE TABLE OF MEASURES

MEASURE	CU. IN. IN ONE GAL.	CU. IN. IN ONE QT.	CU. IN. IN ONE PT.	CU. IN. IN ONE GI.
Dry	($\frac{1}{2}$ pk.) $268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$
Liquid	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$

MEASURES OF TIME

391. **Time** is a measure of duration. Its computations being based upon planetary movements are the same in all lands and among all peoples.

392. The **unit of time** is the ***solar day;*** it includes one revolution of the earth on its axis, and is divided into 24 hours, counting from midnight to midnight again.

393. A **solar year** is the exact time required by the earth to make one complete rotation around the sun, — 365 days, 5 hours, 48 minutes, 49.7 seconds, or about 365¼ days.

394. The solar year is divided in the calendar into 365 days called a *common year,* except every fourth year, when one day is added to the month of February and the year is called a *leap year.* Since the fraction that is disregarded when 365 days is counted as a year is less than one fourth of a day, the addition of a day every fourth year is *not* exactly accurate. The slight error still existing is corrected by excluding from the leap years the centennial years which are not divisible by 400. Hence, to find whether any year is a leap year or not,

Divide the number of centennial years by 400 and all other years by 4; if there is no remainder, the year is a leap year.

Table

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
365 days = 1 common year (yr.).
366 days = 1 leap year.
100 years = 1 century (C.).

$$\overset{\text{yr.}}{1} = \overset{\text{mo.}}{12} = \begin{cases} \overset{\text{da.}}{365} = \overset{\text{hr.}}{8760} = \overset{\text{min.}}{525{,}600} = \overset{\text{sec.}}{31{,}536{,}000}. \\ 366 = 8784 = 527{,}040 = 31{,}622{,}400. \end{cases}$$

Commercial Table

30 days = 1 month (mo.).
12 months = 1 year (yr.).

The 12 months into which we divide the year are called ***calendar months***. They are of variable length, seven of them containing 31 days, four 30 days, and February 28 days, except in leap years, when it has 29 days.

The calendar months, with the number of days they contain, are shown below:

1.	January (Jan.)	31 da.	7.	July	31 da.
2.	February (Feb.)	28–9 da.	8.	August (Aug.)	31 da.
3.	March (Mar.)	31 da.	9.	September (Sept.)	30 da.
4.	April (Apr.)	30 da.	10.	October (Oct.)	31 da.
5.	May	31 da.	11.	November (Nov.)	30 da.
6.	June	30 da.	12.	December (Dec.)	31 da.

Standard Time. In 1883 the principal railroads of the United States and Canada adopted what is known as the "Standard Time System." This system divides the United States and Canada into four sections or time-belts, each covering 15° of longitude, 7½° of which are east and 7½° west of the governing or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt. The governing meridians are the 75th, the 90th, the 105th and the 120th west of Greenwich, and as these meridians are just 15° apart, there is a difference in time of *exactly* one hour between any one of them and the one next on the east, or the one next on the west; the standard meridian next on the east being one hour faster, and the one next on the west one hour slower. The time of the 75th meridian is called **Eastern Time**. The time of the 90th meridian is known as **Central Time**.

The time of the 105th meridian is known as **Mountain Time.** Time in the fourth belt, which is governed by the 120th meridian, and extends to the Pacific coast, is called **Western** or **Pacific Time.** The changes from one time standard to another are made at the termini of roads, or at well-known points of departure, and where they are attended with the least inconvenience and danger. This system has been adopted by most of the principal cities for local use.

MEASURES OF VALUE

395. **Value** is the worth of one thing as compared with another. The general measure of value is *money.*

United States Money

396. **United States money** has been fully treated on pages 58 to 64 inclusive.

Canadian Money

397. **Canadian money** is the legal currency of the Dominion of Canada; it consists of gold, silver, and bronze coins, and paper money.

TABLE

10 mills (m.) = 1 cent (¢).
100 cents = 1 dollar ($).

The mill is not coined. The *gold coins* of Canada are the British sovereign and half-sovereign; the *silver coins* are the 5, 10, 25, and 50-cent pieces; the only *bronze coin* is the cent. The Canadian silver coins are $\frac{37}{40}$ pure metal and $\frac{3}{40}$ copper.

English Money

398. **English** or **sterling money** is the legal currency of Great Britain; it consists of gold, silver, copper, and bills.

399. The **unit** of English money is the *pound sterling,* the value of which in United States money is $4.8665.

TABLE

4 farthings (far.) = 1 penny (*d.*).
12 pence = 1 shilling (*s.*).

$$20 \text{ shillings} = \left\{\begin{matrix}1 \text{ sovereign}\\ 1 \text{ pound}\end{matrix}\right\} (£).$$

£ *s.* *d.* far.
1 = 20 = 240 = 960.

d., *s.*, £, are the initial letters of the Latin words *denarius*, *solidarius*, *libra*, signifying respectively, penny, shilling, and pound.

400. The value in United States money of the different denominations of English money is shown in the following

COMPARATIVE TABLE

1 farthing = $\frac{73}{144}$ cent. 1 shilling = $24\frac{1}{3}$ cents.
1 penny = $2\frac{1}{36}$ cents. 1 pound = $4.8665.

The farthing is but little used except as a fractional part of the penny.

The British gold coins are $\frac{11}{12}$ pure gold and $\frac{1}{12}$ alloy; the silver coins, $\frac{37}{40}$ pure silver and $\frac{3}{40}$ copper; the penny and half-penny pieces are pure copper.

The *gold coins* are the sovereign and the half-sovereign; the *silver coins* are the crown (equal to 5 shillings), the half-crown, the florin (equal to 2 shillings), the shilling, the six-penny piece, the four-penny piece, and the three-penny piece; the *copper coins* are the penny, the half-penny, and the farthing. The guinea (equal to 21 shillings), and the half-guinea are no longer coined.

French Money

401. French money is the legal currency of France; it is a decimal currency, and consists of gold, silver, and bronze coins and national bank notes.

402. The unit of French money is the ***franc***, which is equal to 19.3 cents in United States money.

The franc is also used in Belgium and Switzerland, and under different names in several other countries.

TABLE

10 millimes (m.) = 1 centime (c.).
10 centimes = 1 decime (dc.).
10 decimes = 1 franc (fr.).

fr.		dc.		c.		m.
1	=	10	=	100	=	1000.

403. The value in United States money of the different denominations of French money is shown in the following

COMPARATIVE TABLE

1 centime = $.00193. 1 decime = $.0193. 1 franc = $.193.

The millime is not a coin. The *gold coins* of France are the 5, 10, 50, and 100 franc pieces, which are $\frac{9}{10}$ pure gold and $\frac{1}{10}$ alloy; the *silver coins* are the 1, 2, and 5 franc pieces; also the 25 and 50 centime pieces; they are $\frac{9}{10}$ pure and $\frac{1}{10}$ alloy. The *bronze coins* are the 1, 2, 5, and 10 centime pieces.

French money is read as francs and centimes in the same manner as United States money is read dollars and cents.

German Money

404. **German money** is the legal currency of the German Empire; it consists of gold, silver, and nickel coins, and paper money.

405. The **unit** of German money is the *mark*, which is equal to 23.85 cents in United States money.

TABLE

100 pfennige (Pf.) = 1 mark (Rm.).

The *gold coins* of the German Empire are the 5, 10, and 20 mark pieces; the *silver coins* are the 1 and 2 mark pieces and the 20 and 50 pfennig pieces; the *nickel coins* are the 5 and 10 pfennig pieces; the *copper coins* are the 1 and 2 pfennig pieces. The gold and silver coins are $\frac{9}{10}$ pure and $\frac{1}{10}$ alloy.

ANGULAR MEASURE

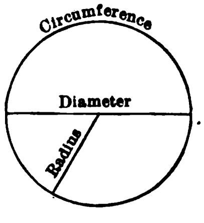
406. **Angular measure** is used in surveying, civil engineering, astronomical calculations, and navigation, for measuring angles, determining directions and location of places, latitude, longitude, difference in time, etc.

407. The **unit** of angular measure is the *degree*, which, in any circle, is measured by $\frac{1}{360}$ of the circumference.

408. A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *center*.

409. The **circumference** of a circle is the curved line bounding it.

410. The **diameter** of a circle is a straight line passing through the center and having its end in the circumference.



411. The **radius** of a circle is a straight line passing from the center to any point in the circumference.

412. Any part of the circumference of a circle is called an **arc**.

413. Every circle, great or small, is divisible into 4 parts called *quarters*, which are divisible into 90 equal parts called *degrees*. Every circle therefore may be divided into 360 degrees.

TABLE

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circle (Cir.).

Cir. ° ′ ″
1 = 360 = 21,600 = 1,296,000.

Minutes of the earth's circumference are called nautical or geographic miles; hence, a degree of the earth's surface at the equator contains 60 geographic miles, or 69⅙ statute miles.

MISCELLANEOUS MEASURES

ENUMERATION TABLE

12 units = 1 dozen (doz.).
12 dozen = 1 gross (gro.).
12 gross = 1 great gross (gt. gro.).

gt. gro. gro. doz. units.
1 = 12 = 144 = 1728.

Two units are often called a *pair*, and 20 units a *score*.

STATIONERS' TABLE

24 sheets (sht.) = 1 quire (qr.).
20 quires = 1 ream (rm.).
2 reams = 1 bundle (bdl.).
5 bundles = 1 bale (bl.).

bl. bdl. rm. qr. sht.
1 = 5 = 10 = 200 = 4800.

DRILL EXERCISE

1. Write two like concrete numbers; two unlike concrete numbers.

2. Write two simple denominate numbers; two compound denominate numbers.

3. Write two like denominate numbers; two unlike denominate numbers.

4. Which is the heavier, a Troy pound or an Apothecaries' pound? a Troy pound or an Avoirdupois pound? Illustrate.

5. Which is the heavier, a Troy ounce or an Avoirdupois ounce? Illustrate.

6. Write two numbers expressing the weight of a dry medical prescription; of gold; of hay; of vinegar; of wheat; of feathers.

7. Write a number expressing volume; surface; distance.

8. Write a number expressing Canadian money value; United States money value; French money value; German money value.

9. Write a number expressing the weight of a liquid medical prescription.

10. Write three articles that are sold by the enumeration table.

11. Write a number expressing a quantity of apples in storage in a certain warehouse; potatoes; onions; beef; fish.

12. Write a number expressing surveyors' square measure; angular measure; surveyors' long measure; cubic measure.

13. Write five numbers expressing area; three expressing capacity; two expressing time.

14. Write the next leap year; the next centennial year.

15. Write the standard unit of weight in the United States in grains.

16. Write the standard unit of English money; of United States money; of French money; of German money.

17. Write the standard unit of dry measure in cubic inches.

18. Write a dry gallon in cubic inches; a liquid gallon; a heaped bushel.

19. Write a degree of the earth's surface at the equator in statute miles; in geographic miles.

20. Write a perch of masonry in cubic inches; a cord of wood in cubic feet; a township in square miles.

21. Express in United States money the difference between a franc and a quarter of a dollar; the difference between a pound sterling and $10.

22. A man has 4 English sovereigns, 3 half-dollars, and 5 francs. Express the total sum in United States money.

23. **Express 5 reams of paper in sheets; 5 quires.**

24. **Express a statute mile in feet; in inches.**

25. **Express in Canadian money the cost of 11 gross of lead pencils at 30¢ a dozen.**

DENOMINATE QUANTITIES

REDUCTION OF DENOMINATE INTEGERS

414. In reduction the *unit* or *denomination* of a number changes, but not the *value*. When the change is from a higher to a lower denomination the process is called *reduction descending*, and when from a lower to a higher, *reduction ascending*.

DRILL EXERCISE

1. Find the cost of 90 ft. of cable at $1.20 per yard.

2. What will 5 sq. ft. of gold leaf cost at 2¢ per square inch?

3. How many inches in 3 yd. 2 in.?

4. How many five-cent pieces should be given for an eagle?

5. How many units in 2 gro.?

6. Find the cost of 2 bu. of apples at 10¢ a peck.

7. In $\frac{5}{80}$ of an acre how many square feet?

8. In $\frac{1}{64}$ sq. mi. how many square rods? in $\frac{1}{32}$ sq. mi.?

9. In 288 Avoirdupois pounds how many Troy pounds? How many Apothecaries' pounds?

10. Express in English money $486.65; in French money $19.30; in German money $23.85.

415. Reduction from a higher denomination to a lower.

416. Example. Reduce £45 5*s.* 8*d.* to pence.

$$\begin{array}{r} \pounds\,45\ 5s.\ 8d. \\ \underline{20} \\ 905s. \\ \underline{12} \\ 10868d. \end{array}$$

Solution. Since £1 is equal to 20*s.*, £45 are equal to 45 times 20*s.*, or 900*s.* 900*s.* with the 5*s.* added is equal to 905*s.*

Since in 1*s.* there are 12*d.*, in 905*s.* there are 905 times 12*d.*, or 10,860*d.* 10,860*d.* with the 8*d.* added is equal to 10,868*d.*, or the required result.

When possible, add mentally the number of lower denomination to the product as shown in the illustration in the margin.

417. From the foregoing illustration the following rule may be derived:

Multiply the units of the highest denomination given by the number of the next lower denomination required to make one of this higher, and to the product add the given units, if any, of the lower denomination.

Proceed in this manner with each successive result until the required denomination is reached.

WRITTEN EXERCISE

Reduce to the lowest denomination named:

1. 3 mi. 17 rd. 3 yd. 1 in.
2. £51 10*s.* 3*d.*
3. 11 bu. 5 pt.
4. 5 bu. 1 pk. 7 qt.
5. 43° 7′ 23″.
6. 17 gal. 2 qt. 1 gi.
7. 5 l. t. 50 lb. 2 oz.
8. 5 cu. yd. 3 cu. ft. 11 cu. in.
9. 175 sq. rd. 15 sq. in.
10. 1 mi. 15 ch. 43 li.
11. 9 T. 5 cwt. 4 lb. 1 oz.
12. 5 lb. 8 oz. 13 pwt.
13. 19 rd. 5 in.
14. 14 sq. rd. 5 sq. yd. 3 sq. ft.
15. 4 A. 31 sq. rd. 5 sq. yd. 3 sq. ft.
16. 5 wk. 367 hr. 5 min. 31 sec.
17. 7 cu. yd. 5 cu. ft.
18. 7 lb. 3 pwt. 4 gr.
19. 1 bbl. 2 gal. 1 pt.
20. 2 mi. 15 rd. 11 ft. 10 in.

418. Reduction from a lower denomination to a higher.

419. Example. Reduce 473 pt. to bushels.

$$\begin{array}{r|l} 2 & 473 \text{ pt.} \\ 8 & \underline{236} \text{ qt.} + 1 \text{ pt.} \\ 4 & \underline{\ 29} \text{ pk.} + 4 \text{ qt.} \\ & \quad 7 \text{ bu.} + 1 \text{ pk.} \end{array}$$

$$473 \text{ pt.} = 7 \text{ bu. } 1 \text{ pk. } 4 \text{ qt. } 1 \text{ pt.}$$

SOLUTION. Since 2 pt. equal 1 qt., 8 qt. 1 pk., and 4 pk. 1 bu., the successive divisors for reducing given pints to bushels are 2, 8, and 4, respectively.

Divide 473 pt. by 2 and the result is 236 qt. with a remainder 1 pt.; divide 236 qt. by 8 and the result is 29 pk. with a remainder 4 qt.; divide 29 by 4 and the result is 7 bu. with a remainder 1 pk. Write the last quotient and the several remainders in order and the required result is 7 bu. 1 pk. 4 qt. 1 pt.

420. Hence the following rule:

Divide the given number by the number of the same denomination required to make one of the next higher denomination, and consider the quotient as units of the higher denomination, and the remainder as units of the lower denomination.

Proceed in like manner with each successive quotient until the required denomination is reached.

The last result and the several remainders written in order will be the answer required.

WRITTEN EXERCISE

Change to units of higher denominations:

1. 72,920 min.

2. 24,840 gi.

3. 7210 pt. dry meas.

4. 40,720 sq. yd.

5. 214,712 in.

6. 60,720 oz. Avoir.

7 52,460 gr. Troy.

8. 24,620 da.

9. 9537 sec.

10. 10,632 sq. rd.

11. 8792 cu. in.

12. 34,832 lb. Avoir.

REDUCTION OF DENOMINATE FRACTIONS

421. When the integral unit of a fraction is a denominate number, the fraction is called a *denominate fraction.*

422. Reduction of denominate fractions from a higher denomination to a lower.

423. Examples. *1.* Reduce $\frac{7}{1860}$ Troy pounds to the fraction of a pennyweight.

$$\frac{7}{\underset{\underset{31}{\cancel{155}}}{\cancel{1860}}} \times \frac{\overset{1}{\cancel{12}}}{1} \times \frac{\overset{4}{\cancel{20}}}{1} = \frac{28}{31} \text{ pwt.}$$

SOLUTION. Denominate fractions may be reduced to lower denominations by multiplication in practically the same manner as denominate integers.

Since 12 oz. equal a pound, and 20 pwt. equal 1 oz., the successive multipliers for reducing pounds to pennyweights are 12 and 20 respectively.

Multiplying $\frac{7}{1860}$ by 12 and 20, by cancellation the result is found to be $\frac{28}{31}$ pennyweight.

2. Reduce $\frac{3}{16}$ of a Troy pound to a compound denominate number.

$$\frac{3}{\underset{4}{\cancel{16}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{9}{4} = 2\tfrac{1}{4} \text{ oz.}$$

$$\frac{1}{\cancel{4}} \times \frac{\overset{5}{\cancel{20}}}{1} = 5 \text{ pwt.}$$

Solution. The successive multipliers are 12 and 20 respectively. Multiplying $\frac{3}{16}$ lb. by 12, the result is $2\frac{1}{4}$ oz. Multiplying $\frac{1}{4}$ oz. by 20, the result is 5 pwt. Therefore, $\frac{3}{16}$ of a Troy pound is equal to 2 oz. 5 pwt.

3. Reduce .3165 of a Troy pound to a compound denominate number.

$$\begin{array}{rl}
.3165 & \text{lb.} \\
12 & \\ \hline
6330 & \\
3165 & \\ \hline
3.7980 & \text{oz.} \\
20 & \\ \hline
15.960 & \text{pwt.} \\
24 & \\ \hline
384 & \\
192 & \\ \hline
23.04 & \text{gr.}
\end{array}$$

Solution. The successive multipliers are 12, 20, and 24 respectively.

Multiplying .3165 lb. by 12, the result is 3.798 oz. Multiplying .798 oz. by 20, the result is 15.96 pwt. Multiplying .96 pwt. by 24, the result is 23.04 gr. Therefore, .3165 lb., as a compound denominate number, is equal to 3 oz. 15 pwt. 23 gr.

ORAL EXERCISE

1. Reduce $\frac{1}{16}$ of a bushel to the fraction of a peck; $\frac{3}{16}$ of a bushel; $\frac{7}{16}$ of a bushel to a compound denominate number.

2. What part of a shilling is $\frac{1}{8}$ of $\frac{7}{20}$ of a pound sterling?

3. What decimal of an inch is .08 of a foot? .016 of a foot?

4. Reduce $\frac{1}{20}$ of a gallon to the fraction of a pint.

WRITTEN EXERCISE

1. Reduce $\frac{3}{40}$ of a Troy pound to grains.

2. How many pennyweights in $\frac{1}{24}$ lb.?

3. Reduce $\frac{3}{8}$ of a mile to integers of lower denominations.

4. Express $\frac{11}{16}$ of an acre as a denominate number.

5. Reduce .1754 of a square mile to lower denominations.

6. Reduce $\frac{3}{11}$ of an acre to lower denominations.

424. **Reduction of a denominate fraction from a lower denomination to a higher.**

425. **Example.** Reduce $\frac{2}{5}$ of a grain to the fraction of a Troy pound.

$$\frac{\cancel{2}}{5}\times\frac{1}{\underset{12}{\cancel{24}}}\times\frac{1}{20}\times\frac{1}{12}=\frac{1}{14400}\text{ lb.}$$

Solution. Denominate fractions may be reduced from a lower denomination to a higher by division in practically the same manner as denominate integers.

Since 24 gr. equal 1 pwt., 20 pwt. equal 1 oz., and 12 oz. equal 1 lb., the successive divisors for reducing grains to pounds are 24, 20, and 12 respectively.

ORAL EXERCISE

1. What part of a foot is $\frac{3}{4}$ of an inch? $\frac{1}{2}$ of an inch?

2. What part of a week is $\frac{1}{6}$ of a day? $\frac{3}{4}$ of a day?

3. What part of a gallon is $\frac{1}{2}$ of a pint? $\frac{1}{8}$ of a pint?

4. What part of a foot is .5 of an inch? .025 of an inch?

WRITTEN EXERCISE

1. Reduce $\frac{4}{5}$ of a penny to the fraction of a pound sterling.

2. Reduce $\frac{7}{8}$ of a shilling to the fraction of a pound sterling.

3. Change 2.4 cwt. to the decimal of a ton.

4. Reduce $\frac{5}{8}$ of an inch to the decimal of a yard.

426. **Reduction of denominate integers to fractions of higher denominations.**

427. **Examples.** *1.* Reduce 9*s.* 6*d.* 3 far. to the decimal of a pound sterling.

(*a*)

$$\begin{array}{r|l} 4 & 3\text{ far.} \\ \hline 12 & 6.75d. \\ \hline 20 & 9.5625s. \\ \hline & £.478125 \end{array}$$

(*b*)

9*s.* 6*d.* 3 far. = 459 far.

£1 = 960 far.

459 ÷ 960 = £.478125

Solutions. (*a*) The successive divisors to reduce farthings to pounds are 4, 12, and 20 respectively. Dividing 3 far. by 4, the result is .75*d.* Putting with this the 6*d.*, the result is 6.75. Dividing 6.75*d.* by 12, the result is .5625*s.* Putting with this the 9*s.*, the result is 9.5625. Dividing by 20, the result is .478125 pounds sterling. Or,

(*b*) In 9*s.* 6*d.* 3 far. there are 459 far., and in £1 there are 960 far. Hence, 9*s.* 6*d.* 3 far. is $\frac{459}{960}$ of a pound sterling. £$\frac{459}{960}$ = £.478125.

2. Reduce 4 yd. 2 ft. 6 in. to the fraction of a rod.

(*a*)

$6 \div 12 = \frac{1}{2}$ ft.

$\frac{5}{2}\ (2\frac{1}{2}) \div 3 = \frac{5}{6}$ yd.

$\frac{29}{6}\ (4\frac{5}{6}) \div 5\frac{1}{2} = \frac{29}{33}$ rd.

(*b*)

4 yd. 2 ft. 6 in. = 174 in.

1 rd. = 198 in.

$174 \div 198 = \frac{174}{198} = \frac{29}{33}$

SOLUTIONS. (*a*) The successive divisors to reduce inches to rods are 12, 3, and $5\frac{1}{2}$ respectively. 6 in. divided by 12 equal $\frac{1}{2}$ ft. Putting with this the 2 ft., the result is $2\frac{1}{2}$ ft. $2\frac{1}{2}$ ft. divided by 3 equal $\frac{5}{6}$ yd. Putting with this the 4 yd., the result is $4\frac{5}{6}$ yd. $4\frac{5}{6}$ yd. divided by $5\frac{1}{2}$ equal $\frac{29}{33}$ rd. Or,

(*b*) 4 yd. 2 ft. 6 in. equal 174 in. 1 rd. equals 198 in. 4 yd. 2 ft. 6 in. is therefore $\frac{174}{198}$ of 1 rd., or $\frac{29}{33}$ rd.

ORAL EXERCISE

1. What part of a dollar is 35¢? of a day is 7 hr.?

2. What part of a gallon is 1 pt.? of a rod is 4 yd.?

3. What decimal of a bushel is 1 pk.? of a gallon is 3 pt.?

4. What decimal of a yard is 2 ft.? of £1 is 12s.?

WRITTEN EXERCISE

1. Reduce 6 cwt. 54 lb. to the decimal of a ton.

2. What decimal of an acre is 2722 sq. ft. 72 sq. in.?

3. What fraction of a pound equals 11 oz. 11 pwt. 18 gr.?

4. Reduce 3 oz. 11 pwt. 12 gr. to the decimal of a pound.

5. 17 cwt. 72 lb. 4 oz. is what fraction of a ton?

ADDITION OF DENOMINATE NUMBERS

428. Denominate numbers may be added, subtracted, multiplied, and divided upon the same general principles by which similar operations are performed in simple numbers. The only variation arises from the fact that in simple numbers ten units of any lower denomination make one of the next higher, while in denominate numbers the scale is not at all uniform.

429. Example. Find the sum of 3 bu. 2 pk. 1 pt., 5 bu. 1 qt., 6 bu. 3 pk. 7 qt. 1 pt., 2 bu. 1 pt.

bu.	pk.	qt.	pt.
3	2	0	1
5	0	1	0
6	3	7	1
2	0	0	1
17	2	1	1

Solution. Write the numbers, as in simple addition, so that the units of the same denominations stand in the same vertical column.

Begin at the right and add. The sum of the pints is 3 pt., which is equal to 1 qt. 1 pt. Write 1 pt. and carry 1 qt.

The sum of the quarts is 9 qt., or 1 pk. 1 qt. Write 1 qt. and carry 1 pk.

The sum of the pecks is 6 pk., or 1 bu. 2 pk. Write 2 pk. and carry 1 bu.

The sum of the bushels is 17, which write as bushels, thus completing the addition.

The result is 17 bu. 2 pk. 1 qt. 1 pt.

WRITTEN EXERCISE

1. What is the sum of £21 5*s.* 7*d.* 2 far., £16 3*s.* 8*d.* 1 far., £14 9*s.* 2 far., £21 8*s.* 12*d.* 2 far., £16 15*s.* 7*d.* 1 far.?

2. What is the sum of 1 mi. 8 rd. 5 yd. 2 ft., 3 mi. 17 rd. 5 yd. 2 ft., 8 mi. 4 yd. 1 ft., 4 mi. 1 yd. 1 ft., 1 mi. 17 rd. 20 ft.?

3. What is the sum of 5 A. 110 sq. rd. 5 sq. ft. 28 sq. in., 1 A. 80 sq. rd. 3 sq. ft. 12 sq. in., 12 A. 16 sq. rd. 2 sq. ft. 48 sq. in., 5 A. 16 sq. rd. 3 sq. ft. 21 sq. in., 8 A. 100 sq. rd. 3 sq. ft. 42 sq. in.

4. Add 436 lb. 4 oz. 15 pwt., 83 lb. 11 oz. 21 gr., 46 lb. 16 pwt., 105 lb. 9 oz. 11 gr.

5. What is the sum of 16 lb. 16 pwt. 16 gr., 100 lb. 1 oz. 5 pwt. 20 gr., 76 lb. 7 oz. 5 pwt. 13 gr., 19 lb. 2 oz. 10 pwt. 20 gr.?

SUBTRACTION OF DENOMINATE NUMBERS

430. Example. From 84 rd. 3 yd. 2 ft. 6 in. take 12 rd. 5 yd. 2 ft. 8 in.

rd.	yd.	ft.	in.
84	3	2	6
12	5	2	8
71	2½	2	10
	½	=1	6
71	3	1	4

Solution. Write the numbers so that the units of the same denominations stand in the same vertical column, and beginning at the right subtract as in simple numbers.

Since 8 in. cannot be subtracted from 6 in., take 1 ft. (12 in.) from 2 ft. and add it to 6 in., making 18 in. 18 in. minus 8 in. leaves 10 in., which write as inches in the remainder.

Inasmuch as 1 ft. was added to 6 in., there is but 1 ft. remaining in the minuend. Since 2 ft. cannot be subtracted from 1 ft., take 1 yd. (3 ft.) from the 3 yd. and add it to the 1 ft., making 4 ft. 4 ft. minus 2 ft. leaves 2 ft., which write as feet in the remainder.

Inasmuch as 1 yd. was added to 1 ft., there are but 2 yd. remaining in the minuend. Since 5 yd. cannot be subtracted from 2 yd., take 1 rd. (5½ yd.) from 84 rd. and add to the 2 yd., making 7½ yd. 7½ yd. minus 5 yd. leaves 2½ yd., which write as yards in the remainder. 83 rd. minus 12 rd. leaves 71 rd., which write as rods in the remainder.

Reducing ½ yd. to lower denominations and adding, the required result is found to be 71 rd. 3 yd. 1 ft. 4 in.

WRITTEN EXERCISE

1. From a barrel containing 36 gal. 1 pt. of oil, there were sold 1 qt. 1 pt. 2 gi. at one time, and at another 21 gal. 2 qt. 1 pt. How much remained unsold?

2. I owned 640 A. of prairie land. After selling 126 A. 45 sq. rd. to A, and 117 A. 37 sq. rd. to B, how much had I left?

3. From ⅞ rd. take 3⅛ ft.

4. Having bought 21$\frac{61}{72}$ lb. of old silver, I used 15 lb. 15 pwt. 15 gr. How much had I left?

5. Having raised 214⅜ bu. of potatoes, I sold 125 bu. 3 pk. What is the remainder worth at 50¢ per bushel?

6. Three men together own 27 T. 75 lb. of hay. If A owns 9¾ T., and B 11 T. 75 lb., how much does C own?

7. An English merchant bought goods amounting to £5926 12*s*. After selling a part of the goods for £4192 12*s*. 9*d*., he took an account of stock and found that he had on hand merchandise worth, at cost prices, £2241 4*s*. 3*d*. Did he gain or lose, and how much?

Finding the Difference between Two Dates

431. Difference of time may be found in either of two ways: (1) by compound subtraction, and (2) by counting the actual number of days from the given to the required date.

432. In all business transactions involving long periods of time, the difference is usually found by compound subtraction, while in transactions involving short periods of time, the difference is usually found by counting the exact number of days.

433. Examples. *1.* A mortgage dated Sept. 22, 1892, was paid Aug. 31, 1903. How long had it run?

yr.	mo.	da.
1903	8	31
1892	9	22
10	11	9

SOLUTION. The later date expresses the greater length of time; hence, write it as the minuend and the earlier date as the subtrahend. August being the 8th month and September the 9th, write 8 and 9 respectively, instead of the names of the months. Subtract as in ordinary denominate numbers, considering 30 days a month and 12 months a year. As near as the time can be expressed in years, months, and days, the mortgage is found to have run 10 yr. 11 mo. 9 da.

2. A note dated June 7 was paid Sept. 5. How many days had it run?

23 da. in June.
31 da. in July.
31 da. in August.
5 da. in September.
90 da. from June 7 to Sept. 5.

SOLUTION. First write the time remaining in June, then the actual number of days in July and August respectively, and finally the number of days in September up to and including Sept. 5. The sum of these days is the required time expressed with exactness. Observe that the aggregate time *excludes* the first and *includes* the last day of the dates.

ORAL EXERCISE

By inspection, find the exact number of days between:*

1. May 3 and June 26.
2. June 25 and Aug. 1.
3. July 2 and Aug. 31.
4. Sept. 20 and Oct. 31.
5. Nov. 10 and Dec. 31.
6. Oct. 16 and Dec. 28.
7. Mar. 1 and Apr. 10.
8. Jan. 9 and Feb. 23.
9. Mar. 20 and Apr. 28.
10. Apr. 3 and June 1.
11. Apr. 15 and June 15.
12. June 2 and Aug. 2.
13. Sept. 5 and Nov. 8.
14. Oct. 15 and Dec. 18.
15. Aug. 9 and Oct. 21.
16. Sept. 19 and Nov. 1.

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* All of the dates given are in the same year unless otherwise specified.

WRITTEN EXERCISE

Find the exact number of days between:

1. Aug. 19, Nov. 23.
2. May 3, Dec. 25.
3. Sept. 29, Aug. 18.
4. Apr. 29, July 25.
5. Apr. 16, Nov. 25.
6. Mar. 18, Nov. 6.
7. May 29, July 17.
8. May 1, Dec. 28.
9. Jan. 1, 1902, Mar. 23, 1902.
10. Jan. 24, 1900, Mar. 2, 1901.
11. Sept. 30, 1903, Mar. 6, 1905.
12. July 27, 1900, Dec. 27, 1902.

By compound subtraction, find the difference between:

13. Jan. 28, 1905, Aug. 31, 1910.
14. Sept. 28, 1889, Sept. 28, 1906.
15. Mar. 5, 1899, July 10, 1903.
16. Feb. 11, 1900, Jan. 6, 1903.
17. June 30, 1901, Aug. 3, 1904.
18. Sept. 18, 1896, Feb. 11, 1904.
19. May 1, 1897, June 2, 1900.
20. May 25, 1883, June 3, 1901.

MULTIPLICATION OF DENOMINATE NUMBERS

434. Example. Multiply 3 bu. 1 pk. 2 qt. 1 pt. by 12.

bu.	pk.	qt.	pt.
3	1	2	1
			12
39	3	6	0

SOLUTION. Begin at the right and multiply as in simple numbers.

12 × 1 pt. = 12 pt. or 6 qt. Write 0 in the place of pints in the product and carry 6 qt.

12 × 2 qt. = 24 qt. 24 qt. + 6 qt. carried from pints equal 30 qt., or 3 pk. 6 qt. Write 6 in the place of quarts in the product and carry 3.

12 × 1 pk. = 12 pk. 12 pk. + 3 pk. carried are 15 pk., or 3 bu. 3 pk. Write 3 in the place of pecks in the product and carry 3.

12 × 3 bu. = 36 bu. 36 bu. + 3 bu. carried are 39 bu. which write in the product. The complete product is therefore 39 bu. 3 pk. 6 qt.

WRITTEN EXERCISE

1. I bought 7 lb. 7 oz. 12 pwt. 18 gr. of old gold at $1.05 per pennyweight. What was the value?

2. What is the cost of 15 chains of gold each weighing 3 oz. 17 pwt. 17 gr. at 7¢ per grain?

3. How much hay in 9 stacks each containing 5 T. 21 cwt. 83 lb.? What is the hay worth at $ 12.75 per ton?

4. A merchant bought 40 lb. 8 oz. of sugar at 6⅔¢ per pound, and 5 times as much coffee at 33⅓¢ per pound. What was the amount of his bill?

5. If a piece of land produces on an average 133⅓ bu. of potatoes per acre, what will be the value of 6 A. 80 sq. rd. at 66⅔¢ per bushel? at 75¢ per bushel? at 50¢ per bushel?

6. A man owning a farm of 400 A. of land sold 125 A. 75 sq. rd. How much is the remainder of the farm worth at $75 per acre?

7. A man bought 30 piles of wood, each containing 4 cd. 80 cu. ft. at $3.33⅓ per cord. He later sold the entire quantity for $560. Did he gain or lose, and how much?

8. Reduce £15 3*s*. 7*d*. 2 far. to United States money.

Solution. Since £1 equals $4.8665, £15 equal $72.9975; since 1*s*. equals 24⅓¢, 3*s*. equal 73¢; since 1*d*. equals 2$\frac{1}{36}$¢, 7*d*. equal $.1414; since 1 far. equals $\frac{50}{99}$¢, 2 far. equal $.0101. Add these separate equivalents together, and the result is $73.879, or $73.88. Or,

Call each 2 shillings $\frac{1}{10}$ of a pound, then 3 shillings equals £.15; call the pence and farthings, reduced to farthings, so many $\frac{1}{1000}$ of a pound. 7 pence plus 2 farthings equals 30 farthings, equals £.030; to this add the £15 and the £.15, and the result is £15.18. Since £1 equals $4.8665, £15.18 equal 15.18 times $4.8665, or 73.87347 = $73.87.

9. Reduce £25 4*s*. 8*d*. to United States money.

DIVISION OF DENOMINATE NUMBERS

435. Example. If 15 yd. of cloth are worth £23 11*s*. 3*d*., what is 1 yd. worth?

	£	*s*.	*d*.
15)	23	11	3
	1	11	5

Solution. The cost of 1 yd. is $\frac{1}{15}$ of the cost of 15 yd. $\frac{1}{15}$ of £23 is £1 with an undivided remainder of £8. Write £1 in the quotient and add the remainder to the next lower denomination. £8 11*s*. = 171*s*. $\frac{1}{15}$ of 171*s*. is 11*s*. with an undivided remainder of 6*s*. Write 11*s*. in the quotient and add the remainder to the next lower denomination. 6*s*. 3*d*. = 75*d*. $\frac{1}{15}$ of 75*d*. is 5*d*., which write in the quotient. 1 yd. of cloth is therefore worth £1 11*s*. 5*d*.

WRITTEN EXERCISE

Find the quotient of:

1. 24 bu. 3 pk. ÷ 9.

2. 16 T. 9 cwt. 24 lb. ÷ 16.

3. £28 10*s.* 5*d.* 2 far. ÷ 6.

4. 25 yr. 5 da. 12 hr. ÷ 8.

5. If 7 lb. 7 oz. 12 pwt. 18 gr. of silver be made into 6 plates of equal weight, what will be the weight of each?

6. A miner having 63 lb. 1 oz. 10 pwt. of gold dust divided $\frac{1}{4}$ of it among his laborers, and had the remainder made into chains averaging 3 oz. 3 pwt. 3 gr. of pure gold each. If he sold the chains for $72.50 each, how much did he receive for them?

7. From the sum of £25 4*s.* 10*d.* and £10 5*s.* 2*d.* take their difference, divide the result by 5, and reduce the quotient to United States money.

8. I sold 98 cd. 96 cu. ft. of wood for $395, and in so doing lost $98.37½. What did the wood cost per cord?

9. Reduce $5164.28 to equivalent English money.

SOLUTION

$5164.28 ÷ 4.8665 = £1061.189.
£.189 × 20 = 3.78*s.*
.78*s* × 12 = 9.36*d.*
.36 *d.* × 4 = 1.44 far.
Hence, $5164.28 = £1061 3*s.* 9*d.* 1 far.

10. Reduce $185 to equivalents in English money; $2500.

WRITTEN REVIEW

1. How many fields, each of 10 A. 56 sq. rd. 21 sq. yd. 5 sq. ft. and 28 sq. in., can be formed from a farm containing 124 A. 40 sq. rd. 16 sq. yd. 8 sq. ft. 48 sq. in.?

2. Reduce $3750 to English money.

3. To the sum of $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{4}{15}$ of an acre, add .0055 of a square mile.

4. From .6375 of an acre take $\frac{14}{15}$ of a square rod.

5. A wheelman ran 71 mi. 246 rd. 1 yd. 2 ft. 6 in. in the forenoon, and 20 mi. 10 rd. 8 in. less in the afternoon. What distance did he run in the entire day?

6. A grocer bought 110 qt. of chestnuts by dry measure, and when selling them used a liquid pint measure. What was his gain if he bought the chestnuts at 6¢ a quart and sold them at 5¢ a pint?

SOLUTION

1 dry quart = 67⅕ cu. in.
1 liquid quart = 57¾ cu. in.
67⅕ cu. in. × 110 = 7392 cu. in.
7392 cu. in. ÷ 57¾ cu. in. = 128.
1 qt. × 128 = 128 qt., or 256 pt.
110 qt. at 6¢ = $6.60, the cost.
256 pt. at 5¢ = $12.80, the selling price.
$12.80 − $6.60 = $6.20, the gain.

7. A blundering clerk bought of a gardener 192 qt. of currants, measuring them by a liquid quart measure and selling them by dry measure. If the currants were bought at 6¢ per quart and sold at 7¢, did he gain or lose, and how much?

8. From a cask of brandy containing 69 gal. 1 pt., and costing $3.75 per gallon, ¼ leaked out and the remainder was sold at 20¢ per gill. What was the amount of the gain or loss?

9. If 2 qt. 1 pt. 1 gi. of oil be consumed per day for the year 1903, what will be its cost for the year at 8¢ per gallon?

10. I bought by Avoirdupois weight $28\frac{8}{10}$ lb. of drugs and from the stock sold by Apothecaries' weight 29 lb. What is the remainder worth at 75¢ per Apothecaries' ounce?

11. A farmer sold 4 loads of hay, weighing respectively 1 T. 2 cwt. 14 lb., 19 cwt. 90 lb., 1 T. 3 cwt. 97 lb., 1 T. 5 cwt., and received for it $16 per ton. How much did he receive?

12. A goldsmith bought 3 lb. 9 oz. 1 pwt. 16 gr. of old gold at 80¢ per pennyweight, and made it into pins of 40 gr. weight each, which he sold at $2 apiece. How much did he gain or lose?

13. 73,920 qt. of cherries were bought at 10¢ per quart dry measure, and sold at the same price liquid measure. How much was thereby gained?

14. A horse requires $\frac{7}{16}$ of a bushel of oats per day. At 9¢ a peck, how much will it cost to feed him oats for July and August?

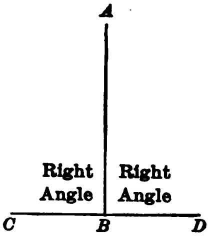
PRACTICAL MEASUREMENTS

DISTANCES.

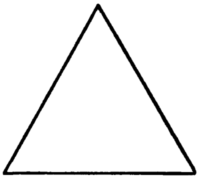
436. An **angle** is the difference in direction of two lines proceeding from a common point called the *vertex*.

437. A **right angle** is an angle formed when one straight line meets another so as to make the adjacent angles equal. The lines forming the angles are said to be *perpendicular to each other*.

In the accompanying diagram ABC and ABD are right angles, and the lines AB and CD are perpendicular to each other.

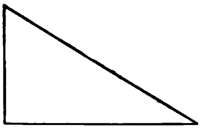


438. A **triangle** is a plane figure with three plane sides and three plane angles. The side on which the triangle stands is the *base*, the opposite corner the *vertex*, and the shortest distance from the vertex to the base, or the base extended, is the *height* or *altitude* of the triangle.



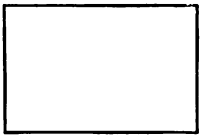
Triangle

439. A **right-angled triangle** is a triangle having a right angle.



Right-angled Triangle

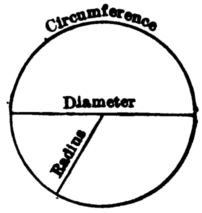
440. A **rectangle** is a plane figure having four straight sides and four square corners. When the four sides are equal, the figure is usually called a *square*.



Rectangle

441. The **perimeter** of any plane figure is the length of the line or lines inclosing it.

442. When a plane figure is bounded by a curved line, every point of which is equally distant from the center, it is called a *circle*. The perimeter of a circle is called its *circumference;* a line passing through the center and terminating in the circumference, the *diameter;* one half of the diameter, the *radius*.



443. To find the circumference of a circle when the diameter is given,

Multiply the diameter by 3.1416.

444. To find the diameter of a circle when the circumference is given,

Divide the circumference by 3.1416.

ORAL EXERCISE

1. How many rods of fence will inclose a farm in the form of an equilateral triangle each side of which is 25 rd.?

2. How many rods of fence will inclose a field 15 rd. long and 10 rd. wide?

3. It required 100 rd. to inclose a garden which is in the form of a perfect square. How many feet in each side of the garden?

4. What is the radius of a circle whose circumference is 314.16 ft.?

5. How many yards of border will be required for the paper in a room, the length and width of whose sides are 18 ft. and 12 ft. respectively?

6. How many posts 16½ ft. apart will be required for the fence of a square field whose sides are each 100 rd.?

7. The perimeter of a rectangle is 72 rd. If the width is 12 rd., what is the length?

8. If it cost $75 to fence a certain square field, how much would it cost to fence a similar field whose sides are double the length of those of the first field?

WRITTEN EXERCISE

1. What is the circumference of a circle whose diameter is 24 ft.?

2. What will be the cost of the posts required to fence a square field whose sides are each 16,500 ft., if the posts are set 1 rd. apart and cost $8.37½ per C?

3. Find the cost of the wire necessary to fence a rectangular field whose length and breadth are 24 rd. and 18 rd. respectively, if the fence contains five wires and the wire is worth ¼¢ per foot.

4. How many feet of fence will inclose a circular field 8.5 rd. in diameter?

5. What is the radius of a circle whose circumference is 7854 ft.?

6. A room is in the form of a rectangle 47½ ft. long and 36⅝ ft. wide. How many feet and inches of molding will be required for its four walls?

AREAS

445. The dimensions of a surface are length and breadth (**364**). In finding the area of any surface having a uniform length and breadth it is necessary to select a measuring unit. This may be any square, each side of which is a unit of length (**370**). The number of square units found in the surface is the required area.

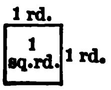
To illustrate, take the following example.

446. Example. What is the area of a garden 5 rd. long by 66 ft. wide?

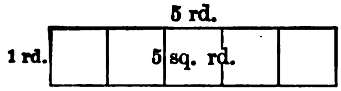
Solution. Select a measuring unit. This may be any square, each side of which is a unit of length.

Since 66 ft. is equal to just 4 rd., take 1 sq. rd. (*a*) as a measuring unit. Then, if lines are drawn as represented in (*c*), the entire surface will be divided into square rods. In the horizontal row (*b*) there will be 5 sq. rd., and since in the entire figure there will be 4 horizontal rows which are equal to (*b*), the area is 4 times 5 sq. rd., or 20 sq. rd. Hence,

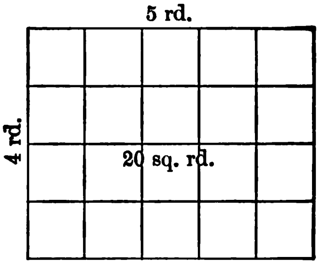
(*a*)



(*b*)



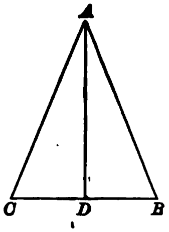
(*c*)



The dimensions of a rectangle reduced to units of the same denomination and multiplied together express the area of a rectangle in square units having the same denomination as the units of length.

447. Example. Find the area of a triangle whose base and altitude are 6 and 8 ft. respectively.

Solution. In the accompanying diagram assume that the base (*CB*) is 6 ft. and the altitude (*AD*) is 8 ft. It will be seen that the altitude divides the triangle into two right-angled triangles, each of which is one half of a rectangle whose sides are 8 ft. and 3 ft. Two triangles, each one half of a rectangle 8 ft. by 3 ft., are equal to one rectangle 8 ft. by 3 ft. The area of the triangle given is, then, the product of these two dimensions, or 24 sq. ft. Hence,



448. To find the area of a triangle, the base and altitude being given,

Multiply the altitude by one half the base.

449. To find the area of a circle, the circumference and diameter being given,

Square the radius and multiply by 3.1416; or square the diameter and multiply by .7854 (¼ of 3.1416).

ORAL EXERCISE

1. How many square feet in the ceiling of a room 25 ft. long and 16 ft. wide?

2. What is the width of a rectangle 12 ft. long if it contains the same area as a square 8 ft. on a side?

3. What is the area of a triangle whose base is 11 ft. and whose altitude is 15½ ft.?

4. A room 15 ft. long and 8 ft. wide has a tile floor. If one tile is 4 in. square, how many tiles in the floor?

5. What is the difference between a square rod and a rod square? between 9 sq. ft. and 9 ft. sq.? between ½ of a square foot and ½ of a foot square?

6. Express in inches the difference between ¼ of a square foot and ¼ of a foot square.

WRITTEN EXERCISE

Find the number of acres in rectangular fields containing dimensions as follows. Perform the multiplications as explained in **91–93**.

1. 36 rd. by 21 rd.
2. 62 rd. by 51 rd.
3. 85 rd. by 64 rd.
4. 72 rd. by 23 rd.
5. 75 rd. by 32 rd.
6. 84 rd. by 23 rd.
7. 47 ch. by 95 ch.
8. 51 ch. by 49 ch.
9. 75 ch. by 52 ch.

10. A field 87½ rd. wide and 240 rd. long produced 27¾ bu. of wheat to the acre. What was the crop worth, at 90¢ per bushel?

11. A farm in the form of a rectangle is 75 rd. wide; if the area is 167.5 A., how long is the farm?

12. I wish to build a shed which will cover 2/5 of an acre of land. If the width of the shed is 42 ft., what must be its length?

13. 17.75 bu. of timothy seed is sown on land at the rate of 6 lb. per acre. What will be the area of the land thus seeded?

14. A hall 7½ ft. wide and 19½ ft. long is covered with oilcloth, at 65¢ per square yard. How much did it cost?

15. A city lot in the shape of a triangle has a base of 90 yd. and an altitude of 120 yd. What is it worth at $2.50 per square foot?

16. What is the area of a semicircle if the radius of the whole circle is 100 ft.?

17. It cost $25 to carpet a square room 20 ft. on a side. At that rate, what will it cost to carpet a square room 40 ft. on a side?

18. A rectangular field containing 54 A. is 30 rd. wide. What will it cost to fence it at 2½¢ a foot?

CARPETING

450. Carpet is sold by the yard. Oilcloth and linoleum are sometimes sold by the square yard.

451. In finding the number of yards of carpet for a room it is always necessary to know whether the strips are to run lengthwise or crosswise.

Merchants will sell fractional lengths, but not fractional widths of carpet; hence, in finding the cost of carpet the number of whole strips must be found and 1 added to this number for all fractions.

452. Where the length of the strip is not an even number of yards, there is usually some waste in matching the figures of the pattern, and since dealers charge for the goods furnished, regardless of the waste, all items of wastage must be included in the cost.

Often carpet may be laid with less waste one way of the room than another; hence, in estimating the cost of carpet it is sometimes desirable to find the cost of the strips running both lengthwise and across the room, and then to make a comparison.

453. Example. How many yards of Brussels carpet $\frac{3}{4}$ of a yard wide, laid lengthwise of the room, will be required to cover a floor 22 ft. by 17 ft. 4 in., if the waste in matching be 6 in. on each strip?

17 ft. 4 in. $= \frac{52}{3}$ ft. $= \frac{52}{9}$ yd.
$\frac{52}{9}$ yd. $\div \frac{3}{4} = \frac{52}{9} \times \frac{4}{3} = \frac{208}{27} = 7\frac{19}{27}$.
$7\frac{19}{27}$ strips is practically 8 strips.
22 ft. + 6 in. = $22\frac{1}{2}$ ft.
$22\frac{1}{2}$ ft. × 8 = 180 ft. = 60 yd.

Solution. Since the strips run lengthwise of the room, to find the number of strips, divide the width of the room by the width of the carpet. $\frac{52}{9}$ yd. divided by $\frac{3}{4}$ yd. equals $7\frac{19}{27}$, the number of times one strip required. Since fractional widths of carpet cannot be bought, drop the fraction and add 1 to the whole number; 8 strips are required, and $\frac{8}{27}$ of a strip may be cut off or turned under. The length of the room is 22 ft. and there is a waste of 6 in. on each strip for matching; hence, the length to be bought for each strip is $22\frac{1}{2}$ ft. If there are $22\frac{1}{2}$ ft. in each strip, in the 8 strips there are 8 times $22\frac{1}{2}$ ft. or 180 ft., or 60 yd.

WRITTEN EXERCISE

1. How many yards of carpet, 1 yd. wide, laid lengthwise of the room, will be required to cover a floor 10.5 yd. long by 6 yd. wide, if no allowance is to be made for matching?

2. What will be the cost of the carpet border for a room $16\frac{1}{2}$ ft. by 21 ft., if the price be $62\frac{1}{2}$¢ per yard?

3. How many yards of carpeting $\frac{3}{4}$ yd. wide will be required to carpet a room 32 ft. long by 25 ft. wide, if the lengths of carpet are laid across the room and 8 in. are lost on each strip in matching the pattern? How many yards if the strips are laid lengthwise and 6 in. are lost in matching? If the carpet is laid in the most economical way, what will be the cost at $2.55 per yard?

4. How many yards of Axminster carpeting $\frac{3}{4}$ of a yard in width, and laid lengthwise of the room, will be required to cover a floor $21\frac{3}{4}$ ft. long and $18\frac{3}{4}$ ft. wide, making no allowance for waste in matching the design?

5. Find the cost of a carpet $\frac{3}{4}$ yd. wide, at $2.50 per yard, for a room 22 ft. by 18 ft., if the strips run lengthwise and there is a waste of $\frac{1}{6}$ of a yard on each strip for matching the pattern.

6. How many yards of ingrain carpeting 1 yd. wide will be required to cover, lengthwise, the floor of a room 26 ft. long by 19 ft.

6 in. wide, the waste being 6 in. on each strip? How many yards of Brussels carpeting 27 in. wide will cover the same room if there is a waste of 18 in. per strip?

7. What will it cost, at $1.15 per yard, to carpet a flight of stairs 11 ft. 4 in. high, the tread of each stair being 10 in. and the riser 8 in.?

8. A hall 8½ ft. wide and 18½ ft. long was covered with oilcloth at 75¢ per square yard. How much did it cost?

PAPERING

454. Wall paper is usually 18 inches wide, and may be bought in single rolls 8 yards long or in double rolls 16 yards long.

455. These dimensions are for the wall paper commonly used in America. Imported papers and some American papers vary in width and length.

In practice there is usually considerable waste in cutting and matching the paper, and it is found more economical to buy double rolls.

There is no uniform rule respecting the allowance to be made for openings, such as doors and windows. Generally paper hangers estimate the number of full strips that would be necessary for the regular surface of the walls, and divide this number by the whole number of strips that can be cut from a full roll of paper. By this method the ends of the rolls are supposed to be utilized for the surface above doors and above and below windows, and other similar irregular places.

456. It is hardly possible to determine in advance the *exact* number of rolls of paper required for the walls of any room, but for practical purposes the following method will be found to approximate accuracy:

From the perimeter of the room deduct the width of the doors and windows.

Find the number of strips necessary for the regular surface of the walls, and divide the result by the whole number of strips that can be cut from a full roll of paper.

The quotient will be the number of rolls required.

For standard rolls there will be twice as many strips of paper required as there are yards in the length of the regular surface to be covered.

Any whole rolls left over after papering may be returned to the seller, but no portion of a roll will ever be taken back.

457. Example. How many double rolls of paper will be required for the sides and ends of a room 24 ft. long, 18 ft. wide, and 8 ft. high, with 1 door and 3 windows, each 3½ ft. wide, making no allowance for waste in cutting?

SOLUTION.

$\overline{24\text{ ft.} + 18\text{ ft.}} \times 2 = 84$ ft., the perimeter of the room.

3½ ft. × 4 = 14 ft., the width of the doors and windows.

84 ft. − 14 ft. = 70 ft., or 23⅓ yd., the length of the regular surface of the walls.

A double roll of paper is ½ yd. wide and 48 ft. long.

23⅓ ÷ ½ = 46⅔, or practically 47, the number of strips necessary for the regular surface.

48 ft. ÷ 8 ft. = 6, the number of strips in each double roll.

47 ÷ 6 = 7⅚, or practically 8 double rolls.

Hence, 8 is the required number of double rolls of paper.

WRITTEN EXERCISE

1. How many single rolls of paper 8 yd. long and 18 in. wide will it take to cover the ceiling of a room 60 ft. long, 45 ft. wide, if there be no waste in matching?

2. What is the cost of paper, at $1.25 per double roll, for a room 18 ft. long, 12 ft. wide, and 9 ft. high above the baseboard, allowing for 1 door and 2 windows, each 3½ ft. wide?

3. How many rolls of paper 8 yd. long and 18 in. wide will be required for the walls of a room 20 ft. long, 15 ft. wide, and having a height of 8 ft. 9 in., allowing for 1 door 3 ft. by 7 ft., and for 2 windows 3 ft. by 6 ft., and a baseboard 9 in. high?

4. At $1.90 per double roll, what will be the cost of papering a parlor 20 ft. square and 8 ft. high from the baseboard, allowing for 1 door 3 ft. by 7 ft. and 3 windows, each 3 ft. by 6 ft.?

5. Allowing for 3 windows each 42 in. by 7 ft., and 2 doors each 4 ft. by 11 ft., what will be the cost, at $1.80 per single roll, of papering a room 24 ft. long, 18 ft. wide, and 12 ft. high from the baseboard?

PAINTING AND PLASTERING

458. The unit of painting and plastering is the *square yard.*

Allowances are frequently made for one half or the whole of the area of the openings and the baseboard; but since there is no uniform custom governing such allowances, a written contract definitely referring to this matter should be drawn up; then complications at the time of settlement will be avoided.

WRITTEN EXERCISE

1. At 11¢ per square yard, what will be the cost to plaster the sides and ceiling of a room that is 30 ft. long, 24 ft. wide, and 14 ft. high above the baseboard, making full allowance for 4 doors 3 ft. 6 in. by 8 ft. 3 in., and 6 windows 3 ft. 8 in. by 7 ft.?

2. At 20¢ per square yard, what will it cost to paint a floor 40 ft. long and 26 ft. wide?

3. At 22¢ per square yard, what will it cost to plaster the sides and ceiling of a room 30 ft. by 24 ft. by 12 ft., if $\frac{1}{8}$ of the surface of the sides is allowed for the doors, windows, and baseboard?

4. What will be the cost at 18¢ per square yard for plastering the ceiling and walls of a room 60 ft. long, 30 ft. wide, and 15 ft. high above the baseboard, allowance being made for 6 doors 4 ft. 6 in. wide by 10 ft. 6 in. high above the baseboard, and 12 windows each 3 ft. 6 in. wide by 8 ft. high?

5. At 21¢ per square yard, what will it cost to paint both sides of a board partition 90 ft. long and 9 ft. 3 in. high?

6. Allowing $\frac{1}{8}$ of the surface of the sides for doors, windows, and baseboard, what will it cost at 12½¢ per square yard to plaster the sides and ceiling of a room 30 ft. long, 18 ft. wide, and 15 ft. high?

ROOFING AND FLOORING

459. The **unit** used in determining the number of square feet in any roof or floor is a *square* 10 ft. on a side, or 100 sq. ft.

460. Shingles are 16 in. long and on an average 4 in. wide. They are usually laid about 4½ in. to the weather, 1 shingle covering 18 sq. in. of roof. At this rate it requires 8 shingles for each square foot of roof, or 800 for each square of 100 sq. ft. Allowing for the waste that is usual in shingling, about 1000 shingles are estimated for each square of 100 sq. ft. Some shingles run better than this, and from 850 to 900 are regarded as an ample number for a square.

461. A **bundle** contains 250 shingles; hence it usually requires about 4 bundles or 1000 shingles for 100 sq. ft. of roof.

WRITTEN EXERCISE

1. I wish to floor and ceil a room 25 yd. long, 15 yd. wide. What will be the cost of the material at $27 per thousand square feet?

2. Find the cost at $45 per thousand square feet of the flooring for a room 40 ft. by 30 ft., the waste being $\frac{1}{6}$ of the area of the floor.

3. Find the cost of laying an oak floor which is 18 ft. by 16 ft., if the labor and other incidentals amount to $25, the price of the lumber is $75 per thousand sq. ft., and an allowance of 42 sq. ft. is made for waste.

4. Counting 1000 shingles for 120 sq. ft., how many will be required to cover the pitched roof of a barn 120 ft. long and 30 ft. wide on each side?

5. Allowing 800 shingles to a square, how many thousand will be required for the roof of a barn 30 ft. wide on each side and 100 ft. long? What will be the cost at $3.50 per thousand?

6. At $10 per square, what will be the cost of the slate for a roof 60 ft. long and 32 ft. wide?

SOLID CONTENTS

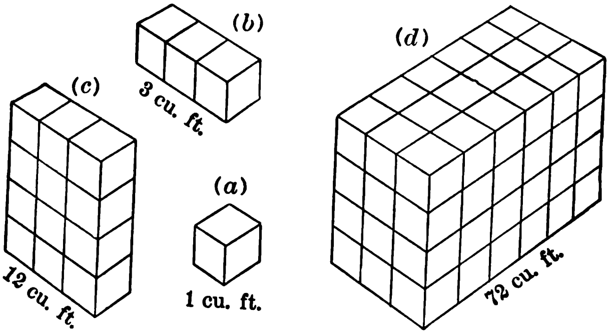
462. A **rectangular solid** is a solid bounded by six rectangular sides or faces. When these sides are squares, the figure is called a *cube*.

463. The dimensions of solids are length, breadth, and thickness (**365**). In finding the solid contents of any solid having a uniform length, breadth, and thickness, it is necessary to select a measuring unit. This may be any cube, each side of which is a unit of length (**382**). The number of cubic units found in the solid is the required *solid contents*.

To illustrate, take the following example.

464. **Example.** What is the volume of a solid 6 ft. long, 4 ft. high, and 3 ft. wide?

SOLUTION. — Select a measuring unit. This may be any cube, each side of which is a unit of length. For convenience, take 1 cu. ft. (*a*) as a measuring unit. Then lines are drawn as represented in (*b*), (*c*) and (*d*). (*b*) contains three times as many cubic feet as (*a*), or 3 cu. ft. (*c*) contains four times as many cubic feet as (*b*), or 12 cu. ft., and the entire figure, or (*d*), contains six times as many cubic feet as (c), or 72 cu. ft. Therefore, the solid contents of the rectangular solid given is 72 cu. ft. Hence,



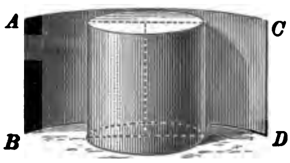
The dimensions of a rectangular solid reduced to units of the same denomination and multiplied together express the solid contents of a solid in cubic units of the same denomination as the units of length.

465. A cylinder is a circular body of uniform diameter, the *bases* of which are parallel circles.

When all the points of one circle are equally distant from all the points of another circle, the circles are said to be *parallel circles.*

466. The **lateral surface** of a cylinder is the surface of its curved sides.

467. The lateral surface of a cylinder is equal to the surface of a rectangular body, the length and height of which are equal to the circumference and height of the cylinder.



Thus, the lateral surface of the cylinder in the accompanying diagram is the area of the rectangle described by *A*, *B*, *C*, and *D* back of the cylinder. Hence,

468. **To find the area of the lateral surface of a cylinder,**

Multiply the circumference of the base by the height of the cylinder.

469. **To find the solid contents of a cylinder,**

Multiply the area of the base by the height of the cylinder.

WRITTEN EXERCISE

1. What will be the cost of a sheet-iron smokestack 40 ft. high and 2 ft. in diameter, at 15¢ per square foot?

2. A bin is 18 ft. long, 4½ ft. wide, and 18 ft. high. How many cubic feet in the bin?

3. At 15¢ per cubic yard, how much will it cost to excavate a cellar 85 ft. long, 43 ft. wide, and 14 ft. deep?

4. How many cubic feet of stone in a walk 195 ft. long, 4½ ft. wide, and 1½ ft. thick?

5. How many cubic feet in a cylinder 10 ft. in diameter and 20 ft. long?

6. A rectangular bin contains 259,200 cu. ft. If it is 40 yd. long and 20 yd. wide, how many feet high is it?

7. Find the cost of digging a round well 25 ft. deep and 8 ft. in diameter, at 35¢ per cubic yard.

BOARD MEASURE

470. In measuring lumber, boards one inch or less in thickness are estimated by the square foot.

Thus, a board 18 ft. long, 12 in. wide, and 1 in. thick contains 18 sq. ft. or 18 ft. *board measure*.

471. In measuring lumber more than one inch in thickness the boards are estimated by the number of square feet of boards, one inch in thickness, to which they are equal.

Thus, a board 12 ft. long, 12 in. wide, and 2½ in. thick contains 2½ times 12 board ft., or 30 board ft.

472. Unless sawed to order, the width of the board is reckoned only to the next smaller half-inch, except in cherry, black walnut, etc., where the price is 15¢ per foot and upward.

Thus, a board 8¼ in. in width is reckoned 8 in.; a board 12⅞ in. in width is reckoned 12½ in.; etc.

473. When the width of a board tapers uniformly, the average width is found by finding one half the sum of the two ends.

Thus, a tapering board 16 ft. long, 12 in. wide at one end, and 6 in. wide at the other, and 1 in. thick, contains $\frac{4}{4}(\overline{12+6} \div 2 \div 12)$ of 16 sq. ft., or 12 board ft.

474. Examples. *1.* How many board feet in 6 pieces of hemlock, 2 in. thick by 6 in. wide by 18 ft. long?

$$6 \times \cancel{2} \times \cancel{6} \times 18 = 108 \text{ board ft.}$$

SOLUTION. Since board feet are equal to square feet one inch in thickness, the length of the board in feet, multiplied by the width of the board in inches, and divided by 12, is equal to the number of board feet in one board, one inch in thickness; but since the board is 2 in. in thickness, 2 times this result is the number of board feet in each board, and 6 times this result, the number of board feet in the 6 boards.

To shorten the work, arrange the factors which are to be multiplied together as shown in the margin, and mentally cancel the 12 in the divisor from any factor or factors in the dividend. 2 × 6 in the dividend is equal to the 12 in the divisor; hence each board contains 18 board ft., and the 6 boards 108 board ft.

2. Find the number of board feet in 6 pieces of hemlock 4 in. thick by 5 in. wide by 16 ft. long.

$$\cancel{6} \times \overset{\overset{2}{\cancel{24}}}{\cancel{4}} \times 5 \times 16 = 160 \text{ board ft.}$$

SOLUTION. Reasoning as in problem 1, 6 × 4 × 5 × 16 ÷ 12 is equal to the number of board feet in the 6 pieces of hemlock. Observe that 6 multiplied by 4 contains 12 twice. Then, 2 × 5 × 16 = 160 board ft. Hence,

475. To find the number of board feet in lumber more than one inch thick,

Express the length in feet and the width and thickness in inches.

The product of these three dimensions, divided by 12, is equal to the number of feet, board measure.

In charging, or billing lumber, the number of pieces are entered first, then the thickness and width in inches, then the feet in length. For example, in recording 6 pieces, 4 in. thick by 6 in. wide and 20 ft. long, the form would be thus: 6 pcs. 4 in. × 6 in.–20 ft., and would be called off by the salesman, "6 *four-by-sixes*–20 ft.," *four-by-sixes* being the name by which he selects and sells stock.

Instead of writing "inches" and "feet," lumber billing clerks use (″) for inches, and (′) for feet; thus, 3 in. by 4 in.–17 ft. long is written, 3″ × 4″–17′.

ORAL EXERCISE

By inspection, determine the number of board feet of lumber in:

1. 5 pcs. 3″ × 4″–16′.

2. 8 pcs. 2″ × 6″–20′.

3. 9 pcs. 4″ × 6″–20′.

4. 10 pcs. 5″ × 7″–12′.

5. 15 pcs. 8″ × 10″–20′.

6. 10 pcs. 2½″ × 8″–18′.

7. 20 pcs. 4″ × 6″–16′

8. 12 pcs. 8″ × 10″–14′.

9. 17 pcs. 6″ × 8″–20′.

10. 20 pcs. 3″ × 4″–14′.

11. 10 pcs. 2″ × 6″–16′.

12. 15 pcs. 3″ × 8″–16′.

13. 25 pcs. 2″ × 6″–20′.

14. 50 pcs. 3″ × 4″–18′.

15. 100 pcs. 2″ × 6″–18′.

16. 4 pcs. 9″ × 10″–16′.

17. 24 pcs. 2″ × 4″–20′.

18. 6 pcs. 4″ × 5″–12′.

19. 10 pcs. 4″ × 6″–16′.

20. 6 pcs. 2″ × 5″–22′.

WRITTEN EXERCISE

1. What is the board measure of 7 planks, each 16 ft. long, 15 in. wide, and 3 in. thick?

2. What will be the cost of plank at $18 per M that will cover a floor 24 ft. by 13 ft., if the plank is 2½ in. in thickness?

3. What will be the cost of 10 sticks 2 in. by 4 in., 10 sticks 2 in. by 6 in., 10 sticks 4 in. by 4 in., and 10 sticks 2 in. by 10 in., if they are each 16 ft. long and the cost is $15 per M?

4. What will be the cost at $15 per M of a tapering board 18 ft. long, 1 in. thick, and 7½ in. wide at one end and 16½ in. wide at the other?

5. Find the amount of the following bill of hemlock, price by the M ft. board measure:

26 pcs. 2″ × 6″–18′ at $12; 24 pcs. 3″ × 4″–20′ at $15;
128 pcs. 8″ × 4″–14′ at $20.

NOTE. Always try to shorten the work by mentally eliminating the 12's from the dividend.

6. Find the amount of the following bill of lumber, price by the thousand ft. board measure:

18 pcs. 3″ × 4″–20′ at $14; 10 pcs. 2½″ × 6″–18′ at $15;
25 pcs. 4″ × 6″–16′ at $12; 12 pcs. 3″ × 5″–20′ at $18.

7. At $32.50 per M, what will be the cost of:
8 scantlings 3″ × 4″–18′; 12 scantlings 4″ × 5″–16′;
8 scantlings 5″ × 6″–14′.

8. At $19.50 per M, what will be the total cost of:
9 boards 1″ × 2″–14′; 6 boards 1½″ × 18″–16′;
15 boards 2″ × 14″–20′; 8 boards 1¼″ × 12″–18′.

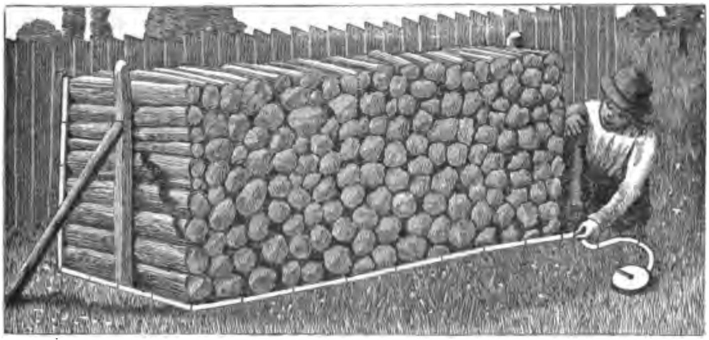
9. At $24 per M, what will be the cost of the lumber required to inclose a field 40 rd. square with a board fence if the boards are 15 ft. long, 5 in. wide, and 1 in. thick, and the fence 5 boards high?

10. At $21.50 per M, what will be the cost of the lumber in a line fence 160 rd. long if the boards are 11 ft. long, 7 in. wide, and 1 in. thick, and the fence 4 boards high?

WOOD MEASURE

476. **A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is called a cord.**

477. A pile of wood 1 ft. long, 4 ft. wide, and 4 ft. high, or $\frac{1}{8}$ of a cord, is called a **cord foot.**



478. **Example.** Find the number of cords of wood in a pile 25 ft. long, 4 ft. wide, and 6 ft. high.

$$\frac{25 \times \cancel{4} \times \overset{3}{\cancel{6}}}{\underset{\underset{16}{\cancel{32}}}{\cancel{128}}} = \frac{75}{16} = 4\tfrac{11}{16} \text{ cords.}$$

SOLUTION. The product of the dimensions of the pile is equal to the number of cubic feet of wood. Since a cord of wood is 8 ft. long, 4 ft. wide, and 4 ft. high, or contains 128 cu. ft., the number of cords in the pile is found by dividing this product by 128. Therefore, the required result is $4\frac{11}{16}$ cords. Hence,

479. To find the number of cords of wood in any pile,

Divide the product of the length, width, and height, expressed in cubic feet, by 128.

WRITTEN EXERCISE

1. How many cords of wood in a pile 108 ft. long, 7 ft. 9 in. high, and 6 ft. wide?

2. From a pile of wood 71 ft. 6 in. long, 9 ft. 4 in. wide, and 6 ft. 8 in. high, 21¾ cords were sold. What was the length of the pile remaining?

3. At $4.75 per cord, what will it cost to fill with wood a shed 34 ft. long, 18 ft. wide, and 10 ft. high?

4. A pile of wood built 10 ft. high and 25 ft. wide must be how long to contain 125 cords?

5. A pile of wood 30 ft. long, 4 ft. wide, and 8 ft. high was sold at $6 per cord. How much was received for it?

PAVING

480. The unit of paving is the *square foot* or the *square yard.*

WRITTEN EXERCISE

1. Find the cost of paving a court 150 ft. long and 120 ft. wide at $3 per square yard.

2. Which would be the more economical way to pave a street 3 mi. long and 1 rd. wide: with granite blocks at $3.65 per square yard, or with asphalt costing 23¢ per square foot, and how much would be saved?

3. Which would be the more economical, and how much: to pave a walk with stone at 22¢ per square foot, or with brick at $1.02 per square yard, if the width of the walk is 4 ft. and the length 200 ft.?

4. How many granite blocks 12 in. by 18 in. will be required to pave a mile of roadway 42 ft. in width?

5. A street 4975 ft. long and 40 ft. wide was paved with Trinidad asphaltum at $2.65 per square yard. What was the cost?

GAUGING

481. The process of finding the contents of any regular vessel in gallons, barrels, bushels, etc., is called **gauging**.

482. In every liquid gallon there are 231 cubic inches. Hence,

483. **To find the *exact* number of gallons in a vessel,**

Divide the number of cubic inches in the vessel by 231.

484. In every bushel, stricken measure, there are 2150.42 cubic inches. Hence,

485. **To find the *exact* number of stricken bushels in any bin,**

Divide the number of cubic inches in the bin by 2150.42.

486. In every bushel, heaped measure, there are 2747.71 cubic inches. Hence,

487. **To find the *exact* number of heaped bushels in any bin,**

Divide the number of cubic inches in the bin by 2747.71.

488. **Approximate Rules.** For practical purposes the rules given below generally approximate accuracy.

489. **Example.** What decimal of a stricken bushel is 1 cu. ft.?

```
                .8+
        ____________
2150.42)1728.000
        1720 336
        ________
           7 664
```

Solution. Since in every bushel there are 2150.42 cu. in., and in every cubic foot 1728 cu. in., a cubic foot is $\frac{172800}{215042}$, or approximately .8 of a bushel. Hence,

490. **To find the *approximate* number of stricken bushels in any number of cubic feet,**

Multiply by .8; and,

To find the *approximate* number of cubic feet in any number of stricken bushels,

Divide by .8.

491. **Example.** What decimal of a cubic foot is a heaped bushel?

```
2747.71)1728.0000(.63-
        1648 626
        ________
          79 3740
          82 4313
          _______
```

Solution. Since a cubic foot contains 1728 cu. in., and a heaped bushel 2747.71 cu. in., a cubic foot may be reduced to a decimal of a heaped bushel as shown in the margin. The result is .63 – cu. ft. Hence,

492. **To find the *approximate* number of heaped bushels in any number of cubic feet,**

Multiply by .63; **and,**

To find the *approximate* number of cubic inches in any number of heaped bushels,

Divide by .63.

493. Example. How many gallons in a cubic foot?

1728 ÷ 231 = 7.48+

Solution. Since a gallon contains 231 cu. in., and a cubic foot 1728 cu. in., the number of gallons in a cubic foot is found by dividing 1728 by 231. The result is 7.48+. Hence,

494. **To find the *approximate* number of gallons in a cistern,**

Multiply the number of cubic feet by $7\frac{1}{2}$ *and from the product subtract* $\frac{1}{100}$ *of the product.*

ORAL EXERCISE

1. Find the approximate number of bushels of grain contained in a box that is 5 ft. long, 4 ft. wide, and 3 ft. high.

2. How high must a box 10 ft. long and 6 ft. wide be built to hold, approximately, 240 bu.?

3. A vat 11 in. long, 7 in. high, and 3 in. wide will contain how many gallons of water?

4. A vat containing 2 gal. of water is 14 in. long and 11 in. wide. How high is it?

WRITTEN EXAMPLES

Find the approximate number of bushels of grain required to fill:

1. A bin 18 ft. × 6 ft. × 4 ft.

2. A bin 13 ft. × 20 ft. × 5 ft.

3. A bin 20 ft. × 8 ft. × 6 ft.

4. A bin 25 ft. × 12 ft. × 8 ft.

5. A bin $12\frac{1}{2}$ ft. × 8 ft. × 6 ft.

6. A bin 20 ft. × 12 ft. × 8 ft.

7. A cubical cistern is 10 ft. on a side. Find the exact number of barrels of $31\frac{1}{2}$ gal. each that it will contain.

8. A farmer exactly filled a bin 9 ft. wide, 12 ft. long, and 7½ ft. deep, with wheat grown from a field yielding 32¼ bushels per acre. How long was the field if its width was 50 rd.?

9. Find the exact number of gallons of water in a well 6 ft. in diameter, when the water is 9 ft. in depth.

10. A cask is 24 in. at the chime, 30 in. at the bung, and 3 ft. long. Find the exact number of gallons that may be put into it, if the cask is already ⅔ full.

STONE AND BRICK WORK

495. The unit of stone work is the *cubic yard* or the ***perch***.

496. **A perch of stone or masonry** is a rectangular solid 16½ ft. long, 1½ ft. wide, and 1 ft. high, and contains 24¾ cu. ft.

The number of cubic feet allowed for a perch of stone or masonry varies in different localities. In some places it is considered 16½ cu. ft., and in other places, 25 cu. ft.

497. The number of bricks in walls is usually estimated by the thousand, and 22 common bricks laid in mortar are counted for each cubic foot of wall.

A common brick is 8 in. long by 4 in. wide by 2 in. thick.

498. In making estimates for stone and brick work, masons take girt measurements. Whether anything is to be deducted for the area of the windows and other openings is generally fixed by contract. In some localities, one half the area of such openings is always deducted, while in others nothing whatever is deducted. In estimating material, however, allowance is generally made for the corners and all openings.

In taking girt measurements the corners are counted twice, but this is considered offset by the extra work required in building corners; the work around openings is also more difficult than straight work.

499. To find the number of perches of stone or masonry in a wall,

Divide the contents of the wall in cubic feet by 24¾.

500. To find the number of bricks for a wall,

Multiply the number of cubic feet in the wall by 22.

WRITTEN EXERCISE

1. A cellar is 24 ft. square inside of the wall, which is 9 ft. high and 2 ft. thick. How many perches of 24¾ cu. ft. each would a mason estimate for the wall?

2. How many cubic yards of masonry in the foundation walls of a house 50 ft. long and 30 ft. wide, outside measurements, if the wall is uniformly 2½ ft. wide and 8 ft. high?

3. How many perches of stone, actual measure, will be required to inclose a field 32 rd. long and 24 rd. wide, with a wall 4½ ft. high and 2½ ft. thick, counting 25 cu. ft. to the perch?

4. What will be the cost, by mason's measure, of building the walls of a block 140 ft. long, 66 ft. wide, and 47 ft. high, outside measurements, at $1.45 per perch of 24¾ cu. ft., if the walls are 18 in. thick and no allowance is made for openings?

5. How many common bricks will be required to erect the walls of a flat-roofed building 120 ft. long, 85 ft. wide, and 22 ft. high, outside measurements, if the walls are 18 in. in thickness and an allowance of 600 cu. ft. is made for openings? (Solve (1) by mason's measure, making allowance for the openings, and (2) by actual measure.)

6. At $2.25 per perch of 24¾ ft., how much will it cost, by mason's measure, to build the walls for a building, the length of which is 49.5 ft. and the width 24¾ ft., the walls to be 14 ft. high from the foundation and 18 in. thick?

7. How many common bricks will be required in building the four walls for a building 90 ft. long, 50 ft. wide, and 60 ft. high, outside measurements, if the walls are uniformly 1½ ft. thick, and 340 cu. ft. is allowed for openings? (Solve (1) by mason's measure, making allowance for the openings, and (2) by actual measure.)

WRITTEN REVIEW

1. In estimating the number of posts necessary for a wire fence to inclose a rectangular field 120 rd. long, it is found that to put the posts 12 ft. instead of 16½ ft. apart will require 180 more posts. What is the field worth at $90 per acre?

2. How high must wood be piled in a shed which is 28 ft. long and 16 ft. wide, to contain 28 cords?

3. What is the cost, at $90 per acre, of a rectangular farm having a length twice its width, if the perimeter is 480 rd.?

4. How much would it cost to plaster the walls and ceiling of a room 25 ft. long, 18 ft. wide, and 12 ft. high, at 27¢ per square yard, making an allowance of 396 ft. for doors, windows, etc.?

5. What will be the cost, at $12 per M, of the boards required for a sidewalk 32 rd. long and 4 ft. wide, if the boards are 16 ft. long, 1 in. thick, and 8 in. wide?

6. Express £15.6 in dollars and cents.

7. At $1\frac{2}{8}$¢ per square inch, what will it cost to bronze a cube the depth of which is 2 ft.?

8. I bought a farm 200 rd. long for $3600. If I paid $72 an acre for the farm, how much will it cost to fence it at 25¢ per rod?

9. What will be the cost of excavating a cellar 45 ft. long, 30 ft. wide, and 8 ft. deep, at 35¢ per cubic yard?

10. Estimating that 150 cu. ft. of air should be allowed for each pupil, how many pupils can be accommodated in a schoolroom 45 ft. long, 30 ft. wide, and 10 ft. high?

11. At 9¢ per square yard, what will it cost to paint the four sides and bottom of a tank 10 yd. long, 16 ft. wide, and 18 ft. deep.

12. At $125 per acre, find the difference in cost between two fields, the first of which contains 80 sq. rd., and the second of which is 80 rd. square.

13. Bought of a produce dealer 900 pounds of wheat at $1 per bushel, and gave in payment a pile of wood 16 ft. long, 6 ft. high, and 4 ft. wide. What price per cord did I receive for the wood?

14. How many cubic feet in 8 pieces of hemlock 24 ft. long, 14 in. wide, and 8 in. thick? How many board feet? What part of a cubic foot is a board foot?

15. A man bought a piece of land 80 rd. square, and after retailing 240 sq. rd., sold the remainder at $90 per acre. How much did he receive?

16. Which would be the more economical, and how much: to pave a sidewalk 1 mi. long and 1 rd. wide with asphalt costing 21¢ per square foot, or with granite blocks costing $2.95 per square yard?

17. What will it cost at $2.50 per yard to carpet a floor 24 ft. long by 17 ft. wide, if the strips, which are $\frac{3}{4}$ of a yard wide, are run lengthwise of the room, and there is a waste of 9 in. on each strip for matching the pattern?

PERCENTAGE AND ITS APPLICATIONS

PERCENTAGE

501. The arithmetical processes in which the basis of comparison is one hundred are termed **percentage.**

502. Per cent, usually written "%," is an abbreviation of the Latin words "per centum," and signifies *by the hundred.*

Thus, eight per cent means eight of every one hundred parts, or .08, and is written 8%; seven and one half per cent means seven and one half of every one hundred parts, or .07½, and is written 7½%.

503. The **essential elements** of percentage are the *base,* the *rate,* and the *percentage.*

504. The **base** is the number upon which the percentage is computed.

505. The **rate** is the number of hundredths of the base to be taken; it is usually expressed as a decimal.

506. The **percentage** is the result obtained by taking a certain per cent of the base; or,

It is the product obtained by multiplying the base by the rate.

In the expression "5% of 500 is 25," the *base* is 500; the *rate,* 5%; and the *percentage,* 25.

507. The **amount per cent** is 100% increased by the rate; or, 1 plus the rate, expressed as a decimal.

508. The **difference per cent** is 100% diminished by the rate; or, 1 minus the rate expressed as a decimal.

509. The **amount** is the base plus the percentage.

510. The **difference** is the base minus the percentage.

511. General Principles. The base may either be an abstract or a denominate number; the rate per cent must always be an abstract number; and the percentage, amount, and difference always have the same name as the base.

512. Since a *per cent* is a number of *hundredths*, it may be expressed either as a *decimal* or as a *common fraction*. The principles of aliquot parts may therefore be used to advantage in many operations in percentage and its applications.

TABLE

PER CENT	DECIMAL VALUE	FRACTIONAL VALUE	PART OF 100%, OR THE BASE	PER CENT	DECIMAL VALUE	FRACTIONAL VALUE	PART OF 100%, OR THE BASE
1%	$.01$	$\frac{1}{100}$	$\frac{1}{100}$	$22\frac{2}{9}\%$	$.22\frac{2}{9}$	$\frac{22\frac{2}{9}}{100}$	$\frac{2}{9}$
$1\frac{1}{4}\%$	$.01\frac{1}{4}$	$\frac{1\frac{1}{4}}{100}$	$\frac{1}{80}$	$28\frac{4}{7}\%$	$.28\frac{4}{7}$	$\frac{28\frac{4}{7}}{100}$	$\frac{2}{7}$
$1\frac{2}{3}\%$	$.01\frac{2}{3}$	$\frac{1\frac{2}{3}}{100}$	$\frac{1}{60}$	$31\frac{1}{4}\%$	$.31\frac{1}{4}$	$\frac{31\frac{1}{4}}{100}$	$\frac{5}{16}$
$2\frac{1}{2}\%$	$.02\frac{1}{2}$	$\frac{2\frac{1}{2}}{100}$	$\frac{1}{40}$	$33\frac{1}{3}\%$	$.33\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$	$\frac{1}{3}$
$3\frac{1}{3}\%$	$.03\frac{1}{3}$	$\frac{3\frac{1}{3}}{100}$	$\frac{1}{30}$	$37\frac{1}{2}\%$	$.37\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$	$\frac{3}{8}$
$6\frac{1}{4}\%$	$.06\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$	$\frac{1}{16}$	$42\frac{6}{7}\%$	$.42\frac{6}{7}$	$\frac{42\frac{6}{7}}{100}$	$\frac{3}{7}$
$6\frac{2}{3}\%$	$.06\frac{2}{3}$	$\frac{6\frac{2}{3}}{100}$	$\frac{1}{15}$	$43\frac{3}{4}\%$	$.43\frac{3}{4}$	$\frac{43\frac{3}{4}}{100}$	$\frac{7}{16}$
$8\frac{1}{3}\%$	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$	$\frac{1}{12}$	50%	$.50$	$\frac{50}{100}$	$\frac{1}{2}$
$9\frac{1}{11}\%$	$.09\frac{1}{11}$	$\frac{9\frac{1}{11}}{100}$	$\frac{1}{11}$	$56\frac{1}{4}\%$	$.56\frac{1}{4}$	$\frac{56\frac{1}{4}}{100}$	$\frac{9}{16}$
10%	$.10$	$\frac{10}{100}$	$\frac{1}{10}$	$62\frac{1}{2}\%$	$.62\frac{1}{2}$	$\frac{62\frac{1}{2}}{100}$	$\frac{5}{8}$
$11\frac{1}{9}\%$	$.11\frac{1}{9}$	$\frac{11\frac{1}{9}}{100}$	$\frac{1}{9}$	$66\frac{2}{3}\%$	$.66\frac{2}{3}$	$\frac{66\frac{2}{3}}{100}$	$\frac{2}{3}$
$12\frac{1}{2}\%$	$.12\frac{1}{2}$	$\frac{12\frac{1}{2}}{100}$	$\frac{1}{8}$	$68\frac{3}{4}\%$	$.68\frac{3}{4}$	$\frac{68\frac{3}{4}}{100}$	$\frac{11}{16}$
$14\frac{2}{7}\%$	$.14\frac{2}{7}$	$\frac{14\frac{2}{7}}{100}$	$\frac{1}{7}$	75%	$.75$	$\frac{75}{100}$	$\frac{3}{4}$
$16\frac{2}{3}\%$	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$	$\frac{1}{6}$	$81\frac{1}{4}\%$	$.81\frac{1}{4}$	$\frac{81\frac{1}{4}}{100}$	$\frac{13}{16}$
$18\frac{3}{4}\%$	$.18\frac{3}{4}$	$\frac{18\frac{3}{4}}{100}$	$\frac{3}{16}$	$83\frac{1}{3}\%$	$.83\frac{1}{3}$	$\frac{83\frac{1}{3}}{100}$	$\frac{5}{6}$
20%	$.20$	$\frac{20}{100}$	$\frac{1}{5}$	$87\frac{1}{2}\%$	$.87\frac{1}{2}$	$\frac{87\frac{1}{2}}{100}$	$\frac{7}{8}$
25%	$.25$	$\frac{25}{100}$	$\frac{1}{4}$	$93\frac{3}{4}\%$	$.93\frac{3}{4}$	$\frac{93\frac{3}{4}}{100}$	$\frac{15}{16}$

DRILL EXERCISE

What common fraction in its simplest form is equivalent to:

1. 1%?	*5.* 20%?	*9.* 100%?	*13.* 16⅔%?	*17.* 125%?
2. 4%?	*6.* 25%?	*10.* 6¼%?	*14.* 33⅓%?	*18.* 150%?
3. 5%?	*7.* 50%?	*11.* 8⅓%?	*15.* 66⅔%?	*19.* 2½%?
4. 10%?	*8.* 75%?	*12.* 12½%?	*16.* 87½%?	*20.* 1⅔%?

Express decimally:

21. 28%.	*24.* 10½%.	*27.* 6⅝%.	*30.* 182%.
22. 35%.	*25.* 7¾%.	*28.* 250%.	*31.* 415%.
23. 50%.	*26.* 9⅛%.	*29.* 137%.	*32.* 106%.

Express as a rate per cent:

33. $\frac{1}{8}$.	*35.* $\frac{1}{10}$.	*37.* $\frac{3}{8}$.	*39.* $\frac{7}{8}$.	*41.* $\frac{2}{3}$.	*43.* $\frac{1}{11}$.
34. $\frac{1}{5}$.	*36.* $\frac{1}{6}$.	*38.* $\frac{5}{8}$.	*40.* $\frac{9}{10}$.	*42.* $\frac{1}{9}$.	*44.* $\frac{1}{20}$.

513. The operations of percentage are based upon the same general principles as the operations of simple multiplication and division, the *base* in percentage corresponding to the *multiplicand* in simple multiplication, the *rate* to the *multiplier*, and the *percentage* to the *product*. Hence, any two of the elements of percentage being given, the other may be found.

514. The formulæ for percentage are derived from the fundamental principles of multiplication and division, as follows:

1. *Multiplicand × multiplier = product; hence, base × rate per cent = percentage.*

2. *Product ÷ multiplicand = multiplier; hence, percentage ÷ base = rate per cent.*

3. *Product ÷ multiplier = multiplicand; hence, percentage ÷ rate per cent = base.*

515. **To find the percentage, the base and rate being given.**

516. **Examples.** *1.* What is 9% of $500?

$500 base.
.09 rate.
$45.00 percentage.

SOLUTION. 9% of a number is .09 of it. Therefore, 9% of $500 is .09 of $500, or $45.

2. What is $12\frac{1}{2}\%$ of $888.80?

(*a*) $888.80 × $.125\%$ = $111.10.

or,

(*b*) $\frac{1}{8}$ of $888.80 = $111.10.

SOLUTIONS. (*a*) $12\frac{1}{2}\%$ of a number is .125 of it; therefore, $12\frac{1}{2}\%$ of $888.80 is .125 of $888.80, or $111.10. Or,

(*b*) $.125\%$ of a number is $.12\frac{1}{2}$, or $\frac{1}{8}$ of it; therefore, $\frac{1}{8}$ of $888.80, or $111.10, is the required result.

517. Hence, to find a percentage of a number,

Multiply the base by the given rate per cent, expressed decimally. Or, *Take such a part of the base as the rate per cent is of 100%.*

DRILL EXERCISE

1. **Formulate a short method for finding $8\frac{1}{3}\%$ of a number.**

SOLUTION. Since $8\frac{1}{3}\%$ of a number is $.08\frac{1}{3}$ or $\frac{1}{12}$ of it, to find $8\frac{1}{3}\%$ of a number, *divide by 12.*

2. Give a short method for finding the percentage when the base is given and the rate is $12\frac{1}{2}\%$; $16\frac{2}{3}\%$; 25%; $33\frac{1}{3}\%$; $9\frac{1}{11}\%$; $28\frac{4}{7}\%$.

3. Formulate a short method for finding 75% of a number; $18\frac{3}{4}\%$ of it; $43\frac{3}{4}\%$ of it; $62\frac{1}{2}\%$ of it; $31\frac{1}{4}\%$ of it.

4. What aliquot part of a number is $93\frac{3}{4}\%$ of it? $83\frac{1}{3}\%$ of it? $66\frac{2}{3}\%$ of it? $22\frac{2}{9}\%$ of it? $87\frac{1}{2}\%$ of it?

5. Express as a rate per cent: $\frac{7}{16}$ of a number; $\frac{3}{8}$ of a number; $\frac{2}{3}$ of a number; $\frac{5}{8}$ of a number; $\frac{9}{16}$ of a number; $\frac{1}{16}$ of a number; $\frac{1}{80}$ of a number; $\frac{1}{40}$ of a number; $\frac{7}{8}$ of a number: $\frac{1}{12}$ of a number.

6. **Give a short method for finding $1\frac{1}{4}\%$ of a number.**

SOLUTION. $1\frac{1}{4}\%$ of a number is $.01\frac{1}{4}$, or $\frac{1}{80}$ of it. Hence, to find $1\frac{1}{4}\%$ of a number, *point off one place and divide by 8.*

7. Give a short method for finding $1\frac{2}{3}\%$ of a number; $2\frac{1}{2}\%$; $3\frac{1}{3}\%$; $11\frac{1}{9}\%$; $14\frac{2}{7}\%$; $6\frac{1}{4}\%$; $56\frac{1}{4}\%$; $6\frac{2}{3}\%$; 20%.

ORAL EXERCISE

By inspection, find the value of:

1. $37\frac{1}{2}\%$ of 160.
2. $62\frac{1}{2}\%$ of 320.
3. $8\frac{1}{3}\%$ of 720.
4. $6\frac{1}{4}\%$ of 960.
5. $14\frac{2}{7}\%$ of 210.
6. 25% of 680.
7. $66\frac{2}{3}\%$ of 930.
8. $2\frac{1}{2}\%$ of 360.
9. $33\frac{1}{3}\%$ of 930.
10. $12\frac{1}{2}\%$ of 880.
11. $6\frac{2}{3}\%$ of 450.
12. $16\frac{2}{3}\%$ of 666.
13. $18\frac{3}{4}\%$ of 480.
14. $31\frac{1}{4}\%$ of 320.
15. $43\frac{3}{4}\%$ of 160.
16. $56\frac{1}{4}\%$ of 800.
17. $62\frac{1}{2}\%$ of 240.
18. 75% of 128.

SHORT METHODS

518. Examples. *1.* What is 36% of $2500?

¼ of $3600 = $900.

SOLUTION. Since 36 times 25 will give the same product as 25 times 36, 36% of $2500 will give the same result as 25% of $3600. 25% is ¼ of a number; therefore, ¼ of $3600, or $900, is the required result.

2. What is 16% of $12,500?

⅛ of 16,000 = $2000.

SOLUTION. 16 times 12½ will give the same product as 12½ times 16; hence, 16% of $12,500 is equivalent to 12½% of $16,000. 12½% is ⅛ of a number; ⅛ of $16,000 is $2000, or the required result.

3. What is 24% of $37,500?

24,000 × ⅜ = $9000.

SOLUTION. 24 times 37½ will give the same product as 37½ times 24; hence, 24% of $37,500 is equivalent to 37½% of $24,000. 37½% is ⅜ of a number; ⅜ of $24,000 is $9000, or the required result.

ORAL EXERCISE

By inspection, find the value of:

1. 12% of $2500.
2. 16% of $2500.
3. 44% of $7500.
4. 48% of $1250.
5. 250% of $64.
6. 75% of $160.
7. 24% of $750.
8. 12½% of $960.
9. 56% of $1250.
10. 16% of $3125.
11. 32% of $5625.
12. 16⅔% of $360.
13. 24% of $62,500.
14. 32% of $3750.
15. 192% of $875.
16. 125% of $888.
17. 1250% of $640
18. 96% of $250.

WRITTEN EXERCISE

1. Having $240,000 to invest, a gentleman bought United States bonds with 33⅓% of it, a home with 25% of the remainder, and invested what still remained equally in farming lands and manufacturing stock. How much did he invest in manufacturing stock?

2. A collector deposited $2400 in coin and 12½% as much in bank bills. What was the amount of his deposit?

3. In a certain barn there are 24,960 bu. of grain, of which 33⅓% is wheat, 12½% oats, and the remainder, barley. How many bushels of each kind of grain are there in the barn?

4. A jobber having 2160 bags of coffee, sold at one time $8\frac{1}{3}$ %, at another, 25 % of what remained, and at a third, sold $33\frac{1}{3}$ % of what still remained. Find the value of what was still left at $18 per bag.

5. A farmer having 156 sheep to shear, agreed to pay for their shearing 4% of the sum received for their wool. If the fleeces average $7\frac{1}{2}$ lb. each, and are sold at 30¢ per pound, how much was paid for shearing?

6. A dealer having bought 240 doz. eggs at 25¢ per dozen, sold $8\frac{1}{3}$ % of them at cost and the remainder at 27¢ per dozen. What was his profit?

7. A farmer having raised 1240 bu. wheat, used 5 % of it for seed and 5 % of it for bread. He then sold to one man $33\frac{1}{3}$ % of the remainder at $1 per bushel, and to another 25 % of what still remained at $1.10 per bushel. How much was received from both sales, and how many bushels were left unsold?

8. A man owning an estate of $200,000 bequeathed 10 % of it to a college, 10 % of the remainder to a church, and divided what still remained equally among his four children. What did each child receive?

519. **To find the rate, the base and percentage being given.**

520. **Example.** A farmer having 480 bu. of wheat, sold 120 bu. What per cent of his wheat did he sell?

(*a*)

$120 \div 480 = .25.$

Solution. (*a*) Since the percentage is the product of the base and rate, the quotient obtained by dividing the percentage by the base will be the rate. Or,

(*b*)

$\frac{120}{480} = \frac{1}{4}.$

$\frac{1}{4}$ of 100 % = 25 %.

(*b*) 120 is $\frac{120}{480}$, or $\frac{1}{4}$, of 480, and since 480 is 100 % of itself, 120, which is $\frac{1}{4}$ of 480, must be $\frac{1}{4}$ of 100 %, or 25 %. Or,

(*c*)

$120 \div 4.80 = 25.$

25 times 1 % = 25 %.

(*c*) Since 480 is 100 % of itself, 1 % of 480 would be $\frac{1}{100}$ part of it, or 4.80. Since 4.80 is 1 % of 480, 120 would be as many times 1 % as 4.80 is contained times in 120, which is 25 times; 25 times 1 % is 25 %.

521. Hence, to find the rate,

Divide the percentage by the base and express the number of hundredths obtained as a rate per cent.

ORAL EXERCISE

What per cent of:

1. **$50 are $5?**

2. **15 hr. are 45 hr.?**

3. **24 bu. are 48 bu.?**

4. **15 A. are 5 A.?**

5. **12 da. are 4 da.?**

6. **$\frac{1}{2}$ bu. is $\frac{1}{4}$ bu.?**

7. **$\frac{1}{8}$ yd. is $\frac{1}{16}$ yd.?**

8. **2.4 are 3.6?**

9. **.12 are .24?**

10. **.24 are .12?**

11. **48 hr. are 36 hr.?**

12. **$160 are $40?**

WRITTEN EXERCISE

1. Of a stock of 800 yd. of prints 240 yd. were sold at one time and 160 yd. at another. What per cent of the whole stock was still unsold?

2. Of a regiment of men entering battle 1040 strong only 260 came out unhurt, $\frac{1}{3}$ of the remainder having been killed. What per cent of the whole regiment was killed?

3. Out of 900 bu. of potatoes put in storage October 15, 45 bu. were found unsound April 1. What per cent of the whole was sound?

4. A merchant's profits for 1903 were $3800, or $200 in advance of his profits for 1902. Find the per cent of increase in the profits of 1903 over those of 1902.

5. From a cask of lard containing 320 lb. 70 lb. were sold at one time and 30% of the remainder at another. What per cent of the whole remained unsold?

6. In a certain school there are 1800 male pupils and 200 female pupils. What per cent more are the male than the female pupils?

7. A merchant failed in business, having resources amounting to $15,000 and liabilities amounting to $75,000. What per cent of his debts can he pay?

8. What per cent more is $\frac{1}{2}$ than $\frac{1}{4}$?

9. What per cent less is $\frac{1}{8}$ than $\frac{1}{4}$?

10. A has 25% less money than B. What per cent has B more than A?

522. **To find the base, the percentage and rate being given.**

523. **Example.** 375 is 12½% of what number?

(*a*)

$$\begin{array}{r} 3\,000 \\ .125\overline{)375.000} \\ \underline{375\,000} \end{array}$$

SOLUTIONS. (*a*) 12½% of a number is equal to .125 of it. If .125 of a number is 375, the whole number may be found by the principles of division of decimals (**305**). Or,

(*b*)

$$\begin{aligned} 12\tfrac{1}{2}\% &= 375. \\ 1\% &= 30. \\ 100\% &= 3000. \end{aligned}$$

(*b*) If 12½% of a number is 375, 1% of the number is $\frac{2}{25}$ of 375, or 30, and 100% or the whole of the number is 100 times 30, or 3000. Or,

(*c*)

12½%, or ⅛, of a number = 375.
⅜ = 8 times 375, or 3000.

(*c*) 12½% of a number is ⅛ of it. If ⅛ of a number is 375, ⁸⁄₈, or the whole number, is 8 times 375, or 3000. Therefore, the required result is 3000.

524. Hence, to find the base,

Divide the percentage by the rate.

ORAL EXERCISE

1. 18 is ¼ of what number?

2. Of what number is 16 25%?

3. 3215 is 41¼% of what number?

4. Of what number is 125 62½%?

5. Of what number is 12½ 5%?

6. 555 is 5% of what number?

7. 19 is 16⅔% of what number?

8. 90 is ¾ of what number?

9. A man's yearly expenses are $150, or 12½% of his income. What is his income?

WRITTEN EXERCISE

1. 2½% of 240 is 25% of what number?

2. On a bill of $1280 $64 discount was allowed. What was the per cent of discount?

3. Jan. 1 I paid William Mason & Co. 75% of my indebtedness to them by a New York draft for $5100. Jan. 8 they sent me goods amounting to $200. Feb. 1 I sent them a check in full of account. What was the amount of the check?

4. Of a shipment of strawberries 10% was damaged, and the remainder, which were sold at 8¢ per quart, brought $146.88. How many quarts were there in the shipment?

5. A grocer, after increasing his stock by goods costing $6448, found that the new purchase was 16% of the old stock on hand. What was the value of his old stock?

6. From his bank account a man checked out ½ of 30% of his money, and after depositing $750 he found that he had in the bank 105% of his original deposit. How much had he in the bank at first?

7. In an orchard 50% of the trees are apple, 10% peach, 20% pear, and the remainder, which are 15 more than 12½% of the whole, are plum. How many trees in the orchard?

8. A floor has a perimeter of 84 ft. If the width is 75% of the length, what will it cost to paint the floor at 22¢ per square yard?

9. The perimeter of the ceiling of a hall is 130 ft. If the width of the hall is 62½% of the length and the height is 80% of the width, and 360 sq. ft. are deducted for openings, what will it cost to paint the walls and ceiling of the hall at 15¢ per square yard?

10. A farmer sold 18 bu. 2 pk. and 1 qt. of tomatoes, which was 5% of his whole crop. How many bushels of tomatoes did he raise?

525. To find the base, the amount and rate of increase being given.

526. Examples. *1.* What number, increased by 87% of itself, is equal to 561?

Solution

Represent the number by 100%.
87% = the increase.
187% = the number, increased by 87% of itself.
561 = the number, increased by 87% of itself.
Therefore, 187% of the number = 561.
1% = $\frac{1}{187}$ of 561, or 3.
100%, or the number, = 100 times 3, or 300.
Hence, the required result is 300.

527. Therefore the following **rule:**

Divide the amount by 1 plus the rate.

ORAL EXERCISE

What number increased by:

1. 10% of itself is 77?

2. 33⅓% of itself is 36?

3. 12½% of itself is 18?

4. 20% of itself is 240?

5. 40% of itself is 28?

6. 50% of itself is 63?

7. 125% of itself is 675?

8. 40% of itself is 280?

9. 190% of itself is 580?

10. 200% of itself is 150?

11. 300% of itself is 1600?

12. 6⅔% of itself is 160?

WRITTEN EXERCISE

1. I sold goods for $750 and gained 25%. What did they cost?

2. I gained 35% by selling goods for $540. What was the cost?

3. A real estate dealer gained 35% by selling a house for $27,000. What was its cost?

4. I sold 945 tubs of butter for $5103, thereby gaining 20%. How much did the butter cost per tub?

5. A drover gained 18⅔% on 33 head of cattle sold for $1096.48. What was the average cost per head?

6. The attendance of pupils at school during May was 954, which was 6% more than attended during April, and this was 80% more than attended during February. What was the attendance for February?

7. The population of a certain town has increased 12½% during the past three years. If the present population is 40,590, what was it three years ago?

8. B sold a farm to C for $12,000, thereby gaining 20%. What was A's cost if in selling the same farm to B he made a profit of 25%?

9. A farmer's wheat crop this year is 15½% greater than last year's crop. What was last year's crop if in the two years he raised 1206⅘ bu.?

10. A certain number plus 17% of itself, plus 6% of 3 times itself, is equal to 270,135. What is the number?

528. To find the base, the difference and rate of decrease being given.

529. Example. What number, decreased by 35% of itself, equals 1300?

SOLUTION

Represent the number by 100%.
35% = the decrease.
65% = the number after decrease.
1300 = the number after decrease.
Therefore, 65% = 1300.
1% = $\frac{1}{65}$ of 1300, or 20.
100% = 100 times 20, or 2000, the required result.

530. Therefore the following rule:

Divide the difference by 1 minus the rate.

ORAL EXERCISE

What number diminished by:

1. 8% of itself equals 184?

2. 16⅔% of itself equals 55?

3. 12½% of itself equals 77?

4. 25% of itself equals 33?

5. 50% of itself equals 27½?

6. ⅖ of itself equals 339?

7. I sold goods for $600 and lost 40%. What did they cost?

8. Brown deposited $850 in a savings bank, which was 15% less than that deposited by his son. How much was deposited by both of them?

9. An agent earned 15% less in May than he did in June. If he earned $370 in the two months, how much did he earn in June?

WRITTEN EXERCISE

1. A boat load of wheat was so damaged that it sold for $8500, which was 15% less than its original value. What was its value before it was damaged?

2. After paying 37½% of his debts a man found that the remainder could be paid with $13,025. What was his original indebtedness?

3. Smith sold two horses for $1500 each, gaining 25% on the first and losing 25% on the second. What did the horses cost him?

4. A liveryman paid $456 for a horse and carriage. If the cost of the carriage was 48% less than the cost of the horse, what was the cost of each?

5. A merchant's sales for Wednesday were 50% greater than his sales for Tuesday, which were 20% less than his sales for Monday. If the sales for the three days aggregated $1500, what were the sales for each day?

6. In selling a suit of clothes for $21.60 a merchant lost 20%. Find the asking price if it was 20% above cost.

7. A man bought a watch, and had $125 remaining. He then bought another watch costing twice as much as the first, and still had left $14\frac{2}{7}$% of his money. How much money had he at first?

8. Divide $1600 between A and B so that B shall have 40% less than A.

WRITTEN REVIEW

1. A benevolent lady gave $10,500 to three charities. To the first she gave $2500, to the second $4500, and to the third the remainder. What per cent did each receive?

2. 25% of B's money equals 75% of A's. How much has A if B has $900?

3. A creditor, after collecting $33\frac{1}{3}$% of a claim, lost the remainder, which was $3918.75. What sum was collected?

4. A has 185% more money than B. How much has each if they together have $9625?

5. The sum paid for two farms was $19,200. $37\frac{1}{2}$% of the sum paid for one equals $62\frac{1}{2}$% of the sum paid for the other. Find the price of each.

6. If a gain of $4775 was realized on a business at the end of the first year, and a loss of $3586.25 was sustained the second year, what was the per cent of net gain or loss for the two years, the investment having been $63,400?

7. After making 7 of the 10 annual payments of the face of a mortgage I find $5850 to be still unpaid. How many dollars have been paid?

8. A has 50% more money than B. What per cent has B less than A?

9. The population of a certain city is 238,375. During the last three years the population has increased yearly 25%. What was the population three years ago?

10. 80% of a mixture of vinegar and water is vinegar. If there were 10 gallons more of vinegar, the mixture would be 85% vinegar. How many gallons of water are in the mixture?

11. From an estate the widow received $9250, which was $\frac{1}{3}$; the remainder was divided among three children aged respectively 15, 12, and 10 years, and they share in proportion to their age. What per cent of the estate did each of the children receive?

12. From a farm containing 180 acres 120 square rods, 50% was sold at one time and 50% of the remainder at another time. What per cent of the whole then remained?

13. After drawing 25% of his deposit from a bank to pay a debt a man finds that he has left in the bank $6756.25. What was the amount of his indebtedness, and how much had he in the bank before drawing the check?

14. A man sold two farms for $8000 each, receiving for one 20% more than it cost and for the other 20% less than it cost. Did he gain or lose by the sale, and how much?

15. During the first, second, and third years a manufacturer realized gains amounting to 25%, 20%, and 45%, respectively, of his original investment. During the fourth year he lost $9000, which was 25% of his original capital. Find his net gain for the four years.

16. A manufacturer's capital was increased during the first year by profits equal to 25% of his original investment; the second year by profits equal to 20% of his capital at the beginning of that year; and the third year it was diminished by a loss equal to 25% of the capital at the beginning of that year. If his profits during the three years exceeded his losses by $8000, what was his original investment?

17. In settling an estate an executor found 7½% of it to be invested in telegraph stock, 15% in railroad stock, 37½% in United States bonds, $16,750 in real estate, and $7350 cash in bank. Find the total value of the estate.

18. A horse is worth 25% more than a carriage, and the carriage is worth 300% of the harness. If the horse is worth $37.50 more than the carriage, what is the value of each?

19. A merchant sold 2 horses for $140 each. On one he gained 25% and on the other he lost $28\frac{4}{7}$%. How much did the horses cost him, and what was the gain or loss?

20. Express .00025 as a per cent; $\frac{1}{80}$% as a decimal; 36.42 as a per cent.

21. Find 3% of 9 t. 7 cwt. 16 lb.

22. I paid for transportation on an invoice of goods $600. I later sold the goods at a profit of 20% on the full cost, receiving $9696.60. What was the first cost of the goods? What per cent was the value of the goods increased by the transportation charges?

23. A last will and testament provided that $\frac{3}{8}$ of an estate distributed should go to the widow and the remainder be so divided among two sons and a daughter that the elder son should receive 10% more than the younger, who should receive 25% more than the daughter. What amount was received by each, the estate being valued at $58,000?

24. A father located his son upon a farm, expending for the farm, stock, utensils, and household furniture $19,512.50. The stock cost twice as much as the household furniture, which cost 75% more than the farm utensils, and the cost of the farm was 140% of the cost of the stock. How much was invested in each?

25. A creditor agrees to receive $962.50 for the full amount of a debt. If this settlement is at the rate of 25¢ on the dollar, what was the original amount of the debt?

26. A manufacturer failing in business finds that his net resources aggregate $12,600. If he can pay 75¢ on the dollar on 20% of his debts, and 60¢ on the dollar on the remainder, what is the amount of his indebtedness.

27. Twice $\frac{1}{2}$ of a number is what per cent of 3 times $\frac{1}{4}$ of it?

28. A farm is composed of 20% more grazing than grain land, and the timber is $\frac{1}{2}$ of the area. How many acres of each are there if after deducting 12 acres for lawn and garden the area of the farm is 1860 acres?

29. A man withdrew 25% of his bank deposit and spent 20% of the money to pay for 20% of his indebtedness to Smith & Co. If his indebtedness to Smith & Co. was $1800, what was the original amount of his bank account?

30. A merchant mixed 100 lb. of coffee at 25¢ per pound with 50 lb. at 30¢ per pound, and sold the mixture for 40¢. What per cent of profit does he make?

531. Applications of percentage. Percentage is applied to two general classes of problems:

1. Those in which time is not an element; as, *Commercial Discounts*, *Gain and Loss*, *Commission*, *Insurance*, *Taxes*, and *Customs*, or *Duties*.
2. Those in which time enters as an element; as, *Interest*, *Bank Discount*, *Present Worth and True Discount*, *Equation of Accounts*, and *Exchange*.

COMMERCIAL DISCOUNTS

532. Discount is an allowance made for the payment of a debt before it becomes due.

533. Commercial discounts are reductions from the *fixed* or *list* prices of articles, the amount of a bill of merchandise, or of any other obligation.

534. Commercial discounts embrace ***trade discounts***, ***time discounts***, and ***cash discounts***.

535. Trade discounts are reductions from the fixed or list prices of articles.

536. Time discounts are reductions from the amount of a bill of merchandise for payment within a definite time.

537. Cash discounts are reductions made for the immediate payment of a bill of merchandise sold on time.

Business houses usually announce their terms upon their billheads; as "Terms: 3 months, or 5% off for cash;" "Terms: 60 days, or 3% discount in 10 days;" etc. When bills are paid before maturity, legal interest for the remainder of the time is usually deducted.

Trade discounts are deducted from the list price when goods are billed. Time discounts are deducted when a bill is paid. Cash discounts are deducted from the amount of the bill when the sale is made.

538. It is customary for manufacturers, jobbers, and wholesale dealers to have fixed price lists for their goods. Trade discounts are usually made to obviate the necessity of changing these price lists from time to time as the market changes. As the market varies, instead of changing their price lists or issuing new catalogues, merchants raise or lower their rates of discount.

539. The fluctuations of the market sometimes give rise to two or more discounts known as a **discount series.** If two or more discounts are quoted, the first denotes a discount off the list price, the second off the remainder, and so on.

540. The list price is called the **gross price,** and the price after the discount has been deducted, the **net price.**

541. Commercial discounts are usually computed by the rules of percentage, the *list price* or the *amount* of the bill or debt corresponding to the *base,* the *per cent of discount* to the *rate,* the *discount* to the *percentage,* and the *net price* or *net amount* of the bill or debt to the *difference.*

542. To find the net selling price, the list price and discount series being given.

543. Examples. *1.* Find the net amount of a bill of $450 after a discount of $33\frac{1}{3}\%$ is made.

SOLUTION

$450 = the list price.
$33\frac{1}{3}\%$, or $\frac{1}{3}$, of $450 = $150, the discount allowed.
$450 − $150 = $300, the net amount of the bill.

2. The list price of a piano is $800. What is the net price if a discount series of 25% and 20% is allowed?

SOLUTION

$800 = the list price.
25%, or $\frac{1}{4}$, of $800 = $200, the first discount.
$800 − $200 = $600, the remainder after the first discount.
20%, or $\frac{1}{5}$, of $600 = $120, the second discount.
$600 − $120 = $480, the net price.

544. Therefore the following rule:

Deduct the first discount from the list price, and each subsequent discount from each successive remainder.

The last remainder is the net selling price.

The order in which the discounts of any series are considered is not material, a series of 25%, 15%, and 10% being the same as one of 15%, 10%, and 25%, or 10%, 25%, and 15%.

ORAL EXERCISE

1. Find the net cost of a piece of glass listed at $3.60 and discounted 25%.

2. A merchant sold a bill of goods amounting to $4.50 on which he allowed 20% discount. What was the net amount of the bill?

3. Find the net amount of a bill of $18, the discount being 11⅑%.

4. What is the net amount of a bill of $450, the discounts being 33⅓% and 20%?

5. A piano listed at $450 is sold less 33⅓% and 10%. What is the net cost to the purchaser?

6. Goods listed at $27 are sold less 33⅓% and 16⅔%. What is the net selling price?

WRITTEN PROBLEMS

Find the net amount of the following bills:

1. $1550 less 33⅓% and 20%.

2. $840 less 25% and 10%.

3. $3500 less 20% and $14\frac{2}{7}$%.

4. $395 less 20% and 20%.

5. A wholesale dealer offers cloth at $2.40 per yard subject to a discount of 25%, 20%, and 5%. How many yards can be bought for $492.48?

6. Find the net price of 2 tons of fence wire listed at 3¢ per pound and sold 20% and 25% off.

7. One drummer offers to sell me $1500 worth of iron pipe at a discount of 25%, 10%, and 10%; another offers to sell me a similar quantity of pipe for the same amount less 20%, 20%, and 5%. Which is the better offer, and what is the difference expressed in dollars?

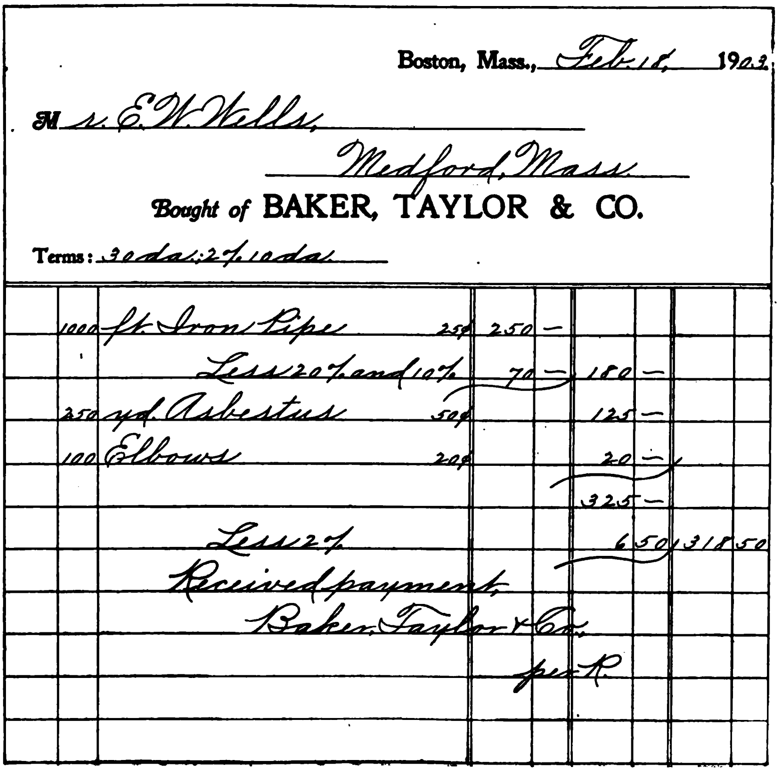
8. Having bought $1500 worth of merchandise at 20% and 25% off, I sold it for $1500 less 15%, 10%, and 20% off. Did I gain or lose, and how much?

9. Books purchased at 25% and 20% off from the list price were sold at the list price. What was the gain per cent? What was the cost of a shipment which sold for $700?

10. A bill of hardware is sold as follows: $25.50 at 20%; $4.50 at 20% and 25%; $153 at $33\frac{1}{3}$% and 10%; $267.50, net. If a further discount of 2% is allowed for immediate payment, what is the net amount of the bill?

545. To find the net amount of a bill to render, the terms and discount series being given.

546. Example. Render a bill for the following transaction: Feb. 18, 1903, E. W. Wells, Medford, Mass., bought of Baker, Taylor & Co., Boston, Mass.: 1000 ft. iron pipe at 25¢ less 20% and



Boston, Mass., Feb. 18, 1903

Mr. E. W. Wells,
Medford, Mass.

Bought of BAKER, TAYLOR & CO.

Terms: 30 da.; 2% 10 da.

1000	ft. Iron Pipe	25¢	250	—				
	Less 20% and 10%		70	—	180	—		
250	yd. Asbestus	50¢			125	—		
100	Elbows	20¢			20	—		
					325	—		
	Less 2%				6	50	318	50
	Received payment,							
	Baker, Taylor & Co.							
	per R.							

10%; 250 yd. asbestus at 50¢; 100 elbows at 20¢. Terms: 30 da., 2% 10 da. The bill is paid for on the date of the sale.

SOLUTION. The iron pipe is subject to a discount series of 20% and 10%; hence, $70 should be deducted from the list price of this part of the bill. The amount of the bill is then found to be $825. If the bill is paid within 10 days, it is subject to a cash discount of 2%. It is paid on the date of the sale; hence, 2% is deducted from $825, and the net amount of the bill is $818.50. (See model, page 201.)

WRITTEN EXERCISE

1. C. A. Martin, Albany, N.Y., bought of Lincoln Bros., Boston, Mass., Feb. 18, 1903, on account 30 da., 2% 10 da.: 1800 ft. iron pipe at 32¢ less 25% and 10%; 65 valves at $2; 130 elbows at 20¢ less 20% and 10%; 290 yd. asbestus at 54¢. Find the net amount of the bill Feb. 20, 1903.

2. Ames, Osborne & Co., Troy, N.Y., sold W. D. Green, Buffalo, N.Y., Feb. 25, 1903, on account 30 da., 2% 10 da.: 135 cultivators at $8.50; 25 doz. bread knives at $12.50; 250 lb. machine bolts at 12½¢; 25 doz. table knives at $11; 15 doz. K hammers at $6.25. Find the net amount due on the bill Feb. 25, if each of the above items is subject to discounts of 20%, 10%, and 5%.

3. E. M. Williams & Co., Bath, Me., sold Smith, Perkins & Co., Rochester, N.Y., Apr. 2, 1904, on account 30 da., 2% 10 da.: 15 cultivators listed at $7.50 each, less 20% and 10%; 25 doz. table knives listed at $9.75, less 10%; 5 doz. pocket knives, 3 doz. at $6.50 and 2 doz. at $7.50, less 33⅓% on each; ¾ doz. cheese knives at $9.75, less 16⅔%. Find the net amount due on the bill 9 days after date.

4. W. D. Brown & Co., Rochester, N.Y., bought of Mason & Hamlin, Buffalo, N.Y., Apr. 1, 1903, on account 60 da., 5% 10 da.: 5 pianos at $450 each, 6 pianos at $575 each, 4 pianos at $250 each, less a discount of 40% from each list price; 10 organs at $125 each, less 25% and 10% from the list price. Find the amount of the bill to render, also the amount to be remitted if the bill is paid April 8, 1903.

5. E. L. Emery, Boston, Mass., sold S. E. Waldruff, Albany, N.Y., Jan. 6, 1904, on account 60 da., 3% 10 da.: 25 sewing machines at $80 each, less 33⅓%; 10 sewing machines at $65 each, less 16⅔%; 5 sewing machines at $80 each, less 10% and 10%. Find the net amount of the bill to render and the amount to remit if paid within 10 da.

6. I bought a phaeton of F. L. Cunningham & Co.; list price $325, less 20% and 10%; terms, 90 da., or 5% off for immediate payment. How much ready money will settle the bill? What is the amount of the trade discount? Of the cash discount?

7. What is gained by selling 25 doz. pairs gloves at $1.25 per pair if they cost $7.50 per doz. less 16⅔% and 10%?

8. An agent bought 15 sewing machines listed at $40 each, less 25% and 10%, and sold them at $45 each. What was the amount of his gain?

547. To find a single rate of discount equivalent to a discount series.

548. Example. What single rate of discount is equal to a discount series 25%, 20%, and 10%?

1.00
.25 (25% or ¼ of 100%).
.75
.15 (20% or ⅕ of 75%).
.60
.06 (10% or $\frac{1}{10}$ of 60%).
.54
100% − 54% = 46%.

Solution. Let 100% represent the gross cost. 25%, or ¼ of 100%, equals 25%, which, subtracted from 100%, leaves 75%. 20%, or ⅕ of 75%, equals 15%, which, subtracted from 75%, leaves 60%. 10%, or $\frac{1}{10}$ of 60%, equals 6%, which, subtracted from 60%, leaves 54%. 100%, the gross cost, minus 54%, the net selling price, equals 46%, the direct discount.

549. Therefore the following rule:

From 100% as the list price deduct each discount in order. Subtract the final remainder from the list price, and the result will be the single discount equivalent to the given series.

SHORT METHODS

550. Since the first of the series of discounts is computed on the basis of 100%, and the second on the difference, or 100% minus the first discount, *the sum of any two discounts is greater than the equivalent direct discount by the product of the two rates per cent.*

Thus, in a discount series of 25% and 20%, the apparent discount is their sum, or 45%; since the second discount is not computed on 100%, but on 75%, 45% is too large by 20% of 25%, or 5%, and the equivalent direct discount is 45% − 5% or 40%. Hence,

551. **To find, mentally, a single discount equivalent to a series of two discounts,**

From the sum of the discounts subtract their product, and the remainder will be the direct discount.

ORAL EXERCISE

By inspection find a single rate of discount equivalent to the following discount series:

1. 20% and 10%.
2. 10% and 10%.
3. 25% and 10%.
4. 30% and 10%.
5. 20% and 5%.
6. 10% and 5%.
7. 40% and 10%.
8. 25% and $33\frac{1}{3}\%$.
9. 20% and $33\frac{1}{3}\%$.
10. 15% and 10%.
11. 20% and $12\frac{1}{2}\%$.
12. 20% and 20%.
13. 25% and 25%.
14. 5% and 5%.
15. 60% and 25%.
16. 25% and 20%.
17. 30% and 20%.
18. 10% and 30%.
19. 35% and 20%.
20. 20% and 15%.
21. 10% and $12\frac{1}{2}\%$.
22. 10% and 6%.
23. 15% and 6%.
24. 25% and 8%.
25. $33\frac{1}{3}\%$ and 6%.
26. 40% and $12\frac{1}{2}\%$
27. $11\frac{1}{9}\%$ and 18%
28. $8\frac{1}{3}\%$ and 24%.
29. $16\frac{2}{3}\%$ and 12%
30. $14\frac{2}{7}\%$ and 35%

552. When a discount series consists of three rates of discount, combine the first two, and then the result obtained and the third.

553. **Example.** Find a single rate of discount equivalent to a series of 25%, 20%, and 10%.

Solution. Combine the first two by saying or thinking, $25\% + 20\% - 5\% = 40\%$, or the single discount equivalent to the series of 25% and 20%.

$40\% + 10\% - 4\% = 46\%$, or the single rate of discount equivalent to the series 25%, 20%, and 10%.

ORAL EXERCISE

By inspection find a single rate of discount equivalent to the following series:

1. 20%, 25%, and 20%.
2. 10%, 10%, and 20%
3. 20%, 5%, and 10%.
4. 30%, 20%, and 10%.
5. 20%, $12\frac{1}{2}\%$, and 10%.
6. 20%, 20%, and 10%.
7. 20%, 15%, and 10%.
8. 25%, $33\frac{1}{3}\%$, and 10%

WRITTEN EXERCISE

1. From a list price I discounted 30%, 25%, and 20%. What per cent better for the purchaser would a single discount of 70% have been?

2. A merchant purchasing a bill of goods was allowed discounts from the list price of 15%, 10%, 10%, and 6%. If the total discount allowed was $352.81, what must have been the asking price of the goods?

3. Goods were sold 25%, 35%, 20%, and 15% off. If the total discount allowed was $334.25, what must have been the net selling price of the goods?

4. The net amount of a bill was $1080 and the discounts were 20%, 25%, and 10%. Find the amount of the discount allowed.

5. Goods sold on account 30 days, 2% 10 days, are paid for on the date of sale. What must have been the gross amount of a bill of goods on which $588 cash was paid if the discount series was 25% and 20%?

6. If the list price on an article is 25% advance on the cost, what other per cent of discount than 10% must be allowed to net 10% gain by the sale?

554. **To find the net cost of goods, the list price and discount series being given.**

555. **Example.** A piano is listed at $450 and the discounts are 20% and 25%. Find the net cost to the purchaser.

$100\% - 40\% = 60\%.$

$60\% \text{ of } \$450 = \$270.$

SOLUTION. Mentally determine a single rate of discount equivalent to a series of 25% and 20%. This is found to be 40%. Represent the gross cost by 100%. 100% minus 40%, the direct discount, leaves 60%, the net cost to the purchaser. 60% of the gross cost is $270, or the net cost to the purchaser.

556. Hence the following rule:

Multiply the list price by the difference per cent, and the product will be the net cost.

When it is not desirable to show the discounts on a bill, the above method will be found the most practicable for finding the net cost.

ORAL EXERCISE

Find the net cost of articles listed at:

1. $200 less 20% and 10%.

2. $300 less 20% and 15%.

3. $600 less 25% and 20%.

4. $200 less 25% and 8%.

5. $1000 less 30% and 5%.

6. $18.50 less 25% and 20%.

WRITTEN EXERCISE

1. Find the net cost of articles listed as follows: lead pipe, 65¢, discount, 40% and 10%; iron pipe, 30¢, discount, 45% and 20%; bath tubs, $12, $10, and $8, discount, 20% and 10%.

2. Five pianos listed at $425 each were sold at a discount of 20% and 25%. If the freight was $27.50 and the drayage $15, what was the net amount of the bill?

3. Find the net cost of an organ listed at $175, subject to a discount of 20% and 10%.

4. Which is the cheaper, and how much, on a bill of $500, a discount series of 60%, 20%, and 10%, or a discount series of 50%, 40%, and 10%?

5. To the net cost of an article $30 was added for freight, making the total net cost $120. What was the list price, the rates of discount being 50% and 5%?

GAIN AND LOSS

557. The **gains** and **losses** arising from business transactions are frequently computed as percentages of the cost.

558. The first cost of goods is called the **prime cost**. The prime cost, increased by all direct outlays incident to the purchase and holding of the goods to the date of sale, such as packing, freight, cartage, storage, commission, etc., is called the **gross** or **full cost**.

559. The actual amount arising from the sale of goods is called the **gross selling price**. The gross selling price, less all charges incident to the sale of the goods, is called the **net selling price**.

560. The difference between the net selling price and the gross cost of the goods is the **gain** or **loss**, — a gain if the selling price is the larger, and a loss if the gross cost is the larger.

561. In ascertaining the gain or loss the operations are usually performed by the rules of percentage, the *gross cost* corresponding to the *base*, the *per cent of gain or loss* to the *rate*, the *gain or loss* to the *percentage*, the *net selling price*, if at a gain, to the *amount*, and the *net selling price*, if at a loss, to the *difference*.

562. **To find the gain or loss, the cost and per cent of gain or loss being given.**

ORAL EXERCISE

By inspection find the gain or loss in each of the following problems:

	Cost	Per Cent of Gain		Cost	Per Cent of Gain		Cost	Per Cent of Loss
1.	$3600	25	*6.*	$280	14 2/7	*11.*	$2500	36
2.	$2500	36	*7.*	$1500	66 2/3	*12.*	$1250	16
3.	$12,500	16	*8.*	$960	33 1/3	*13.*	$480	37 1/2
4.	$250	24	*9.*	$420	28 4/7	*14.*	$3200	31 1/4
5.	$3750	32	*10.*	$500	25	*15.*	$630	18 3/4

16–30. Find the selling price in each of the above problems.

WRITTEN EXERCISE

1. A grocer bought 10 bbl. of sugar, each weighing 330 lb., at 4½¢ per pound, and sold them so as to gain 16⅔%. Find the gain and the selling price.

2. A stock of goods consisting of $25,000 worth of groceries was sold at a loss of 12½%, and 15% of the selling price was in uncollectible accounts. What was the total loss sustained?

3. A produce dealer paid $320 for apples, $90 for onions, and $120 for potatoes. He sold the apples at a gain of 25%, the onions at cost, and the potatoes at 95% of their cost. Did he gain or lose, and how much?

4. A man bought three horses, paying respectively $240, $300, and $500. He sold the first at 125% of its cost, the second at a loss of 10%, and the third at a gain of 15%. Did he gain or lose, and how much?

5. A dry goods merchant bought a bill of goods amounting to $175. He sold 14 2/7% of the bill and realized a gain equal to 50% of the cost of the whole bill. If the remainder of the stock was sold for $100, what was the gain or loss?

563. To find the rate of gain or loss, the cost and gain or loss being given.

ORAL EXERCISE

By inspection find the per cent of gain or loss in each of the following problems:

	Cost	Gain		Cost	Gain		Cost	Loss		Cost	Loss
1.	$125	$12.50	*5.*	$200	$400	*9.*	$1.60	20¢	*13.*	$25	$6.25
2.	$150	$7.50	*6.*	$240	80¢	*10.*	$3.60	40¢	*14.*	$92	$23
3.	$380	$76	*7.*	$100	$250	*11.*	$1.35	45¢	*15.*	$85	$17
4.	$2	$1	*8.*	$250	$100	*12.*	$190	$76	*16.*	$420	$70

17–32. Find the selling price in each of the above problems.

33. What per cent is gained by selling an article for twice its cost? Three and one-fifth times its cost?

34. A speculator bought a quantity of wheat at 80¢ per bushel and sold it at $1 per bushel. How many bushels did he buy if his gain was $20?

35. If a merchant sells 3 lb. of sugar for what 4 lb. cost him, what is his gain per cent?

WRITTEN EXERCISE

1. Wheat bought at 85¢ per bushel is sold at $1.05 per bushel. How many bushels must be handled to realize a profit of $400?

2. If I bought handkerchiefs at $3.25 per dozen and retailed them at 35¢ each, what was my gain per cent?

3. A coal dealer buys his coal at the mines by the long ton. If he sells at an advance of $33\frac{1}{3}\%$ on the cost, and uses the short ton weight, what is his gain per cent?

4. If $\frac{4}{5}$ of an article is sold for what $\frac{3}{4}$ cost, what is the loss per cent?

5. If $\frac{1}{2}$ of an article is sold for what $\frac{1}{3}$ cost, what is the loss per cent?

6. If $\frac{1}{3}$ of an article is sold for what $\frac{1}{2}$ cost, what is the gain per cent?

7. Paper bought at $2.70 per ream is retailed at 1¢ a sheet. What is the per cent of gain?

8. A merchant bought a stock of goods amounting to \$8500, and after disposing of a part of it for \$7500 he took account of the stock remaining unsold and found that at cost prices it was worth \$2700. What was the per cent of gain on the sales?

564. To find the cost, the gain or loss and the per cent of gain or loss being given.

ORAL EXERCISE

Find the cost if:

1. 25% loss = \$30.
2. 20% gain = \$1.50.
3. 30% loss = \$2.10.
4. $\frac{1}{2}$% loss = \$100.
5. $\frac{1}{8}$% gain = \$30.
6. 125% gain = \$3.75.
7. 15% gain = \$150.
8. $14\frac{2}{7}$% gain = \$12.
9. 22% gain = \$880.

10–18. Find the selling price in each of the above problems.

19. What must have been the cost of a stock of goods if the owner, by selling at a gain of $12\frac{1}{2}$%, received \$450 more than the cost?

WRITTEN EXERCISE

1. A dealer sold 35% of a purchase of leather at $14\frac{2}{7}$% gain and the remainder at 5% loss. If his net gain was \$87.50, what must have been the cost?

2. A merchant bought goods and paid freight on them equal to 12% of their cost. He then sold them at $6\frac{1}{4}$% profit on the full cost of the goods, receiving 60% of the price in cash, and a note for the remainder. If the amount of the note was \$1309, what was the first cost of the goods?

3. A dry goods merchant's gain in business for four years aggregated 50% of his capital. If his gain was \$5000 and he withdrew it and his capital and invested the total in a farm, consisting of 375 A., what was the price paid per acre?

4. Having bought a house of A at $12\frac{1}{2}$% less than it cost him, I spent \$430 for repairs and sold it for \$7293, thereby gaining 10% on my investment. How much did the house cost A?

5. A man sold a horse at $33\frac{1}{3}$% profit. He put with the sum received \$50 and bought a piano, which he sold at 20% gain. If his total gain was \$100, what was the cost of the horse?

565. To find the cost, the selling price and the per cent of gain or loss being given.

ORAL EXERCISE

Find the cost when the selling price at:

1. 5% gain = $105. *4.* 125% gain = $225. *7.* 140% gain = $480.

2. 20% gain = $240. *5.* 12½% loss = $140. *8.* 14²⁄₇% loss = $2400.

3. 16⅔% gain = $14. *6.* 33⅓% loss = $360. *9.* 11⅑% loss = $3200.

10–18. Find the gain or loss in each of the above problems.

19. A fruit dealer, after losing 12½% of his apples by frost, had 150 barrels left. If he bought the apples at $2 per barrel and sold at $3, what was his gain?

WRITTEN EXERCISE

1. I sold a house to B at 10% profit. B sold it to C, gaining 15%, and C, by selling it to D for $15,939, gained 20% on his purchase. How much did the house cost me?

2. I sold two watches at the same price. On one I gained 25%, and on the other I lost 25%. If my total loss was $10, what was the cost of each?

3. A sold a stock of silks to B at a gain of 25%; B sold the same stock to C at a gain of 10%. If C's cost was $375 more than A's, what did the silks cost A?

WRITTEN REVIEW

1. What amount of money must an attorney collect in order that he may pay over to his client $1700 and retain 15% for his services?

2. In selling an article for $162 an art dealer cleared 12½%. At what per cent above cost was it marked if the asking price was $176?

3. A man bought a quantity of apples at $2 per barrel. He sold ¼ of them at $3 per barrel, ⅓ of the remainder at $3.25 per barrel, and the remainder, 750 bbl., at $2.50 per barrel. What was his gain?

4. By selling apples at $2.50 per barrel I gained $200. Had I sold them at $2.75 per barrel my rate of gain would have been 37½%. How many barrels did I sell?

5. A dealer bought wheat at 90¢ per bushel. He sold ¾ of it at 33⅓% gain and the remainder at a loss of $25. If his gain on the whole transaction was 22⅔%, how many bushels of wheat did he buy?

6. What per cent of gain must be realized on an engine costing $1928 in order that it may be sold for $2410?

7. A produce dealer bought 24,000 lb. of wheat for $360. He sold it at $1.05 per bushel. What was his gain per cent?

8. A compromised with an insolvent debtor at the rate of 50¢ on a dollar. To obtain an immediate payment he allowed a further discount of 5%. What was the amount of his claim, his total loss having been $10,505.25?

9. An article marked to gain 62½% is sold less 25% and 20%. If a collector was afterwards paid 20% for collecting the account, what was the gain or loss per cent?

10. An article marked 25% above cost is sold at a discount of 16⅔%. If the gain is $25, what is the selling price of the article?

11. If I made a profit of 16⅔% by selling a horse at $7.50 above cost, how much should I have received above cost to realize a profit of 25%?

12. What per cent is gained by buying pork at $17.50 per barrel, and retailing it at 12¢ per pound?

13. Having bought 75 barrels of apples for $187.50, I sold them at a loss of 20%. How much did I receive per barrel?

14. I lost 25% of a consignment of berries. At what per cent of profit must the remainder be sold in order that I may gain 10% on the whole?

15. A Texas farm of 160 acres was bought at $15 per acre; $354 were paid for fencing, $480 for breaking, $626 for a house, and $220 for a barn. At what price per acre must it be sold to realize a net profit of 25% on the investment?

16. If 25% of the selling price is gain, what is the gain per cent?

17. I sell ⅖ of a stock of goods for $27, thereby losing 20%. For what must I sell the remainder to make a profit of 20% on the whole?

18. A banker bought a mortgage at 7½% less than its face value, and sold it for 3% more than its face value, thereby gaining $981.75. What was the face value of the mortgage?

19. If I sell ¾ of an acre of land for what ⅝ of it cost, what will be my gain or loss per cent?

20. B and C each invested an equal amount of money in business; B gained 12½% on his investment, and C lost $5275; C's money was then 42% of B's. How many dollars did each invest?

21. A manufacturing company's per cent of gain on a self-binder was 25% less than that of the general agent; the general agent's profit was 20%, he thereby gaining $25.30. What did it cost to make the machine?

22. For what must hay be sold per ton to gain 16⅔%, if, by selling it at $18 per ton, there is a gain of 25%?

23. A stock of goods is marked 22½% advance on cost, but becoming damaged, is sold at 20% discount on the marked price, whereby a loss of $1186.40 is sustained. What was the cost of the goods?

24. Of a cargo of 8000 bushels of oats, costing 35¢ per bushel, 25% was destroyed by fire. What per cent will be gained or lost if the remainder of the oats is sold at 45¢ per bushel?

25. A grocer bought 200 quarts of berries at 11¼¢ per quart, and 150 quarts of cherries at 6¼¢ per quart. Having sold the cherries at a loss of 30%, for how much per quart must he sell the berries to gain 15% on the whole?

26. Having bought 48 pounds of coffee at the rate of 3½ pounds for 91¢ and 84 pounds more at the rate of 7 pounds for $1.26, I sold the lot at the rate of 9 pounds for $1.53. What was my per cent of gain or loss?

27. Having paid a retailer $138.60 for a set of furniture, I ascertain that by selling to me he gained 12½%, that the wholesaler of whom he bought gained 10%, that the jobber by selling to the wholesaler gained 16⅔%, and that the manufacturer sold to the jobber at 20% above its first cost. How much more than its first cost did I pay?

28. If I pay $3.20 for 20 gallons of vinegar, how many gallons of water must be added that 40% profit may be realized by selling it at 15¢ per gallon?

29. A merchandise account shows that the cost of a stock of goods was $15,000, that the sales to date aggregate $12,000, and that the goods on hand, estimated at cost prices, amount to $4500. Find the per cent of gain or loss on the sales.

30. A sold a horse to B and gained 20%; B sold it to C and gained 25%. If the average gain was $50, what was C's cost?

31. A grocer buys 10 barrels of apples, each barrel containing 2½ bushels at $2 per barrel. If the loss by decay amounts to 20%, at what price per peck must he retail them in order to clear 20%?

32. A grocer mixes 10 pounds of tea costing 36¢ per pound with 8 pounds costing 45¢ per pound. At what price must he sell the mixture to gain 25% upon his outlay?

33. A stock of imported silks bought for £200 was sold for $1167.96. What was the gain per cent?

34. What per cent is gained in buying coal at $4.50 for a long ton and retailing it at $6 a short ton?

35. What per cent is gained on quinine costing $2.90 an ounce and sold at 2¢ a grain?

36. A merchant buys hardware at 25% and 20% off the list prices, and sells at 20% and 10% off the list prices. What per cent of gain does he realize?

37. A manufacturer sells at 20% and 10% off the list prices. His blundering clerk, in making out the bill for the goods, deducted 30%. If the discount deducted was $450, what should have been the net amount of the bill? If the mistake passed unnoticed, what was the buyer's gain per cent on the transaction?

38. A dealer in agricultural implements marked a self-binder at an advance of 25% on the cost. In order to collect an account, he had to pay an attorney 10% of the amount of the debt. If the sale netted him a gain of $25, what was the selling price of the binder?

39. A retailer buys collars at the rate of $1 a dozen, and sells at the rate of 2 for 25¢. What is the per cent of gain?

40. A manufacturer sold a retailer a piano and gained 20%. The retailer compromised with his creditors, paying 75¢ on the dollar. What was the manufacturer's per cent of loss on the transaction if he discounted the amount which he could legally collect 5% for immediate payment?

41. An insolvent debtor pays his creditors 37¢ on the dollar. If his creditors receive $3515, what is their joint loss?

42. A manufacturer sells to a wholesaler at 20% gain; the wholesaler to the retailer at 25% gain; and the retailer to the consumer at 60% gain. Find the cost to the manufacturer of an article for which the consumer pays $2.40 more than twice the cost to manufacture.

MARKING GOODS

566. In marking goods, it is customary for merchants to use a word, phrase, or an arbitrary arrangement of characters called a *key* to represent the ten Arabic numerals. In this way the cost and selling price may be written on an article and yet be unintelligible to all except those who know the key.

567. If letters are used, any word or phrase containing ten *different* letters may be selected. If characters are used, any ten different arbitrary characters may be selected.

568. Merchants generally use two different keys, one to represent the cost and one the selling price.

569. To avoid the repetition of a letter, and to make the key more valuable as a private mark, one or two extra letters called *repeaters* are used to indicate letters which would otherwise be repeated.

To illustrate the method of marking goods, take the following keys:

Cost Mark

S	E	Z	I	R	O	H	T	U	A
1	2	3	4	5	6	7	8	9	0

Repeaters: G and F.

Selling Price Mark

H	T	I	M	S	K	C	A	L	B
1	2	3	4	5	6	7	8	9	0

Repeaters: W and O.

It will be observed that the words *authorizes* and *blacksmith*, spelled backwards, are used to represent the cost and selling price respectively; also that the repeaters in both cases are mere arbitrary letters.

The cost and selling price are generally written one above and the other below a line on a tag, or upon a paster or box.

Thus, if the above keys are used, $\frac{\$\text{O.AF}}{\$\text{C.SB}}$, on a dozen of gloves, would be understood to mean that the cost was $6.00 per dozen, and that the selling price is $7.50 per dozen.

WRITTEN EXERCISE

Using the keys given in **569**, write the cost and selling price of articles costing:

1. $2.50 and selling at 20% gain; $2.40 and selling at 25% gain.

2. $1.80 and selling at 33⅓% gain; $4.20 and selling at 16⅔% gain.

3. 18¢ and selling at 16⅔% gain; $27 and selling at 30% loss.

4. $4.26 and selling at 16⅔% gain; $3.60 and selling at 12½% gain.

5. $425 and selling at 20% gain; $24.90 and selling at 33⅓% gain.

6. $16 and selling at 31¼% gain; $2.40 and selling at 37½% gain.

570. While the unit of measure varies with the quantities and qualities offered for sale, a large number of manufactured products are sold by the dozen. Jobbers and wholesalers buy a great many articles by the dozen. Retailers buy a great many articles by the dozen, but usually sell them by the piece.

571. **To find the cost price of an article, the cost price of a dozen being given.**

572. **Example.** If 1 doz. hats cost $25, what will 1 hat cost?

Solution. Since a dozen hats cost $25, 1 hat will cost $\frac{1}{12}$ of $25, or 2\frac{1}{12}$, which is equal to $2.08⅓ or $2.08.

It is just as easy to divide a number by 12 as it is by any number of one digit. Hence, in dividing by 12, use the short division method. After dividing the figures in the dividend, consider the remainder as twelfths of a number, and mentally reduce it to an approximate decimal.

573. The following table shows the decimal values of the twelfths which may remain after dividing a number by 12,

TABLE OF TWELFTHS

TWELFTHS	SIMPLEST FORM	DECIMAL VALUE	TWELFTHS	SIMPLEST FORM	DECIMAL VALUE
$\frac{1}{12}$		$.08\frac{1}{3}$	$\frac{7}{12}$		$.58\frac{1}{3}$
$\frac{2}{12}$	$\frac{1}{6}$	$.16\frac{2}{3}$	$\frac{8}{12}$	$\frac{2}{3}$	$.66\frac{2}{3}$
$\frac{3}{12}$	$\frac{1}{4}$	$.25$	$\frac{9}{12}$	$\frac{3}{4}$	$.75$
$\frac{4}{12}$	$\frac{1}{3}$	$.33\frac{1}{3}$	$\frac{10}{12}$	$\frac{5}{6}$	$.83\frac{1}{3}$
$\frac{5}{12}$		$.41\frac{2}{3}$	$\frac{11}{12}$		$.91\frac{2}{3}$
$\frac{6}{12}$	$\frac{1}{2}$	$.5$	$\frac{12}{12}$	1	1.

Familiarity with the above table will give facility in dividing numbers by 12.

ORAL EXERCISE

By inspection find the cost of one article when billed by the dozen as follows:

1. Shoes at $18.60.
2. Boots at $42.
3. Hose at $6.60.
4. Hats at $27.
5. Ties at $9.
6. Coats at $116.
7. Scarfs at $1.32.
8. Shirts at $11.60.
9. Hose at $3.90.
10. Gloves at $3.90.
11. Hose at $5.
12. Caps at $16.90.

574. To find the selling price of an article, the cost per dozen and the rate per cent of gain being given.

575. Example. Find the selling price of a pair of gloves so as to net a profit of $33\frac{1}{3}\%$, the cost per dozen being $7.50.

$\frac{1}{3} \times \$7.50 = \$2.50.$
$\$7.50 + \$2.50 = \$10.$
$\$10 \div 12 = \$\frac{5}{6} = 83¢.$

SOLUTION. Since $33\frac{1}{3}\%$, or $\frac{1}{3}$, of the cost is profit, the selling price per dozen will be $\frac{1}{3}$ more than $7.50, or $10. $10 divided by 12 equals 83¢, or the selling price per pair.

576. Therefore the following rule:

Find the selling price of one dozen by adding to the cost of one dozen such a part of the cost as the rate per cent of gain is a part of 100%.

Find the cost per article by dividing the cost per dozen by 12.

WRITTEN EXERCISE

Find the selling price per article:

	Cost per dozen	Gain to be realized		Cost per dozen	Gain to be realized		Cost per dozen	Gain to be realized
1.	$15.00	$33\frac{1}{3}\%$	*7.*	$7.50	$18\frac{3}{4}\%$	*13.*	$36.00	25%
2.	$ 9.00	25%	*8.*	$25.25	20%	*14.*	$116.00	$12\frac{1}{2}\%$
3.	$27.50	$6\frac{1}{4}\%$	*9.*	$17.50	20%	*15.*	$35.50	20%
4.	$28.00	$16\frac{2}{3}\%$	*10.*	$18.90	$33\frac{1}{3}\%$	*16.*	$29.70	$11\frac{1}{9}\%$
5.	$ 4.00	20%	*11.*	$1.50	$16\frac{2}{3}\%$	*17.*	$28.00	$14\frac{2}{7}\%$
6.	$37.50	$8\frac{1}{3}\%$	*12.*	$9.60	$66\frac{2}{3}\%$	*18.*	$56.00	$28\frac{4}{7}\%$

577. To find the price at which goods must be marked to insure a given per cent of gain or loss, the cost and discount series being given.

578. Example. A seal sack cost a manufacturer $240. At what price must it be marked in order that a discount series of 25% and 20% may be allowed and a gain of $33\frac{1}{3}\%$ be realized?

Solution

Let 100% represent the marked price.
A discount series of 25% and 20% is equal to a direct discount of 40%.
100% − 40 = 60%, the amount to be realized.
$240 = the cost of the sack.
$33\frac{1}{3}$, or $\frac{1}{3}$ of $240 = $80, the amount to be gained.
$240 + $80 = $320, the amount to be realized.
Therefore, 60% = $320.
1% = $$5.33\frac{1}{3}$.
100% = $533.33, the marked price.

579. Therefore the following rule:

Add the required gain to or subtract the required loss from the cost and divide by 1 minus the rate of discount.

WRITTEN EXERCISE

1. What must be the asking price of a watch costing $24 in order to insure a gain of $33\frac{1}{3}\%$ and allow the purchaser a discount of 20%?

2. After buying lace at $8 per piece, I so marked it as to allow discounts of 25% and 20% from the marked price, and yet so sell it as to lose but 10% on my purchase. At what price per piece was the lace marked?

3. The cost of manufacturing silk ties being $36 per dozen, how much must they be marked that a gain of $16\frac{2}{3}\%$ may be realized by the manufacturer after allowing a discount of 25% and $12\frac{1}{2}\%$?

4. If a carriage be marked $33\frac{1}{3}\%$ above cost, what per cent of discount can be allowed from the marked price and realize cost?

5. If the list price of an article is 40% advance on the cost, what other per cent of discount than $14\frac{2}{7}\%$ must be allowed to net 10% by the sale?

WRITTEN REVIEW

1. W. A. Briggs & Co. bought of B. A. Altman & Son invoice of silk hats at $100 per dozen, less 30% and 10%. What price per hat must be asked in order to gain $33\frac{1}{3}\%$?

2. If goods are retailed at an advance of 25%, what is the selling price per article of goods costing by the dozen as follows: shoes, $48; boots, $38.40; rubbers, $8.16?

3. Briggs, Slote & Co. imported hosiery and knit goods costing per dozen as follows: Hosiery, $2.40, $2.50, $3.84, and $4.80; knit goods, $7.20, $4.08, $16.32, and $10.40. Determine the selling price per article, goods to be sold at a gain of 25%.

4. Using the word *handsomely* with repeater *R* for the buying key, and the words *black horse* with repeater *W* for the selling key, mark the cost and selling price for the articles in problem 3.

5. What should be the marked price per article of the following goods so as to gain $33\frac{1}{3}\%$ and allow discounts of 10% and 10%? Hats, per dozen, $16.20; gloves, per dozen, $8.10; ties, per dozen, $4.05.

6. At what price should the following goods be marked per article so as to allow discounts of 25% and 20% and still net a gain of $33\frac{1}{3}\%$ on the cost? Hats, $9, $6.30, $15, $21; gloves, $10.80, $12.60, and $8.10.

7. What price each must be asked for cocoanuts costing $4 per C that an allowance of $16\frac{2}{3}\%$ for breakage, 20% for decay, and $11\frac{1}{9}\%$ for bad debts may be made, and still a gain of $33\frac{1}{3}\%$ be realized?

8. Having paid 40¢ per pound for tea, at what retail price must it be marked that I may allow $12\frac{1}{2}\%$ for bad debts and gain 40% on the cost?

9. A publisher's prices are 75% above cost. If he allows his agent a commission of 20% and sells at a discount of 10%, what per cent of gain does he make?

10. What price per pound must be asked for coffee costing 18¢ per pound, in order that the seller may deduct 10% from the asking price for bad debts, allow 16⅔% for loss in roasting, and still gain 20% on the cost?

COMMISSION

580. **Commission** is a compensation allowed by one person, called the principal, to another, called the agent, for the transaction of business. It is usually a percentage of the money involved in the transaction.

Thus, in the purchase of goods, it is usually a percentage of the prime cost; in the sale of goods, a percentage of the gross selling price; in the collection of a debt, a percentage of the amount collected.

581. The person for whom business is transacted is called the **principal**, and the person authorized to transact business for another, the **agent**, **broker**, **commission merchant**, or **collector**, according to the nature of the business transacted.

582. A quantity of goods sent away to be sold on commission is called a **shipment**; a quantity of goods received to be sold on commission is called a **consignment**. The person who sends the goods is called the **consignor**, and the person to whom they are sent, the **consignee**.

583. **Guaranty** is a percentage charged by an agent for assuming the risk of loss from sales made by him on credit, or for giving pledge of the grade of goods bought.

584. **Account sales** is an itemized statement rendered by an agent to his principal, showing in detail the sales of goods and charges thereon, together with the net proceeds remitted or credited.

585. **Account purchase** is an itemized statement rendered by a purchasing agent to his principal, showing the quantity, grade, and price of goods purchased, and all expenses incurred, together with the gross cost of the transaction.

586. The **gross proceeds** of a sale or collection is the total amount received by an agent.

587. The **net proceeds** is the amount remaining after commission and all other charges have been deducted.

588. Computations in commission are performed in accordance with the general rules of percentage, the *gross selling price*, or the *prime cost*, corresponding to the *base;* the *per cent of commission*, either for buying or selling or for guaranty of quality or credit, to the *rate;* the *commission* to the *percentage;* the *total cost* of the goods bought by a purchasing agent to the *amount;* and the *proceeds* to the *difference*.

DRILL EXERCISE

1. Given the amount and the rate, how do you find the base? percentage? difference? why, in each case?

2. Compare the general principles governing commission with those governing abstract percentage.

3. Give the five necessary formulæ for performing the operations in commission.

589. **To find the commission, the cost or selling price and per cent of commission being given.**

DRILL EXERCISE

1. Give a short method for finding 10% commission; $2\frac{1}{2}\%$ commission.

Solution. 10% is $\frac{1}{10}$ of 100%, the whole of a number. Hence, to compute commission at 10%, *find $\frac{1}{10}$ of the base by pointing off one place to the left.* $2\frac{1}{2}\%$ is $\frac{1}{4}$ of 10%. Hence, to find a commission of $2\frac{1}{2}\%$, *point off one place to the left and divide by 4.*

2. Give a short method for calculating a commission of $3\frac{1}{3}\%$; 1%; $\frac{1}{2}\%$; $\frac{1}{4}\%$; $7\frac{1}{2}\%$; $1\frac{2}{3}\%$; 25%; $33\frac{1}{3}\%$; $12\frac{1}{2}\%$; $6\frac{2}{3}\%$; $8\frac{1}{3}\%$.

ORAL EXERCISE

By inspection find the commission in the following problems:

	Gross Selling Price	Rate of Commission		Prime Cost	Rate of Commission
1.	$945.80	10%	*7.*	$8480.80	25%
2.	$724.80	5%	*8.*	$1200.00	7%
3.	$440.40	4%	*9.*	$1500.00	8%
4.	$780.20	5%	*10.*	$2500.00	14%
5.	$750.60	$33\frac{1}{3}\%$	*11.*	$2500.00	16%
6.	$225.00	$3\frac{1}{3}\%$	*12.*	$2978.95	10%

WRITTEN EXERCISE

1. A real estate agent sold a farm of 90 acres at $125 per acre on a commission of 2%. What was the amount of his commission? How much did he turn over to his principal?

2. An agent sold 450 barrels of flour at $6.25 per barrel on a commission of 3⅛%. What was his commission?

3. A collector succeeded in collecting 80% of a doubtful account of $1500. If he charged 7½% commission, how much did he turn over to his principal?

4. My Chicago agent buys for me 4500 bushels of wheat at 83⅛¢ per bushel. How much should I remit him to cover the cost of the wheat and his commission of 5%?

590. **To find the rate of commission, the commission and gross selling price or prime cost being given.**

ORAL EXERCISE

Find the rate of commission in each of the following examples:

	Gross Selling Price	Commission		Prime Cost	Commission
1.	$750	$7.50	*6.*	$105	$35
2.	$216	$6.48	*7.*	$2400	$60
3.	$135	$4.04	*8.*	$125	$25
4.	$150	$5.00	*9.*	$920	$23
5.	$2500	$50.00	*10.*	$200	$24

11-15. Find the net proceeds in problems 1-5 inclusive.

16-20. Find the gross cost in problems 6-10 inclusive.

WRITTEN EXERCISE

1. A lawyer collected a note of $2500 and paid to his principal $2437.50. What was his rate of commission?

2. A commission merchant sold a consignment of 1200 barrels of beef at $14.50 per barrel. After deducting $80 for freight, $20 for storage, and his commission, he remits his principal $16,952 as the net proceeds of the sale. What was his rate of commission?

591. To find the investment or gross sales, the commission and per cent of commission being given.

DRILL EXERCISE

1. Formulate a short method for finding the gross sales when the commission is given and the rate is 10%; 7½%.

SOLUTIONS. 10% equals ⅒ of a number. Hence, if the commission at 10% is given, the gross sales may be found by *multiplying the commission by 10*, or by *removing the decimal point one place to the right.*

7½% increased by ⅓ of itself equals 10%. Hence, to find the gross sales when the commission at 7½% is given, *remove the decimal point one place to the right and add ⅓.*

2. Formulate a short method for finding the gross sales when the commission is given and the rate is 2½%; 3⅓%; 1⅔%; 25%; 33⅓%; 12½%; 16⅔%; 1¼%; 8⅓%; 6¼%.

ORAL EXERCISE

Find the prime cost when:	Find the gross sales when:
1. 5% commission = $27.50.	*7.* 7½% commission = $90.
2. 2½% commission = $22.50.	*8.* 16⅔% commission = $150.
3. 3⅓% commission = $14.20.	*9.* 8⅓% commission = $11.
4. 1⅔% commission = $15.50.	*10.* 25% commission = $140.
5. 16⅔% commission = $75.50.	*11.* 1¼% commission = $110.
6. 7½% commission = $75.75.	*12.* 16% commission = $640.

13–18. Find the gross cost in problems 1–6 inclusive.

19–24. Find the net proceeds in problems 7–12 inclusive.

WRITTEN EXERCISE

1. What must an agent's sales for one year aggregate in order that at 3% commission his yearly income may be $2700?

2. A Mobile factor earned $99.75 by selling cotton at 2⅜% commission. How many bales, averaging 560 pounds, did he sell, the price being 15¢ per pound?

3. I paid a grain dealer 1½% for buying corn for me at 62¢ per bushel. If his commission amounted to $83.70, how many bushels did he buy?

4. An agent charged $433.60 for selling a consignment of canned fruit. If his rate of commission was 2½%, what was the net proceeds?

592. To find the investment and commission when both are included in the remittance by the principal.

593. Example. I sent my agent $1025 with instructions to deduct his commission of 2½% and invest the balance in wheat. How much did he invest, and what was his commission?

SOLUTION

Represent the actual investment by 100%.
2½% = the charges for buying.
100% + 2½% = 102½%, the cost of the investment to the principal.
$1025 = the cost of the investment to the principal.
Therefore, 102½% = $1025.
1% = $10.
100% = $1000, the actual investment in wheat.
$1025 − $1000 = $25, the commission for buying.

ORAL EXERCISE

By inspection find the amount to invest and the commission in each of the following problems:

	Amount Remitted	Rate of Commission		Amount Remitted	Rate of Commission		Amount Remitted	Rate of Commission
1.	$1030	3%	*4.*	$315	5%	*7.*	$515	3%
2.	$105	5%	*5.*	$624	7%	*8.*	$410	2½%
3.	$550	10%	*6.*	$205	2½%	*9.*	$2075	3%

10. I sent my agent a certain amount with which to buy silks, after deducting his commission of 3%. If his commission was $30, what was the amount of my remittance?

WRITTEN EXERCISE

1. How many pounds of wool, at 27¢ per pound, can be bought for $8424, if the agent is allowed 4% for purchasing?

2. I remitted $1306.45 to a Boston agent for the purchase of soft hats. If the agent's commission is 4%, and he makes an added charge of 2% for guaranty of quality, how many dozen hats, at $8.50 per dozen, should he send me?

3. A city merchant remitted his country agent $1093.60 with which to buy butter. If the agent's charges were 3% commission, 5% guaranty, and $13.60 for inspection, how many pounds, at 25¢ per pound, did he buy, and what was his commission?

4. I remitted $300 to an agent for the purchase of hops. If the agent's charges were 5% for purchase and $6 for inspection, how many pounds at 16¢ per pound ought he to buy?

ORAL REVIEW

1. If you sell books to the amount of $240 on 33⅓% commission, what amount do you earn, and what is the net proceeds of the sale?

2. I send you $205 with instructions to expend it for wheat at $1 per bushel, after retaining your commission of 2½%. How many bushels of wheat will you be able to buy, and what will be your commission?

3. If I remit $95 as the proceeds of an account collected by me, how much have I retained, my rate of commission being 5%?

4. What is the amount of sales when the net proceeds are $975 and the commission 2½%?

5. An agent charges 5% commission and receives $250. Find the net proceeds of the consignment.

6. A manufacturer sent his purchasing agent $510 with which to buy leather, after deducting his commission. If the agent received $10 for his services, what was his rate of commission?

WRITTEN REVIEW

1. Rule a sheet of paper, copy the following account sales, and make the necessary extensions, etc.:

ACCOUNT SALES

Boston, Mass., Feb. 23, 1904.

Sold for the Account of

E. W. Marden, Worcester, Mass.

By E. A. Reed & Co., Commission Merchants.

1904							
Jan.	25	100 bbl. S. P. Flour	5.75				
	28	150 bbl. R. P. Flour	6.50				
Feb.	18	200 bbl. S. P. Flour	5.80				
	20	100 bbl. S. P. Flour	6.00				
		Charges					
Jan.	15	Freight, $135	Cartage, $25				
Feb.	20	Storage, $16.50	Insurance, $5.20				
	23	Guaranty, 1%	Commission, 5%				
		Net proceeds,					

2. Prepare an account sales under date of Feb. 24 for 5000 bu. of wheat, sold by E. L. Hardy & Co., Boston, Mass., for the account of Welsh Bros. & Co., Springfield, Mass. Sales: Feb. 1, 500 bu. at $1.02; Feb. 15, 1000 bu. at $1.08; Feb. 19, 500 bu. at $1.05; Feb. 22, the remainder at $1. Charges: freight, $95; cartage, $18; storage, $17 50; insurance, ¼%; guaranty, 1%; commission, 2%.

3. Rule a sheet of paper and copy the following account purchase, making the necessary extensions, etc.

ACCOUNT PURCHASE

Boston, Mass., Feb. 23, 1904.

Purchased by F. R. Berriman & Co.,

For the Account and Risk of

E. L. Brown, Paterson, N.J.

	3	half-ch. G. Tea, 165 lb.	34¢				
	4	half-ch. O. Tea, 240 lb.	41¢				
	5	half-ch. J. Tea, 350 lb.	23¢				
	8	mats J. Coffee, 600 lb.	24¢				
		Charges					
		Cartage	$7.90				
		Commission, 2%					
		Amount charged to your account,					

4. In accordance with the foregoing form prepare an account purchase of tea purchased by W. L. Jordan & Co., Feb. 23, for the account and risk of Adams, Rand & Co. Purchases: 10 half-chests J. Tea, 600 lb., at 38¢; 5 half-chests O. Tea, 250 lb., 55¢; 5 cases C. Tea, 250 lb., at 55¢; 8 half-chests E. B. Tea, 480 lb., 45¢. Charges: cartage, $7.50; commission, 2%.

5. I place a claim of $2580 in the hands of an attorney for collection. If the debtor is a bankrupt having liabilities aggregating $18,000 and resources aggregating $13,500, how much should I receive after my attorney has deducted his commission of 2%?

6. A collector obtained 75% of the amount of an account, and after deducting 12% for fees remitted his principal $495. What was the amount of his commission?

7. A Hartford fruit dealer sent a Lockport agent $1946.70, and instructed him to buy apples at $1.40 per barrel. The agent charged 3% for buying, and shipped the purchase to his principal in six car loads of an equal number of barrels. How many barrels did each car contain?

8. Find the per cent of commission on a purchase if the gross cost is $2048.51, the commission $87.30, the cartage $20, and other charges $1.21.

9. A collector obtained 75% of a doubtful account of $1750. How much was his per cent of commission if, by agreement with the principal, the commission was to be 50% of the net proceeds remitted?

10. A farmer received from his city agent $490 as the net proceeds of a shipment of butter. If the agent's commission is 3%, delivery charges $6.80, and 5% charge is made for guaranty of quality to purchasers, how many pounds, at 27¢ per pound, must have been sold, and how much commission was allowed?

11. An agent sold 2000 bu. Alsike clover seed at $7.85 per bushel, on a commission of 5%, and 1200 bu. medium red at $5.20, on a commission of 2½%, taking the purchaser's 3-months' note for the amount of the sales. If the agent charges 4% for his guaranty of the note, what amount does he earn by the transaction?

12. Find the net proceeds of a sale made by an agent charging 3½% if incidental charges and commission charges were each $41.30.

13. Find the gross proceeds of a sale made by an agent charging 2½% for commission, 5% for guaranty, $17.65 for cartage, $11.40 for storage, and $3.25 for insurance, if the net proceeds remitted amount to $1714.10.

14. I sent $3402.77 to my Atlanta agent for the purchase of sweet potatoes at $1.60 per barrel; his charges were, for commission, 2¼%; guaranty, 3%; drayage, 1¢ per barrel; and freight, $200. How many barrels did he buy, and how much unexpended money was left in his hands to my credit?

15. I received from Duluth a cargo of 16,000 bu. of wheat, which I sold at $1.10 per bushel, on a commission of 4%; by the consignor's instructions I invested the net proceeds in a hardware stock, for which I charged 5% commission. What was the total commission, and how much was invested in hardware?

16. Having sent a New Orleans agent $1835.46 to be invested in sugar, after allowing 3% on the investment for his commission I received 32,400 pounds of sugar. What price per pound did the sugar cost the agent?

17. An agent in Providence, R.I., received $828 to invest in prints, after deducting his commission of 3½%. If he paid 7½¢ per yard for the prints, how many yards did he buy?

18. An agent sold, on commission, 1750 barrels of mess pork at $16.50 per barrel, and 508 barrels of short ribs at $18 per barrel, charging $112.50 for cartage and $5.55 for advertising. He then remitted to his principal $36,000, the net proceeds. Find the rate of commission.

19. Render in full the following account sales, supplying rates per cent for insurance and commission, and showing net proceeds: Feb. 23, Emery Williams & Co., Troy, N.Y., sold for Moody Bros. & Co., Rome, N.Y., 12,000 lb. wool at 35¢, 12,000 yd. woolen goods at 75¢. Charges: freight, $450; insurance, $33; commission, $264.

20. An agent sold wheat on 5% commission and invested the proceeds in barley at 75¢ per bushel on a commission of 5⅝%. If his total commission was $1200, how many bushels of barley did he buy?

INTEREST

594. Interest is that which is paid for the use of money.

595. The **essential elements** of interest are the ***principal***, the ***time***, the *rate*, the *interest*, and the *amount*.

596. The sum upon which interest is charged is termed the **principal**; the period for which the principal bears interest, the **time**; the annual rate charged for the use of the principal, the **rate of interest**; the product of the rate of interest and the time, the **per cent of interest**; the result obtained by taking a per cent of interest of the principal, the **interest**; the sum of the principal and interest, the **amount**.

597. **Legal interest** is interest computed at the rate established by law to apply when no agreement is made. The legal rate of interest, being established by state statutes, varies in the different states.

598. Usury is any rate of interest in excess of the legal rate.

In a number of the states parties may, by special agreement, receive interest at a higher rate than the legal rate.

A person taking a usurious rate of interest is liable to certain penalties regulated by state statutes.

SIMPLE INTEREST

599. Simple interest is interest allowed for the use of the principal only.

600. The term **interest** is always understood to mean *simple interest*. If other forms of interest are meant they are specifically designated; as, compound interest, periodic interest.

601. For convenience, interest is usually computed on the basis of the *commercial year* of 12 months of 30 days each, or 360 days. Interest on this basis is called **common interest**. The practice of taking 360 days as a year, being sufficiently exact for business purposes, has the sanction of law in some states and is generally used in all the states.

602. Simple interest is an application of the principles of abstract percentage with the additional element *time* introduced. The *principal* in interest corresponds to the *base* in percentage; the *per cent of interest*, to the *rate;* the *interest*, to the *percentage;* and the *sum of the principal and interest*, to the *amount*. The solution of problems in interest is therefore dependent upon the general principles of abstract percentage.

603. There are many methods of computing simple interest, but those given herewith are the most rational and simple. The ordinary-day and the bankers' sixty-day methods are particularly adapted to finding the interest when the time is expressed in days, and the six per cent method to finding the interest when the time is expressed in years and months, or years, months, and days.

Ordinary-day Method

DRILL EXERCISE

1. What is the interest on $1 for 1 year at 6%?

2. What part of a commercial year is 60 days? 6 days?

3. How many days will it take $1 to yield 1 cent interest? 1 mill interest?

4. What is the interest on $1 for 60 days at 6%? for 6 days?

5. What is the interest on $1 for 1 day at 6%?

6. What is the interest on $6 for 1 day at 6%? on $18? on $36? on $300? on $1200?

7. What part of the principal is the interest for 6 days at 6%?

8. Give a simple way to find the interest on any principal for any number of days at 6%.

604. General Principles. 1. In 6 days at 6% any principal will yield interest equal to .001 of itself; in 1 day, interest equal to $.000\frac{1}{6}$ of itself.

2. .001 of any given principal is equal to 6 times the interest for 1 day at 6%.

605. Examples. *1.* Find the interest on $750 for 11 da. at 6%.

.750 × 11 = 8.250.

8.250 ÷ 6 = 1.375, or $1.38.

Solution. .001 of the principal, or $.75, is equal to 6 times the interest for 1 day. Hence, 11 times $.75, or $8.25, is equal to 6 times the interest for 11 days. If 6 times the interest for 11 days is $8.25, the actual interest for 11 days must be $\frac{1}{6}$ of $8.25, or $1.38.

2. Find the interest on $875 for 24 da. at 6%.

.875 × 4 = 3.500, or $3.50.

Solution. .001 of the principal, or $.875, is equal to the interest for 6 times 1 day, or 6 days. Since 24 days are 4 times 6 days, 4 times $.875 must be the interest for 24 days. 4 times $.875 = $3.50, the required interest.

606. Hence, the following rule may be derived:

Point off three integral places in the principal, multiply by the number of days, and divide by 6. The result is the required interest at 6%. Or,

When it is seen that the time in days and months is a multiple of 6, point off three integral places in the principal and multiply by $\frac{1}{6}$ of the number of days. The result is the required interest at 6%.

WRITTEN EXERCISE

At 6% per annum find the interest on:

1. $750 for 73 da.
2. $840 for 19 da.
3. $920 for 24 da.
4. $780 for 36 da.
5. $920 for 42 da.
6. $924 for 17 da.
7. $476.87 for 95 da.
8. $925.14 for 72 da.
9. $724.18 for 75 da.
10. $420.10 for 11 da.
11. $500.60 for 7 da.
12. $702.45 for 17 da.
13. $728.16 for 84 da.
14. $846.92 for 108 da.
15. $1246.45 for 24 da.
16. $1432.18 for 36 da.
17. $1945.62 for 18 da.
18. $7514.95 for 12 da.

Using the exact number of days, find the interest at 6% on:

19. $170 from July 15 to Sept. 1; from Apr. 6 to Oct. 9.
20. $1750 from Jan. 1 to Feb. 8; from May 15 to July 9.
21. $2470 from Apr. 7 to July 1; from Apr. 2 to Aug. 1.
22. $7562 from July 2 to Sept. 5; from Mar. 2 to Apr. 30.
23. $2172 from Jan. 2 to July 9; from Sept. 2 to Dec. 1.
24. $2400 from Oct. 1 to Dec. 1; from May 8 to Aug. 1.
25. $2675 from Oct. 5 to Nov. 1; from Sept. 9. to Dec. 1.

607. To find the interest for any number of days at any rate per cent per annum.

608. General Principle. .001 of any given principal is the interest for 1 da. at 36%.

609. Example. What is the interest on $750 for 16 da. (*a*) at 6%? (*b*) at 4%? (*c*) at 9%? (*d*) at 4½%?

(*a*) .750 × 16 = 12.
12 ÷ 6 = 2, or $2.

Solutions. (*a*) .001 of $750 = $.75, or the interest for 6 days. $.75 × 16 = $12, or the interest for 6 times 16 days. $12 ÷ 6 = $2, or the required interest.

(*b*) .750 × 16 = 12.
12 ÷ 9 = 1.33, or $1.33.

(*b*) .001 of $750 = $.75, the interest for 6 days at 6%, or for 1 day at 36%. $.75 × 16 = $12, or the interest for 16 days at 36%. 4% is ⅑ of 36%. Hence, ⅑ of $12, or $1.33, is the interest at 4%.

(*c*) .750 × 16 = 12.
12 ÷ 4 = 3, or $3.

(*c*) The interest on the principal for 16 days at 36% is $12. 9% is ¼ of 36%. Hence, ¼ of $12, or $3, is the interest at 9%.

(*d*) .750 × 16 = 12.
12 ÷ 8 = 1.50, or $1.50.

(*d*) The interest on the principal for 16 days at 36% is $12. 4½% is ⅛ of 36%. Hence, ⅛ of $12, or $1.50, is the interest at 4½%.

610. Hence the following rule may be derived:

Point off three integral places in the principal, multiply by the number of days, and take such a part of the product as the given rate per cent is of 36%.

611. Deducing a rule for each of the ordinary rates of interest, we have the following:

Point off three integral places in the principal, multiply by the number of days, and to find the interest at 6%, *divide by 6;* at 3%, *divide by 12;* at 4%, *divide by 9;* at 4½%, *divide by 8;* at 5%, *divide by 7.2 (8 and .9)*; at 7%, *divide by 6 and add ⅙ of the quotient;* at 7½%, *divide by 6 and add ¼ of the quotient;* at 8%, *divide by 4.5 (9 and .5)*; at 9%, *divide by 4;* at 10%, *divide by 3.6 (6 and .6)*.

WRITTEN EXERCISE

Find the amount.

	Principal	Time	Rate		Principal	Time	Rate
1.	$9000	91 da.	4%	*9.*	$2431.75	35 da.	9%
2.	$1700	73 da.	4%	*10.*	$1862.15	34 da.	9%
3.	$2750	81 da.	4%	*11.*	$2417.50	41 da.	4½%
4.	$2400	15 da.	8%	*12.*	$7500.75	16 da.	4½%
5.	$1750	21 da.	8%	*13.*	$2400	19 da.	4½%
6.	$4200	84 da.	10%	*14.*	$1840.75	71 da.	3%
7.	$2972.50	31 da.	9%	*15.*	$2417.92	76 da.	3%
8.	$1750.90	87 da.	9%	*16.*	$1695.14	93 da.	3%

Bankers' Sixty-day Method

DRILL EXERCISE

1. How many months will it take $1 to yield 1 cent interest at 6%? How many days?

2. What is the interest on $75 for 60 days at 6%? on $250? on $920? on $780? on $240.75? on $21729.75?

3. What part of a number is the interest at 6% for 60 days?

4. What is the simplest way to find the interest on any principal for 60 days?

5. What part of 60 days are 30 days? 20 days? 15 days? 10 days?

6. What is the interest on $84 for 30 days? for 20 days? for 15 days? for 10 days?

7. What is the simplest way to find the interest on any principal for 30 days at 6%? for 20 days? for 15 days? for 10 days?

8. How long will it take $1 at 6% to yield 1 mill interest?

9. What part of the principal is the interest at 6% for 6 days?

10. What is the interest on $175 for 6 days at 6%? on $215? on $240? on $378? on $7560? on $8925.75? on $4928.79?

11. What is the simplest way to find the interest on any principal for 6 days at 6%?

12. What is the interest on $240 for 3 days? 2 days? 1 day?

13. How many months at 6% will it take $1 to yield 10 cents interest? How many days?

14. What part of the principal is the interest at 6% for 600 days?

15. What is the interest on $800 at 6% for 600 days? on $9500? on $2465? on $5619? on $4500? on $217.40? on $924.68? on $275.19?

16. What is the simplest way to find the interest on any principal at 6% for 600 days? 300 days? 200 days? 150 days? 75 days? 120 days? 100 days? 50 days?

17. In how many days will the interest at 6% equal the principal?

18. What is the interest on $1 for 6000 days at 6%? on $24? on $97? on $55? on $372.50? on $920.75? on $860.44?

19. Pointing off 3 integral places in the principal gives the interest for how many days at 6%? 2 places? 1 place? taking the principal for the interest?

20. What is the interest on $5695 for 6 days at 6%? for 60 days? for 600 days? for 6000 days?

612. General Principles. 1. Pointing off 3 integral places from the right in the principal gives the interest at 6% for 6 days.

2. Pointing off 2 integral places from the right in the principal gives the interest at 6% for 60 days.

3. Pointing off 1 integral place from the right in the principal gives the interest at 6% for 600 days.

4. Writing the principal for the interest gives the interest at 6% for 6000 days.

Thus, the interest on $7521 for 6 days at 6% is $7.52; for 60 days, $75.21; for 600 days, $752.10; for 6000 days, $7521.

ORAL EXERCISE

Find the interest at 6% on:

1. $360 for 6 days; for 3 days; for 2 days; for 1 day.

2. $900 for 60 days; for 30 days; for 20 days; for 15 days; for 12 days; for 10 days.

3. $100 for 18 days; for 24 days; for 36 days; for 42 days; for 48 days; for 54 days; for 66 days.

4. $200 for 180 days; for 240 days; for 420 days; for 480 days; for 546 days; for 660 days.

5. $240 for 7 days.

Solution. 7 days are $1\frac{1}{6}$ times 6 days. Hence, to find the interest for 7 days, *point off 3 places from the right in the principal and add* $\frac{1}{6}$. $.24 + $\frac{1}{6}$ of itself = $.28, the required interest.

6. $360 for 7 days; for 8 days; for 9 days.

7. $990 for 4 days.

Solution. 4 days are $\frac{1}{3}$ less than 6 days. Hence, to find the interest for 4 days, *point off 3 places from the right in the principal and subtract* $\frac{1}{3}$. $.99 − $\frac{1}{3}$ of itself = $.66, the required interest.

8. $240 for 4 days; for 5 days; for 40 days; for 50 days; for 40 days; for 80 days; for 90 days; for 70 days; for 50 days.

613. **To find the interest at 6% for aliquot parts of 6 or 60 days, or aliquot parts more or less than 6 or 60 days.**

614. **Examples.** *1.* What is the interest on $1240 for 30 days at 6%?

$12.40
$6.20

Solution. .01 of the principal, or $12.40 is the interest for 60 days. 30 days is $\frac{1}{2}$ of 60 days. Since $12.40 is the interest for 60 days, the interest for 30 days is $\frac{1}{2}$ of $12.40, or $6.20.

2. What is the interest on $2400.60 for 80 days at 6%?

$24.0060
8.0020
$32.0080, or $32.01.

Solution. .01 of the principal, or $24.006, is the interest for 60 days. 80 days are $\frac{1}{3}$ more than 60 days. Hence, $\frac{1}{3}$ more than $24.006, or $32.01, is the required interest.

3. What is the interest on $360 for 5 days at 6%?

$.360
.060
$.30

Solution. .001 of the principal, or $.36 is the interest for 6 days. Since 5 days are $\frac{1}{6}$ less than 6 days, the interest for 5 days is $\frac{1}{6}$ less than $.36, or $.30.

615. Therefore the following rule may be derived:

For 6 days point off three integral places from the right in the principal, and for 60 days, point off 2 integral places.

Take such a part of the interest for 6 or 60 days as the given number of days are a part of 6 or 60 days. Or, *Take such a part of the interest for 6 or 60 days as the given number of days is a part more or less than 6 or 60 days.*

DRILL EXERCISE

1. What aliquot part less than 60 days are 55 days? 50 days? 40 days? 45 days?

2. What aliquot part more than 60 days are 65 days? 70 days? 75 days? 80 days? 90 days?

3. What aliquot part more than 6 days are 7 days? 8 days? 9 days?

4. What aliquot part less than 6 days are 5 days? 4 days?

5. Give a simple way to find the interest at 6% for 80 days; for 90 days; for 70 days; for 50 days; for 45 days; for 40 days; for 5 days; for 7 days; for 4 days; for 8 days; for 9 days.

WRITTEN EXERCISE

1. Find the total amount of interest at 6% on:

$2400 for 60 da.
$1200 for 30 da.
$900 for 20 da.
$440 for 15 da.
$555 for 12 da.
$660 for 10 da.
$720 for 2 da.
$240 for 3 da.
$840 for 6 da.
$840 for 1 da.
$640 for 15 da
$810 for 20 da.

2. Find the total amount of interest at 6% on:

$450 for 20 da.
$720 for 50 da.
$810 for 40 da.
$680 for 45 da.
$820 for 75 da.
$960 for 70 da.
$990 for 80 da.
$370 for 90 da.
$740 for 3 da.
$660 for 5 da.
$750 for 7 da.
$930 for 4 da.

3. Find the total amount of interest at 6% on:

$1152 for 8 da.
$1600 for 20 da.
$519 for 15 da.
$2150.42 for 50 da.
$1700 for 7 da.
$2100 for 90 da.
$975.49 for 70 da.
$832.65 for 90 da.
$439.17 for 50 da.
$3100 for 40 da.
$1350.90 for 10 da.
$759.18 for 1 da.

4. Find the total amount of interest at 6% on:

$1800 for 65 da.
$1200 for 55 da.
$9128.77 for 7 da.
$3160.90 for 2 da.
$7421.18 for 6 da.
$7246 for 40 da.
$8400 for 45 da.
$750 for 200 da.
$640 for 150 da.
$1260 for 300 da.
$799.49 for 600 da
$9600 for 5 da.

616. To find the interest at 6% when the days are an even number of times 6 or 60.

617. Examples. *1.* Find the interest on $690 for 180 da. at 6%.

$6.90
$20.70

SOLUTION. 180 days are 3 times 60 days. .01 of $690, or $6.90, is the interest for 60 days. Since 180 days are 3 times 60 days, 3 times $6.90, or $20.70, is the required interest.

2. Find the interest on $2100 for 54 da. at 6%.

$2.100
$18.90

SOLUTION. 54 days are 9 times 6 days. .001 of $2100, or $2.10, equals the interest for 6 days. Since 54 days are 9 times 6 days, the interest for 54 days will be 9 times $2.10, or $18.90.

613. Therefore the following rule:

Find the interest for 6 or 60 days, and multiply the result by the number of times that 6 or 60 days is contained in the given number of days.

WRITTEN EXERCISE

1. Find the total amount of interest at 6% on:

$925 for 54 da.
$340 for 36 da.
$420 for 40 da.
$19 for 180 da.
$350 for 18 da.
$311 for 66 da.
$710 for 24 da.
$3100 for 240 da.
$4100 for 360 da.
$917 for 420 da.
$700 for 42 da.
$419.20 for 18 da.

2. Find the total amount of interest at 6% on:

$755 for 180 da.
$101.18 for 54 da.
$500.11 for 66 da.
$2100 for 42 da.
$3100 for 54 da.
$1700 for 36 da.
$1100.59 for 48 da.
$317.42 for 120 da.
$179.11 for 180 da.
$430.18 for 240 da.
$710.18 for 420 da.
$111.49 for 18 da.

3. Find the total amount of interest at 6% on:

$519 for 24 da.
$1600.53 for 54 da.
$11 for 540 da.
$210.90 for 180 da.
$1900 for 36 da.
$170.50 for 240 da.
$214.18 for 18 da.
$167.90 for 540 da.
$1100 for 18 da.
$1700 for 120 da.
$210.40 for 66 da
$1100 for 72 da.

4. Find the total amount of interest at 6% on:

$121 for 18 da.
$745 for 600 da.
$600 for 120 da.
$20 for 36 da.
$760 for 240 da.
$500 for 42 da.
$360 for 72 da.
$350 for 180 da.
$900 for 30 da.
$800 for 60 da.
$788 for 66 da.
$89 for 54 da.

619. **To find the interest at 6% for any number of days.**

620. **Examples.** *1.* What is the interest on $660 for 11 days at 6%?

$6.60
1.10
.11
$1.21

Solution. Sometimes aliquot parts may be subdivided so as to make two or more aliquot parts. Subdividing 11 we have 10 and 1, or ⅙ of 60 and ⅙ of 6. The interest for 60 days is $6.60, and for 10 days $1.10. The interest for 6 days is $.66, and for 1 day $.11. Adding the interest for 10 days and the interest for 1 day, the required interest is found to be $1.21.

2. What is the interest on $240 for 53 days at 6%?

$.240
2.160
.04
$2.12

Solution. 53 days are 1 day less than 9 times 6 days. The interest for 6 days is $.24, and for 54 days $2.16. If the interest for 6 days is $.24, the interest for 1 day is $.04. If the interest for 54 days is $2.16, and the interest for 1 day is $.04, the interest for 53 days is the difference between $2.16 and $.04, or $2.12.

3. What is the interest on $240 for 127 days at 6%?

$2.40
4.80
.24
.04
$5.08

Solution. 127 = 60 × 2 + 6 + 1. The interest for 60 days is $2.40. Hence the interest for 120 days is twice $2.40, or $4.80. The interest for 6 days is $.24, and the interest for 1 day is ⅙ of $.24, or $.04. Adding the interest for 120 days, 6 days, and 1 day we have $5.08, or the interest for 127 days.

621. Therefore the following rule:

Find the interest on the principal for 6 days by pointing off three integral places from the right in the principal, and for 60 days by pointing off two integral places.

For any number of days take such a part of the interest for 6 days, or for 60 days, as the given number of days is a part more or less than 6 days, or 60 days; or as many times the interest for 6 days, or 60 days, as the required number of days will contain 6 or 60 any multiple of 6 or 60 days.

WRITTEN EXERCISE

Find the interest at 6% on:

	Principal	Time		Principal	Time		Principal	Time
1.	$900	53 da.	*7.*	$775.10	17 da.	*13.*	$325.50	29 da.
2.	$287.10	47 da.	*8.*	$211	43 da.	*14.*	$211.10	57 da.
3.	$1890	69 da.	*9.*	$500	67 da.	*15.*	$440	25 da.
4.	$14.50	81 da.	*10.*	$450	58 da.	*16.*	$309.09	83 da.
5.	$2[illegible]	91 da.	*11.*	$700.80	126 da.	*17.*	$1200	14 da.
6.	$591.09	31 da.	*12.*	$600	47 da.	*18.*	$100	53 da.

622. **To find the interest at any rate per annum.**

623. **Examples.** *1.* What is the interest on $840 for 54 days at 8%?

$.840
7.560
2.52
$10.08

SOLUTION. The interest at 6% is found to be $7.56. Since 8% is $\frac{1}{3}$ more than the assumed rate 6%, the interest at 8% is $\frac{1}{3}$ more than the interest at 6%. Adding $\frac{1}{3}$ of $7.56 to itself, the result is found to be $10.08, or the interest at 8%.

2. What is the interest on $2100 for 180 days at 5%?

$21.00
63.00
10.50
$52.50

SOLUTION. The interest at 6% is found to be $63. Since 5% is $\frac{1}{6}$ less than the assumed rate 6%, the interest at 5% is $\frac{1}{6}$ less than the interest at 6%. Taking $\frac{1}{6}$ of $63 from itself, the result is found to be $52.50, or the interest at 5%.

624. Hence the following rule:

Add or subtract from the interest at 6% such a part of itself as the given rate is greater or less than 6%.

DRILL EXERCISE

1. Given the interest at 6%, how may the interest at 7% be found?

SOLUTION. 7% is $\frac{1}{6}$ more than 6%. Hence, the interest at 6% increased by $\frac{1}{6}$ of itself is equal to the interest at 7%.

2. Formulate a short method for changing 6% interest to 8% interest; to 5% interest; to $4\frac{1}{2}$% interest; to 9% interest; to 10% interest; to $7\frac{1}{2}$% interest.

3. If the interest at 6% is $60, what is the interest at 7%? at 5%? at 8%? at 4½%? at 7½%?

4. If the interest at 6% is $240, what is the interest at 9%? at 10%? at 3%? at 4½%? at 7½%?

625. General Principles. 1. 6% interest increased by ⅙ of itself equals 7% interest; by ¼ of itself, 7½% interest; by ⅓ of itself, 8% interest; by ½ of itself, 9% interest.

2. 6% interest diminished by ⅙ of itself equals 5% interest; by ¼ of itself, 4½% interest; by ⅓ of itself, 4% interest.

6% interest may be changed to 10% interest by dividing by 6 and moving the decimal point 1 place to the right; to 12% interest by multiplying by 2; to 3% interest by dividing by 2; to any other rate of interest by dividing by 6 and multiplying by the required rate.

WRITTEN EXERCISE

Find the interest on:

1. $1750 for 15 da. at 6%.

2. $1125 for 24 da. at 7%.

3. $742.50 for 30 da. at 6%.

4. $900 for 93 da. at 7½%.

5. $660 for 63 da. at 8%.

6. $136.42 for 33 da. at 9%.

7. $1000 for 21 da. at 10%.

8. $2000 for 12 da. at 5%.

9. $351.23 for 40 da. at 4½%.

10. $1368 for 50 da. at 3%.

11. $93.40 for 150 da. at 6%.

12. $550 for 75 da. at 7%.

13. $842.50 for 45 da. at 6%.

14. $800 for 27 da. at 5%.

15. $1725 for 57 da. at 9%.

16. $125 for 55 da. at 6%.

17. $3741.85 for 6 da. at 7%.

18. $5178 for 9 da. at 9%.

19. $732 for 11 da. at 6%.

20. $1174.51 for 42 da. at 8%.

21. $340 for 70 da. at 10%.

22. $1478 for 80 da. at 6%.

23. $2150 for 96 da. at 4½%.

24. $1200 for 53 da. at 6%.

25. $1500 for 87 da. at 7%.

26. $420 for 41 da. at 5%.

27. $360 for 81 da. at 6%.

28. $2347.50 for 18 da. at 7%.

29. $1112.49 for 25 da. at 8%.

30. $1300 for 13 da. at 6%.

31. $17,000 for 3 da. at 5½%.

32. $195.50 for 33 da. at 10%.

33. $1050 for 43 da. at 7%.

34. $1560 for 44 da. at 7½%.

35. $180 for 47 da. at 6%.

36. $120 for 49 da. at 9%.

37. $60 for 50 da. at 5%.

38. $930 for 83 da. at 6%.

39. $750 for 84 da. at 6%.

40. $550 for 72 da. at 7%.

41. Find the total amount of interest on:

$550 for 18 da. at 6%.
$810 for 40 da. at 7%.
$1000 for 41 da. at 7½%.
$342.50 for 42 da. at 5%.
$1362.50 for 45 da. at 6%.
$250 for 50 da. at 6%.
$593.25 for 80 da. at 7%.
$1966 for 75 da. at 5%.
$450 for 83 da. at 8%.
$990 for 63 da. at 6%.

42. Find the total amount of interest on:

$720 for 9 da. at 10%.
$7500 for 3 da. at 7%.
$216 for 93 da. at 8%.
$504 for 54 da. at 6%.
$600 for 4 da. at 4½%.
$1124 for 15 da. at 3%.
$550 for 45 da. at 7½%
$160 for 27 da. at 6%.
$240 for 31 da. at 8%.
$540 for 41 da. at 9%.

43. Find the total amount of interest on:

$1452 for 8 da. at 3%.
$1728 for 10 da. at 6%.
$2150.42 for 17 da. at 7%.
$519 for 24 da. at 8%.
$1600 for 23 da. at 7½%.
$1400 for 26 da. at 6%.
$1700 for 29 da. at 8%.
$1900 for 37 da. at 7%.
$2100 for 43 da. at 6%.
$3100 for 53 da. at 3%.

SHORT METHODS

626. Interest is a product of which the rate and time are factors. Since the rate, being a constant factor, may be ignored, it will be observed that it will make no difference if, for convenience, the principal in dollars and the time in days be interchanged.

Thus, the interest on $600 for 93 days is the same as the interest on $93 for 600 days. Since the interest for 600 days is $\frac{1}{10}$ of the principal, $\frac{1}{10}$ of $93, or $9.30, is the required interest on $600 for 93 days. The interest on $150 for 88 days is the equivalent of the interest on $88 for 150 days. Since 150 is ¼ of 600, the required result may be found by taking $\frac{1}{10}$ of 88 and dividing the result by 4, obtaining $2.20 as the required interest.

ORAL EXERCISE

By inspection, find the interest at 6% on:

1. $600 for 93 da.
2. $300 for 42 da.
3. $200 for 66 da.
4. $150 for 44 da.
5. $120 for 55 da.
6. $60 for 89 da.
7. $30 for 56 da.
8. $20 for 84 da.
9. $15 for 124 da.
10. $10 for 66 da.
11. $6000 for 139 da.
12. $3000 for 142 da.
13. $2000 for 186 da.
14. $1500 for 64 da.
15. $1000 for 126 da.
16. $750 for 88 da.
17. $1200 for 155 da.
18. $2400 for 11 da.
19. $1800 for 31 da.
20. $3600 for 51 da.
21. $4200 for 11 da.
22. $5400 for 7 da.
23. $240 for 21 da.
24. $360 for 17 da.
25. $420 for 13 da.
26. $4200 for 103 da.
27. $3600 for 108 da.
28. $1200 for 39 da.
29. $3000 for 145 da.
30. $1000 for 246 da.
31. $6000 for 159 da.
32. $1800 for 39 da.
33. $2400 for 51 da.
34. $7200 for 19 da.
35. $4800 for 17 da.
36. $480 for 11 da.

627. When the rate is not six per cent, many times it is desirable to increase or diminish the principal or time, instead of the interest, by the proper fraction.

628. Examples. *1.* Find the interest on $1500 for 84 da. at 8%.

$\frac{1}{3} \times \$84 = \$28.$

SOLUTION. Since 8% is $\frac{1}{3}$ more than 6%, if the principal is increased by $\frac{1}{3}$ of itself, and the interest computed for the given time at 6%, the result will be equal to the interest at 8%. Increasing $1500 by $\frac{1}{3}$ of itself, the result is $2000. Interchanging the dollars and days, the problem in its simplest form is equivalent to $84 for 2000 days at 6%. Since a principal will double itself in 6000 days, it will yield an interest equal to $\frac{1}{3}$ of itself in 2000 days. $\frac{1}{3}$ of 84 equals 28, making the required interest $28.

2. Find the interest on $799.59 for 45 da. at 8%.

$\$7.9959 = \$8.$

SOLUTION. Since 8% interest is $\frac{1}{3}$ more than 6% interest, if we increase the time by $\frac{1}{3}$ of itself and compute the interest on the principal for this time at 6%, the result will be the interest at 8%. 45, increased by $\frac{1}{3}$ of itself, equals 60. .01 of any number is the interest for 60 days. Hence, $8 is the required interest.

3. Find the interest on $844.20 for 80 da. at 4½%.

$8.4420 = $8.44.

SOLUTION. 4½% interest is ¼ less than 6% interest. Hence, if we decrease the time by ¼ of itself and compute the interest on the principal for the remainder at 6%, the result will be the interest at 4½%. 80 days decreased by ¼ of itself equals 60 days. .01 of the principal is the interest for 60 days at 6%. Hence, the required interest is $8.44.

WRITTEN EXERCISE

Find the interest on:

	Principal	Time	Rate		Principal	Time	Rate
1.	$1200.00	79 da.	7½%	*7.*	$799.59	48 da.	7½%
2.	$783.60	45 da.	8%	*8.*	$111.10	48 da.	7½%
3.	$425.80	45 da.	4%	*9.*	$2400.00	59 da.	4½%
4.	$1600.00	35 da.	7½%	*10.*	$2400.00	38 da.	5%
5.	$3200.00	78 da.	7½%	*11.*	$3800.00	73 da.	5%
6.	$2700.00	48 da.	8%	*12.*	$1200.00	66 da.	4%

Six Per Cent Method

DRILL EXERCISE

1. What is the interest on $1 for 1 yr. at 6%? 4 yr.? 5 yr.? 8 yr.?

2. What is the interest on $10 for 2 yr. at 6%? on $30? on $25? on $80 for 5 yr. at 6%?

3. What is the interest on $1 for 1 mo. at 6%? for 2 mo.? for 4 mo.? for 6 mo.? for 8 mo.? for 3 mo.? for 7 mo.?

4. What is the interest on $1 for 6 da. at 6%? for 1 da.?

5. How many mills will $1 yield in 12 da. at 6%? in 24 da.? in 9 da.? in 15 da.?

6. What is the interest on $1 for 1 yr. at 6%? 1 mo.? 1 da.?

7. At 6%, what is the interest on $1 for 1 yr. 2 mo. 6 da.? on $20? on $2000? on $3000? on $1500? on $7500?

8. Give a simple way to find the interest on any principal for any given number of years, months, and days.

629. General Principles. $1 in 1 year at 6% will yield $.06 interest; in 1 month, $.005 interest; in 1 day, $.000⅙ interest.

630. **To find the interest on any principal for any time and rate by the six per cent method.**

631. **Example.** What is the interest on $650 for 2 yr. 4 mo. 12 da. at 6%?

.06 × 2	.12	650
.005 × 4	.02	.142
$.000\frac{1}{6}$ × 12	.002	1 300
	.142	91 00
		92.300

SOLUTION. The interest on $1 for 1 yr. at 6% is $.06, and for 1 mo., or $\frac{1}{12}$ yr., it is $\frac{1}{12}$ of $.06, or $.005, and for 1 da., or $\frac{1}{30}$ of a month, it is $\frac{1}{30}$ of $.005, or $$.000\frac{1}{6}$. If the interest on $1 for 1 yr. is $.06, for 2 yr. it is twice $.06, or $.12. If the interest for 1 mo. is $.005, for 4 mo. it is 4 times $.005, or $.02. If the interest for 1 da. is $$.000\frac{1}{6}$, the interest for 12 da. is 12 times $$.000\frac{1}{6}$, or $.002. If the interest on $1 for 2 yr. is $.12, for 4 mo. $.02, and for 12 da $.002, the interest on $1 for 2 yr. 4 mo. 12 da. is $.142. If the interest on $1 is $.142, the interest on $650 is 650 times $.142, or $92.30.

632. Hence the following rule may be derived:

Multiply the interest on $1 for the given time at 6% by the number of dollars in the principal, and the result is the interest at 6%.

Change 6% to any other rate of interest by **625.**

WRITTEN EXERCISE

Find the interest by the 6% method.

	Principal	Time	Rate		Principal	Time	Rate
1.	$750.50	4 yr. 11 mo.	6%	*4.*	$1116	3 yr. 11 mo.	7%
2.	$3560.00	9 yr. 10 mo.	8%	*5.*	$17,500	2 yr. 1 mo.	4½
3.	$610.15	7 yr. 11 da.	7%	*6.*	$2400	7 yr. 1 mo.	10½%

7. On the 16th of September, 1904, I borrowed $3500 at 8% interest. How much will settle the loan Jan. 1, 1910?

8. My note for $875.25, given 2 yr. 9 mo. 27 da. ago, bearing 4% interest, is due to-day. What is the amount of interest due?

9. July 16, 1903, I borrowed $2750 at 5% interest, and on the same day loaned it at 7½% interest. If full settlement is made Jan. 4, 1905, how much will be gained?

10. Find the amount of interest at 6% by the six per cent method on:

$680, for 2 yr. 6 mo. 10 da.
$1895, for 1 yr. 7 mo. 7 da.
$468, for 5 yr. 5 mo. 1 da
$1000, for 11 yr. 1 mo. 20 da
$645, for 4 yr. 4 mo. 5 da.
$500, for 3 yr. 1 mo. 27 da.
$895, for 5 yr. 11 mo. 11 da.
$1650, for 1 yr. 10 mo. 23 da
$1463, for 9 yr. 1 mo. 9 da.
$365, for 4 yr. 1 mo. 25 da.

EXACT INTEREST

633. **Exact interest** is interest computed for the exact time in days on the basis of 365 days to a common year and 366 days to the leap year. It is used by the United States government and by a few merchants and bankers.

Aside from the uses in government calculations, exact interest is rarely computed. While it is enforcible, being strictly legal, the greater convenience of the 360-day rules so commend them to public favor as to lead to their common use.

634. To change common interest to exact interest.

635. On a basis of 12 periods of 30 days each, a year's interest is taken for too short a period, since a year, exclusive of a leap year, contains 365 days. The time is, therefore, five days or $\frac{5}{365}$, equal to $\frac{1}{73}$ too short, and the interest taken on that basis is proportionately too great.

To correct this error and obtain the exact interest,

Subtract $\frac{1}{73}$ part from any interest computed on the 360-day basis.

WRITTEN EXERCISE

Find the exact interest of:

1. $954 for 63 days at 7%.
2. $630 for 50 days at 6%.
3. $800 for 33 days at 5%.
4. $137.50 for 93 days at 8%.
5. $210.54 for 100 days at 9%.
6. $681.80 for 90 days at 10%.
7. $500 for 48 days at $6%.
8. $1200 for 31 days at 5%.
9. $1500 for 55 days at 7½%.
10. $4500 for 75 days at 8%.
11. $920 from Apr. 15 to July 25, at 6%.
12. $1756.90 from May 5 to Aug. 2, at 6%.
13. $2500.75 from June 25 to Dec. 8, at 6%.
14. $3200 from Oct. 15 to Nov. 25, at 6%.
15. $2500 from Apr. 16 to June 7, at 6%.

PROBLEMS IN INTEREST

636. The four distinct elements considered in interest are the principal, rate per cent, time, and interest or amount. Since the fourth element is practically the product of the first three, if any three of the elements are given, the other may be found in accordance with the general principles of percentage.

637. To find the rate per cent, the principal, interest, and time being given.

638. Example. At what rate per cent must $2100 be loaned for 2 yr. 5 mo. 6 da. to gain $459.90?

SOLUTION

Let 1% equal the rate.
$51.10 = interest on $2100 for the given time at 1%.
$459.90 ÷ $51.10 = 9.
The interest at 1% is contained in the given interest 9 times.
Therefore the required rate is 9 times 1%, or 9%.

639. From the above solution the following rule may be derived:

Divide the given interest by the interest on the given principal for the given time at 1%.

ORAL EXERCISE

Find the rate of interest:

	Principal	Interest	Time		Principal	Interest	Time
1.	$600	$72	2 yr.	*4.*	$200	$24	4 yr.
2.	$500	$60	3 yr.	*5.*	$400	$16	6 mo.
3.	$300	$60	5 yr.	*6.*	$100	$24	8 yr.

WRITTEN EXERCISE

1. At what rate will $1260 yield $13.44 interest in 96 days?

2. The interest for $2400 for 1 yr. 8 mo. 6 da. is $262.60. Find the rate of interest.

3. If I pay $518.75 interest on $1250 for 5 yr. 6 mo. 12 da., what is the rate per cent?

4. A lady deposited in a savings bank $3750, on which she received $93.75 interest semiannually. What per cent of interest did she receive on her money?

640. To find the time, the principal, interest, and rate of interest being given.

641. Example. **In what time at 8% will $2000 gain $400 interest?**

SOLUTION

Let 1 year represent the time.

8% of $2000 = $160, the interest on the given principal for 1 year.

400 ÷ 160 = 2.5.

Since the interest for 1 year is contained in the given interest 2.5 times, the required interest must be 2.5 times the assumed time.

1 year × 2.5 = 2.5 year, or 2 years 6 months, the required time.

642. From the above solution the following rule may be derived:

Divide the given interest by the interest on the principal for 1 year at the given rate per cent.

ORAL EXERCISE

Find the time in each of the following problems:

	Principal	Interest	Rate		Principal	Interest	Rate
1.	$700	$84	6%	*4.*	$750	$ 7.50	6%
2.	$250	$90	4%	*5.*	$900	$67.50	5%
3.	$400	$ 4	4%	*6.*	$600	$ 6.00	6%

WRITTEN EXERCISE

1. How long will it take $360 to gain $53.64 at 6%?

2. How long should I keep $466.25 at 8% to have it amount to $610.48?

3. A debt of $1650 was paid with 5½% interest on Aug. 30, 1888, by delivering a check for $2316.85. At what date was the debt contracted?

643. To find the principal, the interest, rate of interest, and time being given.

644. Example. What principal will yield $400 interest in 2 yr 6 mo. at 8%?

Solution

Let $1 represent the principal.

$.20 = the interest on $1 for 2 yr. 6 mo. at 8%.

400 ÷ .20 = 2000.

The interest on the required principal is 2000 times the interest on the assumed principal.

Therefore the required principal is 2000 times $1, or $2000.

ORAL EXERCISE

Find the principal in each of the following problems:

	Interest	Rate	Time		Interest	Rate	Time
1.	$40	6%	6 yr. 8 mo.	*4.*	$24	9%	8 mo.
2.	$42	6%	3 yr. 6 mo.	*5.*	$32	6%	6 mo. 12 da.
3.	$50	7½%	240 da.	*6.*	$50	7½%	24 da.

WRITTEN EXERCISE

1. What principal at 7% will gain $1080 in 3 yr. 6 mo.?

2. What principal at 4% will yield $455 in 3 yr. 6 mo. 18 da.?

3. A dealer who clears 12½% annually on his investment is forced by ill health to give up his business. He lends his money at 7%, by which his income is reduced $1512.50. How much had he invested in his business?

645. To find the principal, the amount, rate per cent, and time being given.

646. Example. What principal will amount to $508 in 4 yr. 6 mo. at 6%?

Solution

Let $1 represent the principal.

$1.27 = the amount of $1 for 4 yr. 6 mo.

$508 = the amount of a certain principal for 4 yr. 6 mo.

$508 ÷ $1.27 = 400.

Since the given amount is 400 times the assumed amount, the required principal must be 400 times the assumed principal.

400 times $1 = $400, the required principal.

647. From the above solution the following rule may be derived:

Divide the given amount by the amount of $1 for the given time and rate.

ORAL EXERCISE

Find the principal in each of the following problems:

	Amount	Time	Rate		Amount	Time	Rate
1.	$1120	2 yr.	6%	*5.*	$1025	3 mo.	10%
2.	$2080	6 mo.	8%	*6.*	$1212	60 da.	6%
3.	$4090	90 da.	9%	*7.*	$218	2 yr.	4½%
4.	$3120	6 mo.	8%	*8.*	$367.50	3 yr.	7½%

WRITTEN EXERCISE

1. Find the principal that will amount to $3360 in 3 yr. at 4%.

2. What sum of money put to-day at 6% interest will amount in 7 mo. 12 da. to $4148?

3. Owed a debt of $5310 due in 1 yr. 6 mo. 18 da. I deposited in a bank that allowed me 4% interest a sum sufficient to cancel my debt when due. Find the sum deposited.

4. I borrowed a certain sum for 2 yr. 6 mo. with the understanding that I was to pay interest at the rate of 8%. If at maturity I gave my check for $2400, what was the sum loaned me?

PERIODIC INTEREST

648. **Periodic interest** is simple interest on the principal and on any interest remaining unpaid.

649. When interest is payable annually, it is called *annual interest;* when payable semiannually, *semiannual interest;* when payable quarterly, *quarterly interest;* etc.

650. In some states annual and other periodic interest is sanctioned by law, but in many states it cannot be legally enforced. To secure periodic interest in any state, it must be specified by contract.

Periodic interest is sometimes secured by a note or a series of notes; in such cases the principal only is secured by one of the series (if not by mortgage or otherwise), while each of the other notes is drawn for one interest payment, and matures on the date at which such payment is due. By such arrangement, periodic interest can be enforced in states where it would otherwise be regarded as illegal.

651. To find periodic interest.

652. Example. Find the interest on $400 for 2 yr. at 6%, payable semiannually.

SOLUTION

$48 = the simple interest for the whole time.
$12 = the semiannual interest.
1 yr. 6 mo. = the period for which 1st interest remained unpaid.
1 yr. = the period for which 2d interest remained unpaid.
6 mo. = the period for which 3d interest remained unpaid.
3 yr. = the period for which one semiannual interest draws interest.
$2.16 = the simple interest on $12 for 3 yr.
$48 + $2.16 = $50.16, the semiannual interest due.

653. From the above analysis the following rule may be derived:

To the simple interest on the principal for the full time add the interest on one period's interest for the aggregate time for which the payments of interest were deferred.

WRITTEN EXERCISE

1. Find the quarterly interest on $1600 for 2 yr. at 6%.

2. What is the difference between the simple and the annual interest of $2000 for 3 yr. at 6%?

3. Find the amount of interest due at the end of 4 yr. 9 mo. on a note for $1155 at 6%, interest payable annually, but remaining unpaid.

4. On a note of $1750, dated Aug. 1, 1898, given with interest payable annually at 10%, the first three payments were made when due. How much remained unpaid, debt and interest, Jan. 1, 1905?

COMPOUND INTEREST

654. Compound interest is the interest on the principal and on the principal increased by the interest at the expiration of regular intervals.

655. Interest may be added to the principal annually, semiannually, or quarterly, according to agreement.

Compound interest is not recoverable by law, but a creditor may receive it if tendered without incurring the penalty of usury; and a new obligation may also be taken at the maturity of a compound interest claim for the amount so shown to be due, and such new obligation will be valid and binding.

Most savings banks allow compound interest on balances remaining on deposit for a full interest term.

656. To find compound interest.

657. Example. Find the compound interest on $600 for 3 yr 6 mo. at 6%.

SOLUTION

$600 × $.06 = $ 36.00, the interest for the first year.
$600 + $36 = $636.00, the amount for the first year.
$636 × $.06 = $ 38.16, the interest for the second year.
$636 + $38.16 = $674.16, the amount for the second year.
$674.16 × $.06 = $ 40.45, the interest for the third year.
$674.16 + $40.45 = $714.61, the amount for the third year.
$714.61 × $.03 = $ 21.44, the interest for 6 mo.
$714.61 + $21.44 = $736.05, the amount for the full time.
$736.05 − $600 = $136.05, the compound interest for the full time.

658. Hence the following rule may be derived:

Find the amount of the principal and interest for the first period and make that the principal for the second period, and so proceed to the time of settlement.

Subtract the principal from the last amount, and the remainder will be the compound interest.

If the time contains fractional parts of a period, *find the amount due for the full periods, and to this add its interest for the fractional period.*

WRITTEN EXERCISE

1. What is the compound interest on $1200 for 4 years at 7% if the interest is compounded annually?

2. What is the compound interest on $600 for 3 years at 5% if the interest is compounded quarterly?

3. Find the compound interest on $400 for 4 years at 4%, interest payable semiannually.

4. Find the amount at compound interest on $500 for 3 yr. 4 mo. at 5%, interest payable annually.

5. Find the compound interest at 6% on $2000 for 1 yr. 5 mo., interest payable quarterly.

Compound Interest Table

Showing the amount of $1 at compound interest at various rates per cent for any number of years, from 1 year to 50 years, inclusive.

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.
1	1.0100 000	1.0150 000	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178
8	1.0828 567	1.1264 926	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332
29	1.3345 039	1.5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 8986
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 1345
39	1.4741 225	1.7872 103	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599
40	1.4888 637	1.8140 184	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3351	4.9930 6145
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527
44	1.5493 176	1.9253 330	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8898	6.5705 2824
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7680	4.2562 1944	2.3960 6459	6.8333 4937
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335

COMPOUND INTEREST TABLE

Showing the amount of $1 at compound interest, at various rates per cent, for any number of years, from 1 year to 50 years, inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5668 743	1.6771 001	1.7715 610
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425
11	1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390
20	2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434
39	5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	49.7851 811
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 819	37.3175 320	54.7636 992
43	6.6374 3818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853
48	8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338
49	8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529

Application of Compound Interest Table

659. To find the amount of any given principal for any given number of years:

Multiply the given principal by the amount of $1 at the given rate, as shown by the table.

For periods beyond the scope of the table, *multiply together the amounts shown for periods, the sum of which will equal the time required.*

For example, to find the compound amount of $1 for 100 years:

Multiply the amount for 50 years by itself; to find the amount for 75 years, multiply the amount for 50 years by the amount for 25 years; etc.

If interest is to be compounded semiannually, *take one half the rate for twice the time.* If interest is to be computed quarterly, *take one quarter of the rate for 4 times the time;* etc.

660. To find the compound interest on principals of $100 or less, use four of the decimal places shown in the table; **on principals of $1000 or less,** use five of the decimal places; and so on.

WRITTEN EXERCISE

By the use of the compound interest table solve the following problems:

1. Find the amount of $1750 compounded annually for 10 yr. 6 mo. at 5%.

2. Find the compound interest on $800 for 16 yr. 4 mo. 18 da. at 8%, interest compounded annually.

3. Find the compound interest at 6% on $500 for 1 yr. 3 mo., interest payable quarterly.

4. Find the compound interest at 8% on $1200 for 2 yr. 5 mo., interest payable quarterly.

5. What principal will, in 8 yr. at 5%, amount to $4107.26, if interest is compounded semiannually?

ORAL REVIEW

1. What principal in 2 yr. at 6% will yield $24 interest? $72 interest? $84 interest?

2. What principal in 8 yr. at 5% will amount to $280?

3. What sum of money invested at 6% will in 2 yr. 3 mo. yield $135 interest? $405 interest?

4. At what rate would $300 in 3 yr. yield $45 interest?

5. A lady deposited in a savings bank $750, on which she received $15 interest semiannually. What per cent of interest did she receive on her money?

6. To satisfy a debt of $400 which had been standing 2 yr., I gave my check for $440. What was the rate of interest charged?

7. How long will it take $100 to gain $12 at 6%?

8. How long must $550 be on interest at 7% to amount to $570?

9. In what time will money bearing 8% interest double itself?

Solution. In order to double itself the interest accumulated must be equal to 100% of the principal. Since the principal increases 8% per annum, it will require as many times one year to increase 100%, or to double itself, as 8% is contained times in 100%, or 12½ times, equal to 12 yr. 6 mo.

10. In what time will any sum bearing interest at 4% double itself? at 5%? at 10%? at 9%?

11. How many days will $6000 require to yield $71 interest?

12. In how many days will the interest at 6% be one half of the principal? double the principal?

WRITTEN REVIEW

1. A note of $1260 is 151 days past due. What amount will settle the note and interest, money being worth 6% per annum?

2. The interest on a certain sum in 12½ years is ½ of that sum. What is the rate of interest?

3. A certain principal placed at simple interest for 64 days amounts to $606.40. If the same principal would amount to $624.90 in 249 days, what is the rate of interest? What is the principal?

4. What monthly rent should be charged for a house costing $10,240 in order that 6% interest on the investment may be realized?

5. At the end of five years the accrued interest on a certain principal is found to be ⅕ of the sum drawing interest. What is the rate of interest?

6. A merchant charges interest at the rate of 6% per annum on overdue accounts. He received a check for $1205.40 in settlement for an overdue account of $1200. How long overdue was the account?

7. A man borrowed $12,000 at 5% and with it immediately bought a house which he rented for $1800 per year. What was his yearly per cent of net gain or loss? In how many years will his net gains aggregate the sum borrowed?

8. May 16 I bought 300 barrels of flour, at $7 per barrel; July 28 I sold 50 barrels, at $8 per barrel; Oct. 30, 100 barrels, at $6.75 per barrel; and Feb. 13 following, the remainder, at $7.80 per barrel. Allowing interest at 6%, what was my gain?

9. Find the simple interest on £40 8*s.* 5*d.* for 2 years at 6%. Give the answer in United States money.

10. Find the compound interest, by the table, on $2000 for 4 years at 6%, interest payable quarterly.

11. A man invested $16,000 in business, and at the end of 3 yr. 3 mo. withdrew $22,240, which sum included investment and gains. What yearly per cent of interest did his investment pay?

12. Find the interest of that sum for 11 yr. 8 da. at $10\frac{1}{2}$%, which will, at the given rate and time, amount to $1715.08.

13. Sold an invoice of crockery on a credit of 2 months; the bill was paid 3 mo. 18 da. after the date of purchase, with interest at 8%, by a check for $1963.45. How much was the interest?

14. A man having $21,000 invested it in real estate, from which he received a semiannual income of $787.50. He sold this property at cost and invested the proceeds in a business which yielded him $472.50 quarterly. How much greater rate per cent per annum did he receive from the second investment than from the first?

15. In order to engage in business, I borrowed $3750 at 6%, and kept it until it amounted to $4571.25. How long did I keep the money?

16. In what time will interest at 8% equal $\frac{8}{5}$ of the principal?

17. A building which cost $10,500 rents for $87.50 per month. What annual rate of interest on his investment does the owner receive if he pays yearly taxes amounting to $102.50; insurance, $21.25; repairs, $136.80; and janitor's services, $56.95?

18. A merchant sold a stock of glassware on one month's credit; the bill was not paid until 3 mo. 21 da. after it became due, at which time the seller received a draft for $4716.21 for the bill and interest thereon at the rate of 5%. Find the selling price of the goods.

19. Oct. 12, 1904, I purchased 2700 bushels of wheat at $1.05 per bushel, and afterwards sold it at a profit of 6%. On what date was the wheat sold if my gain was equivalent to 10% interest on my investment?

20. I am offered a house that will rent for $27 per month, at such a price that, after paying $67.20 taxes and other yearly expenses amounting to $24.85, my net income will be 8½% on my investment. What is the price asked for the house?

PRESENT WORTH AND TRUE DISCOUNT

661. The **present worth** of a debt payable at a future time without interest is such a sum as being put at simple interest at a legal rate will amount to the given debt when it becomes due.

662. The **true discount** is the difference between the face of the debt due at a future time and its present worth.

To illustrate, suppose A owes B $212 to be paid for one year after date. Should A care to cancel the indebtedness at once, the sum which he ought to pay should be such that, if put out at legal interest by B, it would at the end of the year amount to $212.

Suppose that B can receive 6% on his money. At this rate $1 put at simple interest would, at the end of one year, amount to $1.06. If $1 in one year amounts to $1.06, it will take as many times $1 to amount to $212 as $1.06 is contained times in $212, or 200 times. Hence, $200 must be the sum which A ought to pay now to cancel a debt which at the end of one year amounts to $212.

The $200 to be paid is called the *present worth*, and the difference between $212 and $200, or $12, the *true discount*.

663. Computations in present worth and true discount come under the case of interest problems, in which the amount, the rate per cent, and the time are given, to find the principal or interest. The *debt* corresponds to the *amount;* the *rate per cent* agreed upon to the *rate;* the *time* intervening before the maturity of the debt to the *time;* the *present worth,* which is the unknown term, to the *principal;* and the *true discount* to the *interest.*

664. **To find the present worth and true discount of a debt.**

665. **Example.** Find the present worth and true discount of a debt of $ 545 payable in 1 yr. 6 mo., when money may be loaned at 6%.

SOLUTION

Let $1 represent the present worth of the debt.
$0.09 = the interest on $1 for 1 yr. 6 mo. at 6%.
$1.09 = the amount of $1 for 1 yr. 6 mo. at 6%.
$545 ÷ $1.09 = 500.

Since the present worth of $1.09 is $1, the present worth of $545, which is 500 times $1.09, must be 500 times $1, or $500.

$545 – $500 = $45, the true discount.

666. Hence the following rule may be derived:

Divide the amount of the debt at maturity by the amount of $1 for the given time and rate and the quotient will be the present worth.

Subtract the present worth from the amount of the debt and the remainder will be the true discount.

ORAL EXERCISE

1. Find the true discount on $ 330 due 2 years hence at 5%.

2. What principal is that which in 2 years at 5% will amount to $ 210?

3. What is the present worth of $ 224 to be paid in 2 years if the money is loaned at 6%?

4. Which is the better, and how much, to buy a piano for $ 636 on 12 months' time, or to pay $ 580 cash, money being worth 6%?

5. If money is worth 6%, what cash offer will be equivalent to an offer of $ 102.50 for a bill of goods on 5 months' credit?

6. A merchant marks an article with two prices, one for cash, $ 48, and the other for credit of 6 months, $ 51.50. If money is worth 6%, which is the better price for the buyer? for the seller?

WRITTEN EXERCISE

1. Which is greater, and how much, the interest or the true discount on $ 516 due in 1 year 8 months, if money is worth 10% per annum?

2. Which is the better, and how much, to buy flour at $6.75 per barrel on 6 months' time, or to pay $6 cash, money being worth 6%?

3. When money is worth 5% per annum, which is preferable, to sell a house for $40,000 cash, or for $42,000, due in 1 year?

4. A farmer offered to sell a pair of horses for $420 cash, or for $475 due in 15 months without interest. If money is worth 8% per annum, how much would the buyer gain or lose by accepting the latter offer?

5. If money is worth 6%, what cash offer will be equivalent to an offer of $1546 on a bill of goods on 90 days' credit?

6. An agent paid $840 for a traction engine, and after holding it in stock for 1 year, sold it for $933.80 on 8 months' credit. If money was worth 6%, what was his actual gain?

7. Find the difference between the interest and true discount on $4160 for 1 year at 4%.

8. A stock of moquette carpeting bought at $1.95 per yard on 8 months' credit was sold for cash on the day of purchase at $1.80 per yard. If money was worth 6% per annum, what per cent of gain or loss did the seller realize?

9. William is now 16 years old. How much money must be invested for him at 6% simple interest in order that he may have $19,500 principal and interest when he celebrates his 21st birthday?

10. What amount of goods bought on 6 months' time, or 5% off for cash, must be purchased in order that they may be sold for $4180 and net the purchaser 10% profit on paying cash and getting the agreed discount off?

11. A merchant bought a bill of goods for $2150 on 6 months' credit and the seller offered to discount the bill 5% for cash. If money is worth 7½% per annum, how much would the merchant gain by accepting the seller's offer?

12. On Jan. 1, 1903, I paid $800 for a debt of $832 payable on a certain future date. If money was worth 8% per annum and my payment was the present worth of the claim, when was the debt due?

13. What amount of goods bought on 4 months' time, 10% off if paid in 1 month, 5% off if paid in 2 months, must be purchased in order that they may be sold for $11,480 and ½ of the stock net a gain of 15%, and the remainder a gain of 20% to the purchaser, if he pays his bill within 1 month, and gets the agreed discount off?

14. The true discount on $4160 for 1 year is $160. What is the rate of interest?

15. I sold my farm for $10,000, the terms being one fifth cash and the remainder in four equal semiannual payments with simple interest at 5% on each from date. Three months later the purchaser settled in full by paying with cash the present worth of the deferred payments on the basis of 10% for the use of the money. How much cash did I receive in all?

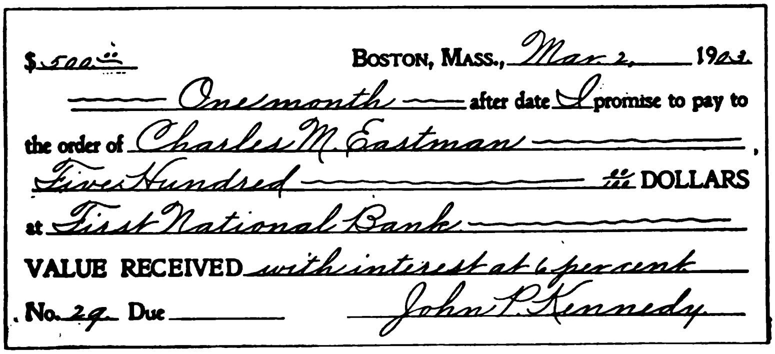
Note. Before finding the present worth of interest-bearing debts, add to the face of the debt the simple interest for the time to the date of maturity.

NEGOTIABLE PAPER

667. Negotiable paper is paper which may be transferred from one owner to another by indorsement or delivery, or both; as, promissory notes, drafts, and bills of exchange.

668. A promissory note, or a **note**, is a written promise on the part of one person to pay another a certain sum of money on demand or at a certain specified future time.

669. A negotiable note is one which is made payable *to the order* of a designated payee, or *to the bearer.*



500\frac{00}{}$ BOSTON, MASS., Mar. 2, 19[illegible]

One month after date I promise to pay to

the order of Charles M. Eastman,

Five Hundred $\frac{00}{100}$ DOLLARS

at First National Bank

VALUE RECEIVED with interest at 6 per cent

No. 29 Due John P. Kennedy

670. The parties whose names appear on a note when it is made are the *maker* and the *payee.*

671. The **maker** is the person who promises to pay and whose name is signed to the note. The **payee** is the person named in the body of the note and the one to whom the money is to be paid.

When a note is made payable to a certain person, *or bearer*, or to *the bearer*, any person who is the lawful holder of the note is the payee.

672. The **face** of a note, draft, etc., is the sum for which it is given.

In the foregoing form, John P. Kennedy is the maker; Charles M. Eastman, the payee; and $500, the face of the note.

When no place of payment is named, a note is payable at the residence or place of business of the maker.

When a note contains the words "with interest" or "with use" and does not specify the rate, it draws interest at the legal rate for the place where it is drawn up. A note containing an interest clause will bear interest from its date unless other time be specified. When the words "with interest" or "with use" are omitted, the note does not draw interest, and is called *non-interest-bearing*. Non-interest-bearing notes become interest-bearing after maturity.

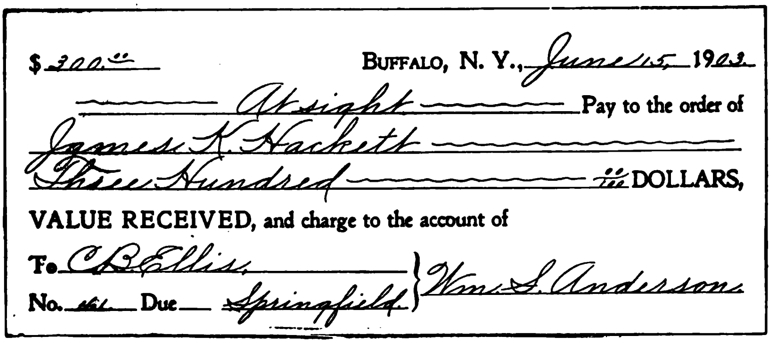
673. A **non-negotiable note** is one that is drawn payable only to the party named in the note.

If the words "to the order of" were omitted in the foregoing form, the note would be non-negotiable, and not transferable except by assignment.

674. A **draft** is a written order by one person on another for the payment of a specified sum of money to a third person, or to his order.

675. With reference to time of payment there are two kinds of drafts, — *sight drafts* and *time drafts*.

676. A **sight draft** is one drawn payable "at sight"; that is, when it is presented to the drawee for payment.



$300.00 BUFFALO, N. Y., June 15 1903

At sight Pay to the order of

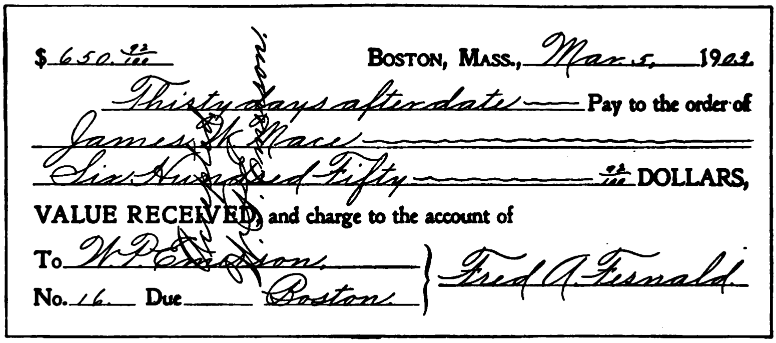
James K. Hackett

Three Hundred 00/100 DOLLARS,

VALUE RECEIVED, and charge to the account of

To C. B. Ellis, } Wm. S. Anderson.
No. 61 Due Springfield. }

677. A **time draft** is one payable on a specified date, a certain time after date, or a certain time after sight.



$650.00/100 BOSTON, MASS., Mar. 5, 1902

Thirty days after date Pay to the order of

James W. Mace

Six Hundred Fifty 00/100 DOLLARS,

VALUE RECEIVED, and charge to the account of

To W. P. Emerson, } Fred A. Fernald.

No. 16 Due Boston }

678. The **drawer** is the person who writes or draws the draft. The **drawee** is the person on whom the draft is drawn. The **payee** is the person to whom the draft is to be made payable.

In the above draft Fred A. Fernald is the drawer; W. P. Emerson, the drawee; and James W. Mace, the payee.

A draft, being simply an order to pay money, is not binding on the drawee without his consent.

By accepting the above draft W. P. Emerson has expressed his willingness to pay it. The draft is then an *acceptance*, and W. P. Emerson the *acceptor*.

The drawee's liability begins with his acceptance of the draft. By accepting the above draft W. P. Emerson has practically created a promissory note with himself as maker.

The acceptance on drafts payable after sight should always be dated to make it possible to determine the maturity of the paper.

A draft payable "after date" begins to mature from the date of the paper.

A draft payable "after sight" begins to mature from the date of the acceptance of the paper.

To *honor* a draft is to accept it or pay it upon presentation.

679. An **indorsement** is that which is written on the back of a note or bill which pertains to the transfer or payment thereof.

The person who indorses or writes his name on the back of a note or bill is called the *indorser*, and the person to whom the note or bill is indorsed, the *indorsee*.

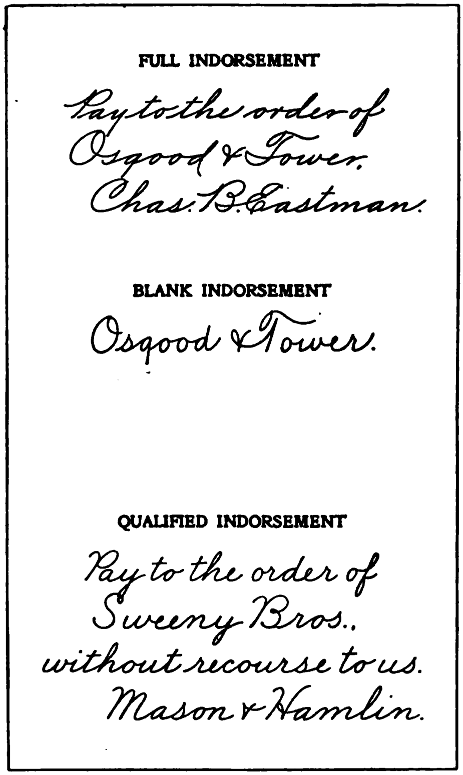
680. The purchaser of negotiable paper has a valid claim against the original and subsequent parties thereto if he can show:

1. That he gave value for it.
2. That he bought it before maturity.
3. That to the best of his knowledge and belief no fraud was connected with it.

681. Indorsements are made on notes and drafts for three purposes: 1. To secure their payment. 2. To effect their transfer 3. To make memorandum of a partial payment.

682. **The principal forms of indorsement** in common use in business transactions are: 1. *Full.* 2. *Blank.* 3. *Qualified.*

683. A **full indorsement** is one in which the indorser directs the payment of the instrument to be made to the order of a particular party. A **blank indorsement** is one in which the indorser merely writes his name on the back, thus making the instrument payable to the bearer. A **qualified indorsement** is one in which the indorser relieves himself from the ordinary responsibility of an indorser by placing over his name the words "without recourse." A qualified indorsement may be in blank or in full.



FULL INDORSEMENT

Pay to the order of
Osgood & Tower.
Chas. B. Eastman.

BLANK INDORSEMENT

Osgood & Tower.

QUALIFIED INDORSEMENT

Pay to the order of
Sweeny Bros.,
without recourse to us.
Mason & Hamlin.

For greater security, checks, notes, and drafts should generally be indorsed in full. A check, note, or draft used for remittance purposes should always be indorsed in full.

When paper is indorsed in blank, the lawful holder of it may, if he chooses, write over the name of the indorser the words necessary to convert the blank indorsement into a full indorsement.

684. A **protest** is a written, or partly written and partly printed, declaration made by a notary public, of the demand and refusal to pay or accept negotiable paper.

685. A note is **dishonored** if the maker refuses payment at maturity; a draft, if the drawee does not accept or pay it upon presentation.

If the holder of a note or draft fails to give notice of dishonor by the maker or acceptor, the indorsers of the paper are not liable for its non-payment.

BANK DISCOUNT

686. A **commercial bank** is a corporation chartered by law for the receiving and loaning of money, for facilitating its transmission, and, in case of banks of issue, for furnishing a circulating medium.

687. **Bank discount** is a deduction from the sum due upon a negotiable paper at its maturity for the cashing or buying of such paper before it becomes due.

688. In true discount the present worth is taken as the principal; in bank discount the future worth is taken as the principal.

689. The **time** in bank discount is the period included from the date of the paper to its maturity.

690. The **term of discount** is the time from the date of discount to the date of maturity.

Banks generally charge interest for the exact number of days.

In a few cities both the day of date and day of maturity are included in the term of discount.

In this text the term of discount is found by taking the *exact number of days* from the date of discount to the date of maturity.

691. The **discount** is the interest at the legal rate for the term of discount; it is always paid in advance.

A fixed sum may be taken as the discount, but this is unusual.

692. The **maturity** of negotiable paper is the date upon which it is due.

The maturity of a note or draft payable after date is determined by adding the time to the date of the note or draft; the maturity of a draft payable after sight by adding the time to the date of the acceptance. When the time of a note or draft is expressed in months, simply the number of months, regardless of the number of days included, is counted; but when the time is expressed in days, the exact number of days, regardless of the number of months included, is counted; *e.g.* a note or draft due two months from date matures on the corresponding day of the second month following, providing there is a sufficient number of days in that month to make this possible. If there are not days enough, the note or draft is due on the last day of that month. Thus, notes and drafts dated Jan. 28, 29, 30, and 31, and payable one month after date, would mature Feb. 28; but if payable 30 days after date, they would mature Feb. 27, 28, Mar. 1, and Mar. 2, respectively.

In a great many of the states the added day for leap year is not counted, and a note or draft payable 30 days after Feb. 25 in any leap year would be considered due Mar. 27 instead of Mar. 26, as would be the case if the extra day were counted. In this text the extra day for leap years is counted.

693. The **proceeds** of a note or other negotiable paper is the amount paid to the one offering it for discount; it is equal to the face of the paper less the discount.

694. The **value** of a note at its maturity is its face if the note does not bear interest; and its face plus the interest for the time if the note does bear interest.

695. **Days of grace** are the three days sometimes allowed by law for the payment of a note or time draft after the expiration of the time specified in the note or draft.

Days of grace have been abolished in so many states that all problems in this text omit them.

The laws of the different states with regard to paper maturing on Sunday or a legal holiday are not uniform. Some states provide that when notes or drafts mature on Sunday, they shall be paid on Saturday, and that if Saturday be a legal holiday, then they shall be paid on Friday; also that if a legal holiday occurs on Monday, payment must be made on the preceding Saturday. In other states if notes or drafts fall due on Sunday or on a legal holiday, they must be paid on the next succeeding business day, etc. The laws of the different states should be carefully studied and fully observed in order that all contingent parties may be held responsible.

696. Computations in bank discount may be made in accordance with the general principles of percentage or interest, the *value of the note* corresponding to the *base* in percentage, or the *principal* in interest; the *per cent of discount* to the *rate* or the *per cent of interest;* the *bank discount* to the *percentage* or the *interest;* and the *proceeds* to the *difference.*

ORAL EXERCISE

1. What is the maturity of a note dated Oct. 31 and payable in 60 days? in 90 days?

Solution. Since 60 days is equal to 2 months of 30 days each, to find the maturity of a note due 60 days from Oct. 31, count ahead two months, obtaining Dec. 31 as the due date; but as December has 31 days instead of 30, to find the actual due date deduct this one day from the date of maturity, obtaining Dec. 30 as the result.

Following this line of reasoning, when the time is 90 days count ahead three months, and the due date is Jan. 31; but since two of the months, January and December, each have an extra day, deduct two days from the approximate due date, obtaining Jan. 29 as the actual date of maturity.

By inspection, find the maturity of each of the following notes:

	Date	Time		Date	Time
2.	Jan. 29, 1903	1 mo.	*7.*	Sept. 30, 1903	60 da.
3.	Jan. 30, 1904	1 mo.	*8.*	Oct. 8, 1904	90 da.
4.	Jan. 28, 1903	1 mo.	*9.*	Apr. 5, 1903	1 mo.
5.	Jan. 31, 1904	30 da.	*10.*	May 30, 1904	60 da.
6.	Oct. 31, 1903	1 mo.	*11.*	Apr. 15, 1904	6 mo.

By inspection, find the maturity of the following time drafts:

	Date	Time after Date		Date	Time after Date		Date	Time after Date
12.	Mar. 5	30 da.	*14.*	Sept. 1	2 mo.	*16.*	Jan. 1	3 mo.
13.	July 16	90 da.	*15.*	May 31	1 mo.	*17.*	Dec. 28	60 da.

By inspection, find the maturity of each of the following drafts:

	Date accepted	Time after Sight		Date accepted	Time after Sight		Date accepted	Time after Sight
18.	Aug. 9	10 da.	*20.*	July 16	90 da.	*22.*	July 3	3 mo.
19.	Dec. 12	1 mo.	*21.*	Nov. 30	30 da.	*23.*	Feb. 28	1 mo.

WRITTEN EXERCISE

Find the maturity and term of discount in the following problems:

	Date of Note	Time	Date discounted		Date of Note	Time	Date discounted
1.	Jan. 15	3 mo.	Jan. 25	*5.*	Sept. 20	3 mo.	Sept. 20
2.	Mar. 8	30 da.	Mar. 15	*6.*	June 5	60 da.	June 18
3.	Apr. 7	90 da.	May 3	*7.*	Aug. 8	30 da.	Aug. 15
4.	Mar. 20	2 mo.	Apr. 1	*8.*	Oct. 1	3 mo.	Oct. 20

Find the maturity and term of discount in the following problems:

	Date of Draft	Time after Sight	When accepted	When discounted
9.	Feb. 12	10 da.	Feb. 12	Feb. 13
10.	Oct. 6	30 da.	Oct. 7	Oct. 12
11.	Dec. 24	60 da.	Dec. 29	Dec. 29
12.	Nov. 30	3 mo.	Dec. 1	Dec. 7

Find the maturity and term of discount in the following problems:

	Date of Draft	Time after Date	When accepted	When discounted
13.	Mar. 3	90 da.	Mar. 4	Mar. 8
14.	June 29	3 mo.	July 1	July 10
15.	Jan. 15	30 da.	Jan. 17	Jan. 17
16.	Feb. 14	1 mo.	Feb. 15	Feb. 16

697. **To find the bank discount or proceeds of a note, the face, time, and rate per cent being given.**

698. **Examples.** *1.* Find the bank discount and proceeds of a note for $ 638 due in 54 days.

SOLUTION

Face = $638.
Bank discount = the simple interest on the face for 54 days, or $5.74.
Proceeds = $638 – $5.74, or $632.26.

2. A note for $540, dated Jan. 10, 1903, payable in 3 months, with interest at 6%, is discounted Feb. 27 at 6%. Find the bank discount and proceeds.

SOLUTION

Face = $540.
Interest = $8.10.
Value of the note at maturity = $540 + $8.10, or $548.10.
Bank discount = the interest on $548.10 for 42 days, or $3.84.
Proceeds = $548.10 − $3.84, or $544.26.

699. Hence the following rule may be derived:

Compute the interest on the value of the note for the time and rate, and the result is the bank discount.

Subtract the bank discount from the value of the note, and the result is the proceeds.

WRITTEN PROBLEMS

Find the bank discount and proceeds of the following notes:

	Face	Date of Note	Time	Date of discount	Rate of discount
1.	$2400	Apr. 2	60 da.	Apr. 18	6%
2.	$1200	Apr. 8	2 mo.	Apr. 20	6%
3.	$3250	June 15	60 da.	June 30	4½%
4.	$500	Mar. 20	2 mo.	Apr. 5	5%
5.	$2500	May 8	3 mo.	June 16	6%
6.	$3600	July 8	90 da.	Aug. 7	6%
7.	$1500	Aug. 5	60 da.	Aug. 5	6%
8.	$2400	Sept. 5	2 mo.	Sept. 6	8%
9.	$4500	Oct. 2	30 da.	Oct. 8	4½%
10.	$2400	Nov. 1	30 da.	Nov. 5	8%

11. Apr. 18, J. M. Cox & Co. borrowed of National Bank of Redemption $1200 on their note at 60 days, indorsed by W. C. Williams. Write the note and show the indorsement. Find the proceeds, the rate of discount being 6%.

Find the date of maturity, the term of discount, the discount, and the proceeds of the following notes and drafts:

12. $1200.00. BOSTON, MASS., Apr. 15, 1903.

Ninety days after date I promise to pay to the order of Smith Bros. & Co., Twelve Hundred Dollars.

Value received. THOMAS BROWN, JR.

Discounted May 1 at $4\frac{1}{2}\%$.

13. $4500.00. BOSTON, MASS., Mar. 3, 1903.

Ninety days after date we promise to pay to the order of E. M. Williams & Co., Forty-five Hundred Dollars at National Bank of Redemption. Value received.

CHARLES E. COLE.
FRANK E. SPRING.

Discounted Apr. 2 at 6%.

14. $900.00. DENVER, COLO., Nov. 25, 1903.

Three months after date I promise to pay to the order of Smith Bros. & Co., Nine Hundred Dollars at the Erie County National Bank. Value received.

W. L. SUNDERLAND.

Discounted Jan. 1, 1904, at 8%.

15. $660.90. NEW ORLEANS, LA., May 5, 1903.

Ninety days after date I promise to pay to the order of H. H. Douglas, Six Hundred Sixty and $\frac{90}{100}$ Dollars, with interest at 6%.

Value received. CLAYTON S. MEYERS.

Discounted June 1 at 5%.

16. $2400.00. ST. PAUL, MINN., Aug. 31, 1903.

Six months after date I promise to pay to the order of John W. Bell, Twenty-four Hundred Dollars, with interest at 8%, after one month. Value received.

OLIVER JONES.

Discounted Sept. 5, 1903, at 8%.

17. $800.00. CLEVELAND, O., Jan. 31, 1903.

One month after date we promise to pay to the order of Hale & Bly, Eight Hundred Dollars, with interest at 5%.

Value received. HART & COLE.

Discounted Feb. 10, 1903, at 6%.

18. $1200.00. SPRINGFIELD, MASS., Mar. 5, 1903.

Four months after date we jointly and severally promise to pay to the order of Shaw Bros. & Co., Twelve Hundred Dollars, with interest from date. Value received.

RAYMOND D. DANN,
CHARLES L. KINSLEY.

Discounted May 1 at $4\frac{1}{2}\%$.

19. $2576.25. CHICAGO, ILL., May 20, 1903.

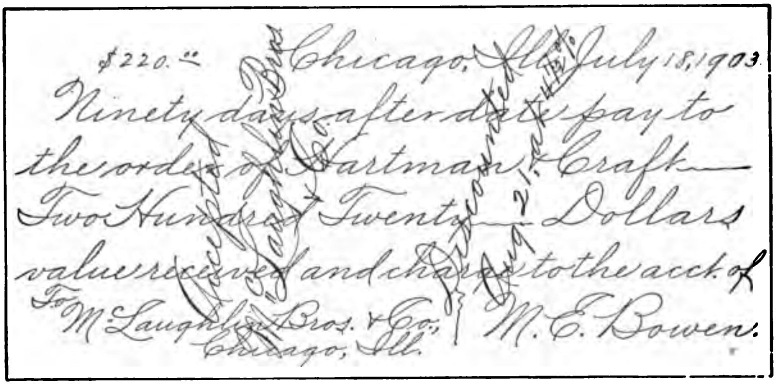
At ninety days' sight pay to the order of Ourselves, Twenty-five Hundred Seventy-six and $\frac{25}{100}$ Dollars, value received and charge to the account of

To SPEAR BROS. & Co.,
San Francisco, Cal.

F. E. ROGERS & Co.

Accepted May 25, 1903. Discounted May 30, 1903, at 6%.

20.

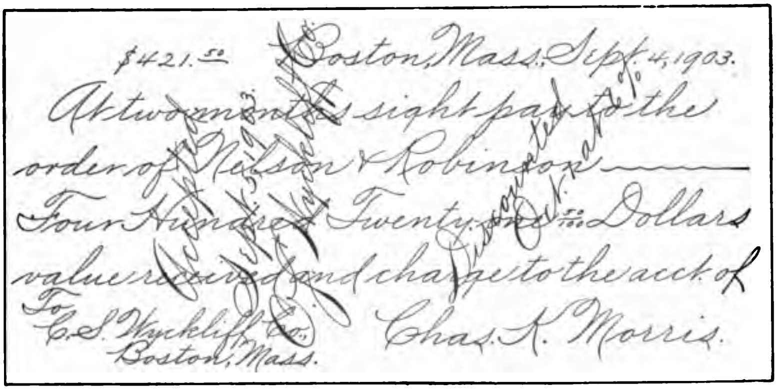


$220.00 Chicago, Ill. July 18, 1903.
Ninety days after date pay to
the order of Hartman & Craft
Two Hundred Twenty Dollars
value received and charge to the acct. of
To McLaughlin Bros. & Co.,
Chicago, Ill.
M. E. Bowen.

Accepted
McLaughlin Bros.

Discounted
Aug. 21, at 4½%

21.



$421.50 Boston, Mass. Sept. 4, 1903.
At two months' sight pay to the
order of Nelson & Robinson
Four Hundred Twenty-one $\frac{50}{100}$ Dollars
value received and charge to the acct. of
To C. S. Wyckliff & Co.,
Boston, Mass.
Chas. K. Morris.

Accepted
Sept. 5
C. S. Wyckliff & Co.

Discounted
Oct. 1, at 5%

22. $ 795$\frac{50}{100}$. ROCHESTER, N. Y., May 15, 1904.

Sixty days after date pay to the order of Ourselves, Seven Hundred Ninety-five and $\frac{50}{100}$ Dollars, value received, and charge to the account of

BOWEN, MERRILL & CO.

TO GRAY & SALISBURY, Buffalo, N. Y.

Accepted May 20, 1904. Discounted May 21, 1904, at 6%. Collection charges $\frac{1}{10}$%.

23. A note for $1200, dated Boston, Mass., Mar. 4, 1903, and payable 3 months after date, was discounted Apr. 5 at 5%. Find the bank discount and proceeds.

24. What is the proceeds of a note for $ 3500, dated Feb. 2, 1903, and due in 4 months, without interest, if the note is discounted Apr. 20, at 4% ?

25. Find the date of maturity, the bank discount, and proceeds of a note for $1800, dated Feb. 18, 1903, payable in 90 days, and discounted Apr. 25, 1903 at 6%.

26. Paul Harmon's bank account is overdrawn $3596.11. He now discounts at 6%: a 90-day note for $450, a 60-day note for $1754.81, a 30-day note for $851.95, a 20-day note for $345.25, a 10-day note for $100; proceeds of all to his credit at the bank. What is the condition of his bank account after he receives these credits?

27. C. H. Good & Co.'s bank account is overdrawn $7,564.19. They discount at 6% : a 90-day note for $ 3975.21, a 60-day note for $1546.19, and a 20-day note for $2546.85; proceeds of all to their credit at the bank. What is the condition of their bank account after they receive credit as above?

28. Assuming that the model note, page 259, was discounted March 4, 1903 at 6%, find the bank discount and net proceeds.

29. Assuming that the model draft, page 261, was discounted March 9, 1903 at 5%, find the bank discount and net proceeds; collection charges, $\frac{1}{8}$%.

700. **To find the face of a note, the proceeds, time, and rate of discount being given.**

701. **Example.** What must be the face of a note, payable in 30 days, in order that when discounted at 6% the proceeds will be $895.50?

SOLUTION

Let $1 represent the face of the note.
$.005 = the bank discount on $1 for 30 da.
$1 – $.005 = $.995, the proceeds of $1 due in 30 days.
$895.50 ÷ $.995 = 900.
The given proceeds is 900 times the proceeds of $1.
Therefore the required face is 900 times $1, or $900.

702. Hence the following rule:

Divide the proceeds of the note by the proceeds of $1 at the given rate for the given time.

WRITTEN EXERCISE

1. I wish to borrow $650.08 of a bank. For what sum must I issue a 90-day note to obtain the amount, discount being at 6%?

2. A man wishes to borrow $594 cash. For how much must he draw a note, so that when discounted at 6% on 60 days' time, without interest, the proceeds will be the sum wanted?

3. A 30-day 6% interest-bearing note was discounted 10 days after it was drawn up. If the rate of discount was 6% and the bank discount $13.40, what was the face of the note?

4. A merchant bought goods to the amount of $2376. For how much must he draw his 60-day note, without interest, that when discounted at 6% he may pay for his purchase with the proceeds?

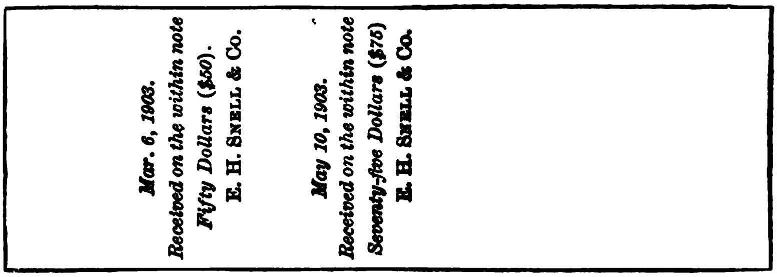
5. A note dated Mar. 15, 1903, payable in 3 months with interest at 7½%, was discounted Apr. 16, 1903, at 10%. If the proceeds were $2404.25, what must the face have been?

6. You have $650.80 to your credit at a bank; you give your check for $1872.40, after which you discount a 30-day note for $850.80, proceeds to your credit at the bank. You then discount a 90-day note, made by H. C. Davis, proceeds to your credit, when you find yourself indebted to the bank $24.74. If discount be at 6%, what must have been the face of the note made by Davis?

PARTIAL PAYMENTS

703. A **partial payment** is a payment of a part of the amount due on a note, mortgage, or other interest-bearing obligation.

Partial payments should be acknowledged by indorsement on the back of a note, as follows:



Mar. 6, 1903.
Received on the within note
Fifty Dollars ($50).
E. H. Snell & Co.

May 10, 1903.
Received on the within note
Seventy-five Dollars ($75)
E. H. Snell & Co.

Sometimes special receipts are given for partial payments on notes and other similar obligations.

A debtor, or his authorized agent, may make a payment, either in part or in full, of any obligation, and such payment may be received by the creditor, or his authorized agent.

704. Various rules are in use for finding the balance due on claims on which partial payments have been made, but only the *United States Rule* and the *Merchants' Rule* have more than local application.

UNITED STATES RULE

705. The **United States Rule** is very generally used when partial payments are made on interest-bearing obligations that run for more than one year. It is the rule that has been adopted by the Supreme Court of the United States, and by most of the separate states.

706. General Principles. The United States Rule recognizes the following general principles:

1. Accrued interest must be paid before the principal may be diminished.

2. Interest must not be charged upon interest.

707. **To find the balance due by the United States Rule for partial payments.**

708. **Example.** A note, the face of which was $2500, bearing interest at 6%, was given Nov. 1, 1899, and settled Aug. 5, 1904. Find the balance due, the following payments having been made: Dec. 5, 1900, $600; Jan. 5, 1902, $500; May 1, 1903, $100; July 5, 1904, $800.

SOLUTION

Face of note		$2500.00
Interest from Nov. 1, 1899, to Dec. 5, 1900 (1 yr. 1 mo. 4 da.)		164.17
Amount due Dec. 5, 1900, time of first payment . .		2664.17
Payment of Dec. 5, 1900		600.00
New principal, or amount to draw interest from Dec. 5, 1900,		2064.17
Interest from Dec. 5, 1900, to Jan. 5, 1902 (1 yr. 1 mo.) .		134.17
Amount due Jan. 5, 1902, time of second payment . .		2198.34
Payment Jan. 5, 1902		500.00
New principal, or amount to draw interest from Jan. 5, 1902		1698.34
Interest from Jan. 5, 1902, to May 1, 1903 (1 yr. 3 mo. 26 da.)	$134.73	
The interest exceeds the payment, and a new principal is not formed.		
Interest from May 1, 1903, to July 5, 1904 (1 yr. 2 mo. 4 da.)	120.02	
Total interest due July 5, 1904		254.75
Amount due July 5, 1904, the time of the fourth payment		1953.09
Sum of payments May 1, 1903, and July 5, 1904 . .		900.00
New principal, or amount to draw interest from July 5, 1904		1053.09
Interest from July 5, 1904, to Aug. 5, 1904 (1 mo.) . .		5.27
Amount due Aug. 5, 1904, the final date of settlement .		1058.36

CONDENSED FORM FOR WRITTEN WORK

DATES			INTEREST PERIODS			PRINCIPALS	INTERESTS	AMOUNTS	PAYMENTS
Yr.	Mo.	Da							
1899	11	1							
1900	12	5	1 yr.	1 mo.	4 da.	$2500.00	$164.17	$2664.17	$600
1902	1	5	1	1	0	2064.17	134.17	2198.34	500
1903	5	1	1	3	26	1698.34	134.73		100
1904	7	5	1	2	4	1698.34	120.02	1953.09	800
1904	8	5	0	1	0	1053.09	5.27	1058.36 *Ans.*	

709. Rule. *Find the amount of the principal to the time when the payment or the sum of the payments shall exceed the interest then due.*

From this amount deduct the payment or the sum of the payments made.

Consider the remainder as a new principal, and proceed as before to the time of settlement.

WRITTEN EXERCISE

1. On a mortgage for $650, made Aug. 10, 1894, and bearing interest at 6%, payments were indorsed as follows: Feb. 2, 1896, $100; June 20, 1898, $50; Nov. 1, 1900, $250. How much was due Mar. 31, 1903?

2. On a claim for $3000, dated Sept. 15, 1900, bearing interest at 8%, payments were made as follows: Jan. 1, 1901, $300; July 20, 1901, $500; Feb. 2, 1902, $125; Apr. 20, 1903, $1800. How much was due at final settlement Jan. 1, 1904?

3. On the note below indorsements were made as follows: Apr. 1, 1901, $300; Nov. 20, 1902, $1000; Mar. 18, 1903, $600; Mar. 15, 1904, $1100. What was the balance due Jan. 1, 1905?

$4000.00 Chicago, Ill., Mar. 15, 1901.

On demand we promise to pay to the order of Williston, Burgess & Hart, Four Thousand Dollars, at Union National Bank, with interest at 6%.

Value received. Houghton, Dutton & Co.

4. A note of $1500, dated June 20, 1902, bearing interest at 4%, had payments indorsed upon it as follows: Dec. 5, 1902, $300; Apr. 2, 1903, $30; July 20, 1903, $500; Dec. 31, 1903, $400. Find the amount due Apr. 1, 1904.

5. On a mortgage for $4500, dated May 1, 1899, and bearing interest at 7%, the following payments were made: Feb. 2, 1900, $100; Aug. 5, 1900, $75; Aug. 5, 1901, $2000; Dec. 20, 1902, $300. How much was due at final settlement Apr. 1, 1903?

6. On the note below payments were indorsed as follows: Oct. 1, 1901, $750; Apr. 1, 1902, $150; Oct. 1, 1902, $365.90; Mar. 25, 1903, $150; Nov. 1, 1903, $200. How much was due Apr. 6, 1904?

$2500.00 Boston, Mass., Apr. 6, 1901.

Two years after date for value received, I promise to pay to the order of C. W. Frey & Co., Twenty-five Hundred Dollars, with interest at 6%. F. M. ELLERY.

MERCHANTS' RULE

710. When partial payments are made on interest-bearing notes that run for one year or less, the amount due at final settlement is usually found by the Merchants' Rule.

711. General Principles. The Merchants' Rule recognizes the following general principles:

1. The face of the note draws interest to the time of settlement.
2. Interest is allowed on each payment from the time such payment is made to the date of settlement.

The Merchants' Rule is varied in its use by different creditors, and hence is rather more an agreement, founded upon custom or otherwise between debtor and creditor as to mode of settlement, than a strict rule of law.

712. To find the balance due by the Merchants' Rule for partial payments.

713. Example. A note for $900, dated May 5, 1903, payable on demand, shows that the following payments have been made: June 20, $200; Aug. 15, $300; Dec. 1, $200. What is due Dec. 31, 1903, money being worth 6%?

SOLUTION

Face of note		$900.00
Interest from May 5, 1903, to Dec. 31, 1903 (7 mo. 26 da.)		35.40
Amount of note at date of final settlement, Dec. 31, 1903		935.40
First payment	$200.00	
Interest on payment from June 20 to Dec. 31 (6 mo. 11 da.)	6.37	
Second payment	300.00	
Interest on payment from Aug. 15 to Dec. 31 (4 mo. 16 da.)	6.80	
Third payment	200.00	
Interest on payment from Dec. 1 to Dec. 31 (30 da.)	1.00	
Value of payments at the date of final settlement		714.17
Balance due Dec. 31, the date of final settlement		$221.23

CONDENSED FORM FOR WRITTEN WORK

DATES		INTEREST PERIODS		PRINCIPAL	PAYMENTS	INTERESTS	AMOUNT OF PRINCIPAL	AMOUNTS OF PAYMENTS	BALANCE
Mo.	Da.	Mo.	Da.						
5	5	7	26	$900		$35.40	$935.40		$935.40
6	20	6	11		$200	6.37		$206.37	
8	15	4	16		300	6.80		306.80	
12	1	0	30		200	1.00		201.00	714.17
12	31								$221.23 *Ans.*

714. RULE. *Find the amount of the principal to the date of settlement regardless of any payments made.*

Find the amount of each payment from the time it was made to the time of settlement.

Subtract the sum of the payment amounts from the amount of the principal and the result will be the balance due.

WRITTEN EXERCISE

1. A note for $2100 dated Apr. 15, 1903, payable on demand, with interest, bears the following indorsements: June 20, $300; Sept. 1, $200; Nov. 25, $750; Dec. 18, $300. What is due Jan. 31, 1904, money being worth 7%?

2. What is the balance due Apr. 15, 1904, on a note for $525 dated Jan. 1, 1903, bearing 6% interest if the following indorsements were made thereon: Mar. 2, 1903, $75; July 15, 1903, $200; Sept. 20, 1903, $100; Jan. 3, 1904, $50?

3. A note for $1600 dated Mar. 2, 1903, payable in 6 months with interest at 6% has the following indorsements: Apr. 3, $450; June 15, $320; July 19, $179.85; Aug. 3, $400. What is due at the maturity of the note?

4. A note for $950.75 dated Mar. 8, 1903, bears the following indorsements: Apr. 16, $250; June 8, $150; Aug. 2, $200; Sept. 30, $100; Nov. 2, $90. What was due Dec. 8, 1903, at 6%? At 5%? At 8%?

5. A note for $1900 dated Jan. 25, 1903, was indorsed as follows: Apr. 2, $900; May 3, $750; July 3, $100. What remained due Sept. 25, 1903, money being worth 6%?

EQUATION OF ACCOUNTS

715. Equation of accounts is the process of finding the date when the balance of an account can be paid without loss or gain to either party.

716. Accounts having items on but one side, either debit or credit, are called *simple accounts*, and the equating of such accounts is called *simple equation*, or *equation of bills.*

717. Accounts having both debit and credit items are called *compound accounts*, and the equating of such accounts is called *compound equation*, or *equation of accounts.*

718. The **term of credit** is the time that must elapse before a debt becomes due.

If the term of credit is given in days, the exact number of days must be added to the date of the purchase or sale; if given in months, the number of months, regardless of the number of days included, must be added to the date of the purchase or sale.

719. Book accounts bear legal interest after they become due; and notes, even if not containing an interest clause, bear interest after maturity.

720. The **average term of credit** is the time that must elapse before several debits due at different times may be equitably discharged in one sum.

721. The **average date of payment**, the **equated date**, or **due date** is the date on which payment or settlement may be equitably made.

To illustrate, suppose A stands charged as follows:

Jan. 10, $200.
Jan. 20, $200.
Jan. 30, $200.

If the first charge were not paid on Jan. 10, it would be subject to interest from Jan. 10 to the date of settlement; if the third charge were paid before Jan. 30, discount should be allowed for the number of days between the date of settlement and Jan. 30; if the second charge were paid Jan. 20 no interest would be charged or discount allowed. It will be seen that the interest on the first charge from Jan. 10 to Jan. 20 is equal to the discount on the third charge from Jan. 20 to Jan. 30. Hence, the whole account can be equitably paid on Jan. 20, *the average date of payment.*

722. The **focal date** is any assumed date of settlement with which the dates of several accounts are compared for the purpose of finding the average term of credit, or due date.

Any date may be used as a focal date, and the result will be the same. This is true because all items are equally affected when a different focal date is used.

Any rate per cent may be used and the result will be the same. As a matter of convenience always use 6%, and base all computations on the commercial year of 360 days.

723. Only personal accounts are equated. The occasion for equating personal accounts arises from two causes:

1. If any item of any account is not paid when due, the holder of the account suffers a loss.
2. If an item is paid before it is due, the holder of the account realizes a gain.

724. Accounts are equated to ascertain a date when the settlement may be made without loss or gain to either the holder of the account or the maker of it.

725. The **face value** of an item is always to be used in equating accounts.

An item not subject to a term of credit is worth its face value the day it is dated. This is always true of an interest-bearing note.

Items subject to a term of credit and non-interest-bearing notes are worth their face value at maturity.

DRILL EXERCISE

1. How long may $2 be kept to balance the use of $4 for 10 days? $1 for 20 days? $6 for 15 days?

2. The use of $40 for 1 month is equivalent to the use of what sum for 2 months?

3. If I use $6 of B's money for 30 days, how much of my money should he use for 10 days in return for the accommodation?

4. If I pay one half of an account 10 days before the whole account is due, how long after the whole account is due may I have in which to pay the balance?

5. If I pay $10 of an account 30 days before it is due, how long may I keep $5, the balance account, after maturity? $20?

6. Jan. 10 Mason & Brown sold F. E. Rogers on account 30 days merchandise amounting to $400. (*a*) When is the account due? (*b*) If on Jan. 25 F. E. Rogers paid $200, on what date is the balance due?

SOLUTIONS. (*a*) The account is due 30 days after Jan. 10, or Feb. 9.

(*b*) If a payment of $200 was made on Jan. 25, Mason & Brown have had the use of $200 for 15 days; hence, they should extend the time on the remaining $200 15 days beyond the original maturity. Feb. 9 plus 15 days equals Feb. 24, the date on which the remaining $200 of the account should be paid.

7. May 5 F. C. Clark & Co. sold Charles H. Jones & Son on account 30 days merchandise amounting to $400. May 20 Charles H. Jones & Son made a payment of $100 on account. On what date should the balance be paid without loss or gain?

SOLUTION. The original charge matures 30 days after May 5, or June 4. The amount paid, $100, is $\frac{1}{3}$ of the amount remaining unpaid, $300. Since $100 is paid 15 days before it is due, the $300 may be kept $\frac{1}{3}$ of 15 days, or 5 days, after it is due. June 4 plus 5 days equals June 9, the date on which the remaining $300 should be paid without loss or gain.

8. Nov. 1 George B. Thayer & Co. sold E. M. Williams on account 30 days merchandise amounting to $600. (*a*) When is the account due? (*b*) If a payment of $300 is made on Nov. 1, on what date is the balance due?

9. Nov. 1 W. D. Lyman sold F. C. Hill on account 30 days merchandise amounting to $600. If a payment of $200 is made on Nov. 11, on what date is the balance due? A payment of $100? A payment of $400?

10. Mar. 20 W. K. Frey purchased a bill of goods of you amounting to $300. If no term of credit is given, how much was legally due Mar. 30?

11. Smith & Brown bought goods of you as follows:

Mar. 20, $300.
Mar. 30, $300.

(*a*) How much is legally due on the above account Mar. 30? (*b*) On what day can the amount of the account, $600, be paid without interest?

SOLUTIONS. (*a*) If the account is settled Mar. 30, on the charge of Mar. 20 there is 10 days' interest due. The interest on $300 for 10 days is $.50, which, added to the amount of the account, $600, makes the amount legally due $600.50.

(*b*) 10 days' interest on $300 is equivalent to 5 days' interest on $600; therefore if the whole account had been paid 5 days before Mar. 30, or Mar. 25, only the face of the account, $600, would have been due.

Proof. The interest on the first charge from Mar. 20 to Mar. 25 is $.25, and the discount on the second charge from Mar. 25 to Mar. 30 is $.25. The interest and discount being equal, the face of the account, $600, can be paid Mar. 25 without loss or gain to either party.

12. If I owe $200 due May 1, and $400 due May 31, at what time can both debts be equitably paid?

SIMPLE EQUATIONS

726. To find the equated time of an account when the items are all on one side and are subject to no terms of credit.

727. Example. Robert S. Campbell is charged on the books of Spencer, Mead & Co. as follows:

Nov. 1, To mdse., $ 60.
Nov. 7, To mdse., 120.
Nov. 13, To mdse., 180.
Nov. 19, To mdse., 240.

On what date may the amount of the account, $600, be paid without interest?

Date	Items	Time	Interest
Nov. 1	$ 60	18 da.	$.18
Nov. 7	120	12 da.	.24
Nov. 13	180	6 da.	.18
Nov. 19	240	0 da.	.00
	$600		$.60

The interest on $600 for 1 da. = $.10.

$.60 ÷ $.10 = 6, or 6 da.

Nov. 19 – 6 da. = Nov. 13, 1903, the due date.

Solution. For convenience assume that the account is being settled Nov. 19. The amount of the first charge, $60, would then draw interest from Nov. 1 to Nov. 19, or for 18 days. The interest on $60 for 18 days equals $.18. The amount of the second charge, $120, would draw interest from Nov. 7 to Nov. 19, or for 12 days. The interest on $120 for 12 days equals $.24. The amount of the third charge would draw interest from Nov. 13 to Nov. 19, or for 6 days. The interest on $180 for 6 days equals $.18. The amount of the fourth charge, being dated on the assumed date of settlement, will not draw interest. Adding the several interest charges, the amount is found to be $.60.

It will be seen that if the account were settled Nov. 19, a payment of $600.60 would be required; but the question is not, "What is the cash balance due Nov. 19?" but "When may the amount of the account, $600, be paid without interest?" Hence, we have given the principal, interest, and rate to find the time in days. The interest on $600 for 1 day is $.10. $.60 is 6 times $.10. Therefore, there are 6 days' interest due on $600 Nov. 19, and to pay the amount of the account without interest, settlement must be made 6 days before Nov. 19, or Nov. 13.

Proof. To prove that $600 may be paid on Nov. 13 without interest, it is necessary to show that the interest on the charges before Nov. 13 is equal to the discounts to be allowed on the charges made after that date.

Solution

The items that fall due before Nov. 13 are the following:

Dates	Items	Interest Periods	Interests
Nov. 1	$ 60	12 da.	$.12
Nov. 7	120	6 da.	.12
		Total interest due Nov. 13,	$.24

The item that falls due after Nov. 13 is the following:

Date	Item	Interest Period	Interest
Nov. 19	$240	6 da.	$.24

The discount on the item charged after Nov. 13 equals $.24.

Since the interest on the items due before Nov. 13 is equal to the discount to be allowed on the item due after Nov. 13, it is proved that on Nov. 13 only the face value of the account was due. Hence the equated date is Nov. 13.

728. From the foregoing explanations the following rule may be derived:

Select the latest date as the focal date.

Find the exact time in days from the date of each item to the focal date and compute the interest on these items for the time as found.

Divide the sum of the interest on the several items by the interest on the face of the account for one day, and the quotient will be the number of days average time.

Count back from the focal date the number of days so found and the date thus reached will be the date on which the face of the account may be paid without loss or gain to either party.

In finding the average term of credit in days, fractions of a day of one half or greater are counted as a full day; fractions less than one half day are rejected.

Any date may be taken as a focal date, the only question involved being a balance of interest or discount, but except for illustrative purposes by the teacher or test exercises for advanced pupils, the selection of any date except the latest date for the focal date is not recommended.

The selection of the latest date as the focal date removes the objection often raised in case an earlier date or the earliest date be chosen; namely, that the account is not likely to be settled before it was made.

WRITTEN EXERCISE

Find the equated date of the following accounts of Lord, Taylor & Lord. Prove the work of each problem.

1. John E. Parker & Co., Dr.

1903		
Oct. 5,	To mdse. .	$100.
Oct. 9,	To mdse. . .	$150.
Oct. 16,	To mdse. . .	$200.
Oct. 25,	To mdse. . .	$100.
Oct. 31,	To mdse. .	$200.

2. C. A. Ruggles, Dr.

1903		
Nov. 1,	To mdse. . .	$150.
Nov. 8,	To mdse. . .	$200.
Nov. 15,	To mdse. . .	$120.
Nov. 21,	To mdse. . .	$150.
Nov. 28,	To mdse. .	$200.

3. Patterson, Gould & Co., Dr.

1903		
June 6,	To mdse. . .	$500.
June 20,	To mdse. . .	$200.
June 29,	To mdse. . .	$150.
July 2,	To mdse. . .	$300.
July 18,	To mdse. . .	$200.

4. T. L. King, Dr.

1903		
Dec. 1,	To mdse. . .	$450.
Dec. 15,	To mdse. . .	$300.
Dec. 30,	To mdse. . .	$150.
1904		
Jan. 8,	To mdse. . .	$200.
Jan. 18,	To mdse. . .	$300.

729. To find the equated date when the items are all on one side and terms of credit are given.

730. Example. John Pierce is debited on the books of Benedict & Schuier as follows:

1903			
Oct. 1,	To mdse.,	30 da.,	$600.
Oct. 8,	To mdse.,	10 da.,	300.
Oct. 9,	To mdse.,	15 da.,	240.
Oct. 20,	To mdse.,	10 da.,	360.

When is the face amount of the account due by equation?

Dates	Terms of Credit	Due Dates	Items	Interest Periods	Interests
Oct. 1	30 da.	Oct. 31	$600	00 da.	$.00
Oct. 8	10 da.	Oct. 18	300	13 da.	.65
Oct. 9	15 da.	Oct. 24	240	7 da.	.28
Oct. 20	10 da.	Oct. 30	360	1 da.	.06
			$1500		$.99

On Oct. 31, 1903, $1500 + $.99, or $1500.99 is due on the account.

Since there is interest due on Oct. 31, 1903, the face amount of the account was due before Oct. 31, 1903.

The interest on the face amount of the account, $1500, for 1 day is $.25.

The total interests due on the face amount of the account Oct. 31, 1903, is $.99.

$.99 ÷ $.25 = 4.

Therefore it will take $1500 4 days to yield $.99 interest.

4 days' interest is due Oct. 31, 1903; hence the face amount of the account was due 4 days before Oct. 31, 1903.

Oct. 31, 1903, minus 4 days equals Oct. 27, 1903, the equated date of payment.

PROOF. To prove that Oct. 27 is the equated date of payment, it is necessary to show that the interest due on the items before that date is equal to the discount to be allowed on the items due after that date.

SOLUTION

Dates	Items	Interest Periods	Interests
Oct. 18	$300	9 da.	$.45
Oct. 24	240	3 da.	.12
		Total interest,	$.57

Dates	Items	Discount Periods	Discounts
Oct. 30	$360	3 da.	$.18
Oct. 31	600	4 da.	.40
		Total discount,	$.58

In finding the average term of payment there was a fractional number of days. Since all fractions less than one half day are dropped and all fractions one half day or more are called another whole day, if the difference between the interest and the discount is less than one half the interest or discount on the face amount of the account for 1 day, the equated date used is proved to be correct. In the above proof the difference between the interest and discount is $.01, or less than one half the interest on the amount of the account for 1 day; hence Oct. 27, 1903, is proved to be the equated date of payment.

731. From the foregoing explanations the following rule may be derived:

To the date of each item add the term of credit and proceed as in **728.**

WRITTEN EXERCISE

Find the equated date of the following accounts of C. W. Allen & Co. Prove the work of each problem.

1. **Greene & Sloan, Dr.**

1903		
Jan. 2,	**To mdse., 30 da.,**	**$200.**
Jan. 28,	**To mdse., 60 da.,**	**300.**
Apr. 15,	**To mdse., 30 da.,**	**150.**
Apr. 2,	**To mdse., 60 da.,**	**240.**

2. **Groves & Co., Dr.**

1903		
Jan. 30,	**To mdse., 1 mo.,**	**$ 300.**
Feb. 28,	**To mdse., 60 da.,**	**300.**
Mar. 25,	**To mdse., 2 mo.,**	**1200.**
June 20,	**To mdse., 30 da.,**	**1200.**

3. **Gibbons & Stone, Dr.**

1903		
May 8,	**To mdse., net,**	**$250.**
May 25,	**To mdse., 30 da.,**	**200.**
June 18,	**To mdse., 30 da.,**	**420.**
June 30,	**To mdse., 30 da.,**	**480.**

4. **Steven Brackett, Dr.**

1903		
Oct. 5,	**To mdse., 30 da.,**	**$319.50.**
Oct. 20,	**To mdse., 4 mo.,**	**750.00.**
Dec. 1,	**To mdse., 2 mo.,**	**280.50.**
Dec. 31,	**To mdse., 1 mo.,**	**415.90.**

5. **F. H. Harper & Sons, Cr.**

1903		
Jan. 20,	**To mdse., 3 mo.,**	**$180.**
Jan. 25,	**To mdse., 3 mo.,**	**420.**
Mar. 8,	**To mdse., 3 mo.,**	**480.**
Mar. 18,	**To mdse., 3 mo.,**	**120.**

6. **F. H. Spencer & Co., Cr**

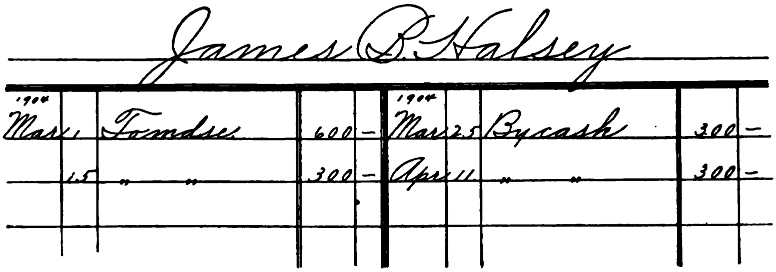
1903		
Sept. 18,	**To mdse., 1 mo.,**	**$1000.**
Sept. 30,	**To mdse., 60 da.,**	**200.**
Nov. 8,	**To mdse., 60 da.,**	**420.**
Dec. 1,	**To mdse., 30 da.,**	**180.**

7. **Shepard & Norwell Co. sold C. W. Davis the following bills of merchandise: May 3, 1903, $2400 on 30 days' credit; June 20, 1903, $180 on 2 months' credit; Aug. 20, 1903, $1200 on 30 days' credit; Aug. 31, 1903, $420 on 60 days' credit. On what date may the several bills be equitably paid in one sum?**

COMPOUND EQUATIONS

732. **To find the equated date when the account has both debits and credits and no terms of credit are given.**

733. Examples. *1.* Find the equated date for paying the balance of the following account:



James B. Halsey

1904 Mar.	1	To mdse.	600	—	1904 Mar.	25	By cash	300	—
	15	" "	300	—	Apr.	11	" "	300	—

SOLUTION

Take Apr. 11, 1903, as the focal date.

Debits

Dates	Items	Interest Periods	Interests
Mar. 1	$600	41 da.	$4.10
Mar. 15	300	27 da.	1.35
	$900		$5.45

Credits

Dates	Items	Interest Periods	Interests
Mar. 25	$300	17 da.	$.85
Apr. 11	300	00 da.	.00
	$600		.85

$900 – $600 = $300, the balance of the account.

$5.45 – $.85 = $4.60, the interest due the holder of the account Apr. 11, 1904.

The interest on $300 for 1 day = $.05.

$4.60 + $.05 = 92.

Therefore $300 will yield $4.60 interest in 92 days.

Apr. 11, 1904, James B. Halsey not only owes the balance of the account, $300, but 92 days' interest on this amount.

Therefore the face of the account was due 92 days before Apr. 11, 1904.

Apr. 11, 1904, – 92 days = Jan. 10, the equated date of payment.

PROOF. To prove the correctness of the above work it is necessary to show that the payment of $300 on Jan. 10, 1904, will result in neither a loss nor gain to either debtor or creditor. This may be done by equating the account again with Jan. 10 as the focal date.

SOLUTION

Debits

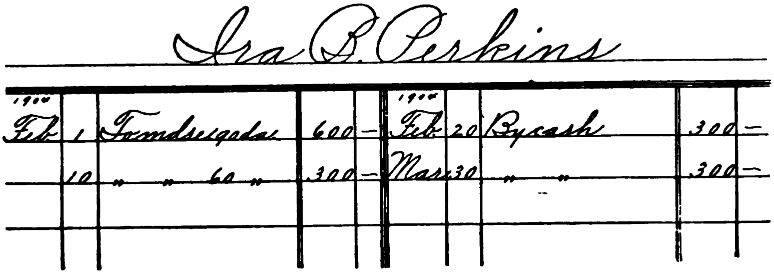
Dates	Items	Interest Periods	Interests
Mar. 1	$600	51 da.	$5.10
Mar. 15	300	65 da.	3.25
		Total debit interest,	$8.35

Credits

Dates	Items	Interest Periods	Interests
Mar. 25	$300	75 da.	$3.75
Apr. 11	300	92 da.	$4.60
		Total credit interest,	$8.35

From the above statement we see that if the holder of the account had, on Jan. 10, received cash for each item charged in the account, he would have had the use of $600 for 51 days and $300 for 65 days, and would have gained $8.35 interest; but the first payment of $300 was made Mar. 25, and the second Apr. 11. By not receiving payment on Jan. 10 he would not have the use of $300 for 75 days and 92 days; hence he would have lost $8.35 interest. Since this loss is canceled by a gain of $8.35, it is shown that the balance of the account may be equitably settled Jan. 10.

2. Find the equated date for paying the balance of the following account:



Ira B. Perkins

1904				1904			
Feb.	1	To mdse. 90 da.	600 —	Feb.	20	By cash	300 —
	10	" " 60 "	300 —	Mar.	30	" "	300 —

SOLUTION

Take May 1, 1904, as the focal date.

Debits

Dates	Terms of Credit	Due Dates	Items	Interest Periods	Interests
Feb. 1	90 da.	May 1	$600	00 da.	$ .00
Feb. 10	60 da.	Apr. 10	300	21 da.	1.05
			$900		$1.05

Credits

Dates	Items	Interest Periods	Interests
Feb. 20	$300	71 da.	$3.55
Mar. 30	300	32 da.	1.60
	$600		$5.15

$900 − $600 = $300, the balance of the account.
$5.15 − $1.05 = $4.10, the interest due Ira B. Perkins May 1.
The interest on $300 for 1 day = $.05.
$4.10 ÷ $.05 = 82.

Therefore, $300 will yield $4.10 interest in 82 days.
On May 1 there is 82 days' interest due Ira B. Perkins.
Hence, the face of the account is not due until 82 days after **May 1.**
May 1 plus 82 days equals July 22, the equated date of payment.

Proof

Debits

Due Dates	Items	Interest Periods	Interests
May 1	$600	82 da.	$8.20
Apr. 10	300	103 da.	5.15
			$13.35

Credits

Due Dates	Items	Interest Periods	Interests
Feb. 20	$300	153 da.	$7.65
Mar. 30	300	114 da.	5.70
			$13.35

The above statement shows that on July 22, 1904, the loss suffered by the holder of the account is canceled by the gain realized. Therefore, the account is proved to be equitably due July 22, 1904.

In proving the equation of accounts, the equitable settlement of which is found to come at a date within the account or between its extreme dates, the difference between the interest and discount of the debit items from their respective dates to the due dates must be offset or balanced by the difference between the interest and discount on the credit items from their respective dates to the due date, within one half cent of the interest or discount on the balance for one day.

734. General Principles. *1.* If the larger interest is on the larger side of the account, it shows that the holder of the account has suffered a loss because items were not paid when due; if on the smaller side, it shows that the holder of the account has gained because items were paid before they were due.

2. A loss is offset by dating back; a gain is offset by dating forward.

735. Hence the following **rule:**

Find the balance of the account and also the excess of interest from the latest date as the focal date.

If the balance of account and excess of interest are on the same side, date back; if on opposite sides, date forward.

ORAL EXERCISE

1. Oct. 1, Henry Ball & Co. sold F. E. Gorham a bill of merchandise amounting to $800. Terms: 30 days. Oct. 11, F. E. Gorham made a payment of $400 on account. When is the balance, $400, equitably due?

Solution. By the terms of the contract the account would mature Oct. 31. Since a payment of $400 is made 20 days before maturity, Henry Ball & Co. have gained the use of $400 for 20 days. To offset this gain they should allow F. E. Gorham 20 days beyond the original maturity of the account in which to pay the balance. Oct. 31 plus 20 days is equal to Nov. 20, the date on which the balance of the account may be equitably paid.

Find the time for equitably paying the balance of the following accounts. Terms: cash.

	Dr.	*Cr.*
2.	Oct. 1, $800;	Oct. 11, $400.
3.	Apr. 12, $300;	Apr. 17, $150.
4.	July 5, $900;	July 20, $300.

Find the time for equitably paying the balance of the following accounts. Terms: 30 da.

	Dr.	*Cr.*
5.	Oct. 1, $600;	Oct. 6, $200.
6.	May 10, $400;	May 6, $200.
7.	June 15, $800;	June 25, $600.

WRITTEN EXERCISE

Find the equated date of payment of each of the following accounts. Prove all work.

1. E. M. Eldred & Co.

1903					1903				
Jan.	20	To mdse.	600		Feb.	8	By cash	300	
Feb.	25	To mdse.	300		Mar.	20	By cash	300	

2. VICTOR H. BROWN & CO.

1903					1903				
Jan.	15	To mdse.	600		Jan.	25	By cash	1000	
	30	To mdse.	300		Feb.	15	By cash	200	
Feb.	8	To mdse.	600						
	20	To mdse.	300						

3. B. N. SHERWOOD & SON

1903					1903				
Apr.	8	To mdse.	420		Apr.	18	By cash	240	
	20	To mdse.	180			20	By cash	60	
May	15	To mdse.	540		June	2	By cash	300	
June	2	To mdse.	60						

4. W. I. PARKER

1903					1903				
Aug.	5	To mdse.	200		Sept.	8	By cash	240	
	20	To mdse., 2 mo.	360		Oct.	5	By 60-da. note	240	
Sept.	15	To mdse., 30 da.	360				(no interest)		

5. REED & HAMLIN

1903					1903				
June	20	To mdse., 30 da.	300		July	1	By cash	100	
	30	To mdse., 60 da.	180		Aug.	1	By cash	100	
Aug.	1	To mdse., 30 da.	480		Sept.	1	By cash	100	
Sept.	20	To mdse., 30 da.	120		Oct.	1	By cash	100	
					1904				
					Jan.	1	By cash	100	

NOTE. Interest may be computed on one of the five similar credit items for the aggregate number of days.

EQUATION OF ACCOUNTS SALES

736. An account sales is equated in practically the same manner as an ordinary ledger account. The agent's charges constitute the debits of the account, and the gross sales the credits.

The agent's charges include freight, cartage, storage, commission, insurance, advertising, guaranty, etc.

737. When equating accounts sales, agents generally consider such charges as freight, cartage, storage, and insurance, as not due until they have been paid.

738. When goods are sold promptly, agents usually consider commission and guaranty as due on the date of the last sale. When the sales are large and there are long intervals between them, the commission or guaranty is considered due on the average due date of the sales.

When goods are sold for cash, or on short time, the account sales is seldom averaged.

WRITTEN EXERCISE

1. Find when the net proceeds of the following account sales are due by equation. Consider the commission as due on the date of the last sale.

Boston, Mass., Oct. 8, 1903.

Parker, Montgomery & Co.

Sold for the account of W. D. Sprague,
Buffalo, N.Y.

1903		*Sales*					
Sept.	23	95 bbl.	$5.^{60}$, cash				
Oct.	1	200 bbl.	$5.^{75}$, 1 mo.				
	18	65 bbl.	$5.^{80}$, 60 da.				
Nov.	3	110 bbl.	$5.^{80}$, 30 da.				
	25	130 bbl.	$5.^{75}$, cash				
		Charges					
Sept.	24	Freight		62	50		
	20	Cartage		30			
Oct.	28	Cash advanced		2000			
Nov.	15	Cooperage		5			
	25	Commission, 4%					

2. Using the foregoing form for a model, arrange the following narrative in the form of an account sales and find when the net proceeds are due by equation. Consider the commission as due on the average due date of the sales.

R. J. Briggs & Co., Boston, Mass., sold for the account and risk of B. Sommers & Co., Chicago, Ill., 1000 bbl. potatoes as follows: Nov. 2, 1903, 400 bbl. peach blows at $ 3, cash; Dec. 1, 1903, 300 bbl. pink eyes at $ 3.50, 30 da.; Jan. 1, 1904, 100 bbl. peach blows at $ 3.60, cash; Jan. 25, 1904, 200 bbl. pink eyes at $ 3.50, 30 da. The charges were as follows: Nov. 1, 1903, freight, $ 350; Nov. 1, 1903, cartage, $ 50; Nov. 1, 1903, insurance and advertising, $ 100; commission and guaranty, 3%.

CASH BALANCE

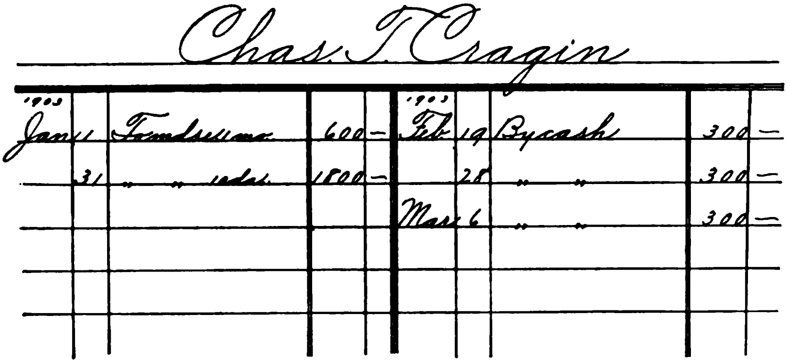
739. **Cash Balance** treats of showing the balance or amount due on an account at any given date.

740. The cash balance of an account on which interest is not charged is the difference between the two sides of the account in the ledger. The cash balance of an account on which interest is charged is the difference between the two sides of the account after interest has been added to the items past due, or deducted from the items not due at the date of settlement.

741. Each item of an account equitably draws interest from the time it becomes due to the date of settlement, and each item paid before maturity is equitably entitled to discount for the time from the date of payment to the date it is due.

Whether interest is charged on the items of a running account or not is usually regulated by the custom of the business, or an agreement between the parties thereto. As a rule, retailers do not charge interest on the items of running accounts, but frequently the balance of a closed account is considered interest-bearing from the date the balance is brought down. Wholesale dealers usually charge interest on the items of an account at the expiration of the time specified in the terms of credit.

742. **Example.** When money is worth 6% per annum, what is the cash balance due on the following account June 23, 1903?



Chas. T. Cragin

1903 Jan.	1	To mdse 1 mo	600	—	1903 Feb.	19	By cash	300 —
	31	" " 10 da.	1800	—		28	" "	300 —
					Mar.	6	" "	300 —

Solution

Debits

Dates	Terms of Credit	Due Dates	Items	Interest Periods	Interests
Jan. 1	1 mo.	Feb. 1	$ 600	142 da.	$ 14.20
Jan. 31	10 da.	Feb. 10	1800	133 da.	39.90
			$ 2400		$ 54.10

Credits

Dates	Items	Interest Periods	Interests
Feb. 19	$ 300	124 da.	$ 6.20
Feb. 28	300	115 da.	5.75
Mar. 6	300	109 da.	5.45
	$ 900		$ 17.40

$ 2400 + $ 54.10 = $ 2454.10, the amount due on account June 23, 1903, had no payments been made.

$ 900 + $ 17.40 = $ 917.40, the value of the payments on June 23, 1903.

$ 2454.10 – $ 917.40 = $ 1536.70, the cash balance of the account June 23, 1903.

743. From the foregoing explanation the following rule may be derived:

Find the maturity of each item of the account.

Compute the interest on each item from the date it becomes due to the date of settlement.

To the sum of the debit items add the sum of the debit interests; also to the sum of the credit items add the sum of the credit interests.

Subtract these totals and the result is the cash balance.

WRITTEN EXERCISE

1. If money be worth 7% per annum, what is the cash balance due on the following account, July 1, 1903?

HENRY HARRISON & Co.

1903					1903				
Jan.	31	To mdse.	450		Jan.	2	By mdse.	600	
Mar.	30	To mdse.	450		Feb.	13	By cash	300	
					Mar.	29	By mdse.	300	

2. What is the cash balance of the following account, Apr. 1, 1903, if the money be worth 8% per annum?

BENJAMIN TRACY & SON

1902					1902				
Aug.	4	To mdse., 1 mo.	200		Oct.	1	By cash	150	
Sept.	1	To mdse., 2 mo.	400		Nov.	1	By cash	150	
Oct.	31	To mdse., 4 mo.	600		Dec.	1	By cash	150	
Dec.	3	To mdse.	300		1903				
					Jan.	1	By cash	150	
					Feb.	1	By cash	150	
					Mar.	1	By cash	150	

3. Equate the following account and find the cash balance due Apr. 1, 1903, if money be worth 7% per annum. Prove the work.

BROWN, SHIPLEY & Co.

1902					1903				
Sept.	9	To mdse.	600		Jan.	2	By cash	500	
Oct.	1	To mdse., 2 mo.	300		Mar.	16	By 2-mo. note		
Dec.	13	To mdse., 1 mo.	150				(on interest)	100	
1903					Apr.	30	By 3-mo. note		
Jan.	31	To mdse., 1 mo.	450				(no interest)	300	
					May	1	By cash	200	

NOTE. To find the cash balance of an equated account.

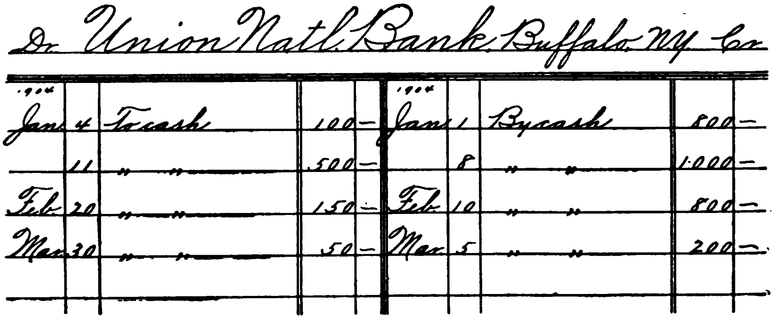
Find the difference between the equated date of payment and the date of settlement, and compute the interest on the balance of the account for this time. The sum of the interest thus found and the balance of the account is the cash balance due at the date of settlement.

BANKERS' CASH BALANCE

744. Many bankers balance their accounts with their correspondents at regular intervals, — monthly, quarterly, semiannually, or yearly, allow interest on all sums that have been credited, charge interest on all sums that have been debited, and bring the cash balance down to a new account to subsequently draw interest the same as the regular items in the account.

745. Some bankers and trust companies balance their accounts with depositors at regular intervals and allow interest on the balances credited.

746. **Example.** Find the balance due on the following account Apr. 1, 1904, settlements being made quarterly with interest at 6%.



Dr Union Natl. Bank, Buffalo, N.Y. Cr

1904					1904				
Jan	4	To cash	100	—	Jan	1	By cash	800	—
	11	" "	500	—		8	" "	1000	—
Feb	20	" "	150	—	Feb	10	" "	800	—
Mar	30	" "	50	—	Mar	5	" "	200	—

BANK ACCOUNT CURRENT

Dates		Debits		Credits		Credit Balances		Days	Credit Interests	
1904										
Jan.	1			800		800		3		40
	4	100				700		4		47
	8			1000		1700		3		85
	11	500				1200		30	6	
Feb.	10			800		2000		10	3	33
	20	150				1850		14	4	32
Mar.	5			200		2050		25	8	54
	30	50				2000		2		67
		800		2800					24	58
				800						
				2000+	24	.58 = 2024	.58			

Solution. By arranging the debit and credit items in the order of their dates the balance of the account at each of the dates may easily be determined. The account shows a credit balance of $ 800 from Jan. 1 to Jan. 4, or for 3 days, when the amount is reduced to $ 700 by the charge of $ 100. The interest on $ 800 for 3 days is $ .40. The account shows a credit balance of $ 700 from Jan. 4 to Jan. 8, or for 4 days, when the amount is increased to $ 1700 by the credit of $ 1000. The interest on $ 700 for 4 days is $ .47. Continuing in this manner to Apr. 1, it is found that on that date the account shows a credit balance of $ 2000, and that on the daily balances there has accumulated $ 24.58 interest. The cash balance of the account is then found to be $ 2000 plus $ 24.58, or $ 2024.58.

Note. Had there been a debit balance on any of the above dates, two extra columns would have been required in the operation, — one for the debit balances and one for the debit interests. The difference between the debit and credit interests would then be the balance of accrued interest.

747. Hence the following rule may be derived:

Arrange the debits and credits in the order of their dates and find the balance of the account at each date.

Find the interest on each balance for the period that it remains unchanged.

If the balance of interest and the balance of the account are on the same side, take their sum; if on the opposite sides, take their difference.

The result obtained is the cash balance due.

WRITTEN EXERCISE

1. Find the balance due on the following bank account July 1, 1903, at 4%.

Dr. Central National Bank, Springfield, Mass. Cr.

1903					1903				
Apr.	15	To cash	200		Apr.	1	By cash	1200	
	20	To cash	200		May	4	By cash	900	
May	20	To cash	300		June	1	By cash	500	
June	10	To cash	300			20	By cash	420	

2. Find the balance due on the following bank account, Apr. 1, 1903, at 3%.

Dr. MERCHANTS NATIONAL BANK, Rochester, N.Y. Cr.

1903					1903				
Jan.	2	To cash	600		Jan.	1	By cash	1500	
Feb.	8	To cash	480			31	By cash	1200	
	21	To cash	240		Feb.	15	By cash	120	
Mar.	20	To cash	180		Mar.	31	By cash	400	

3. The Security Trust Company, Rochester, N.Y., allows interest to its depositors on daily balances at 4% per annum, payable quarterly. Find the cash balance of the following account with George W. Snyder, Apr. 1, 1903: Jan. 1, 1903, deposited $900; Jan. 8, drew out $200; Jan. 12, deposited $750; Jan. 15, drew out $475; Feb. 9, deposited $721.90; Feb. 24, drew out $121.90; Mar. 15, deposited $795.98; Mar. 30, drew out $400.

SAVINGS-BANK ACCOUNTS

748. A **savings bank**, as its name implies, is an institution organized for the purpose of encouraging economy and thrift and caring for the savings of the people.

749. **The deposits in savings banks are practically payable on demand.**

Savings banks generally reserve the right to require depositors to notify them from 30 to 60 days before making a withdrawal.

750. The **interest term** is the time between dates at which dividends of interest are declared.

Dividends of interest are usually declared semiannually.

751. If interest is not withdrawn, it is placed to the credit of the depositor on the books of the bank, and draws interest the same as any regular deposit. In this way savings banks pay their depositors compound interest.

No interest is allowed on fractional parts of a dollar.

752. The **interest days** are the days on which interest is allowed to commence.

753. Savings banks are not uniform in their practice of allowing interest on deposits made after the beginning of the interest term. In some savings banks deposits begin to draw from the first of each quarter; in others, from the first of each month. The latter method is preferable for persons having a small income.

Monthly interest days usually begin on the first day of each month; quarterly interest days on Jan. 1, Apr. 1, July 1, and Oct. 1; semiannual interest days on Jan. 1 and July 1.

754. Nearly all savings banks allow interest on only those sums that have been on interest for the full time between the interest days.

Thus, if the interest begins quarterly, only those sums that have been on deposit for the full quarter draw interest; if monthly, only those sums that have been on deposit for the full month draw interest.

755. Savings banks furnish each depositor with a small book called a *pass book,* in which are entered all amounts deposited and all amounts withdrawn, together with the interest credited to the depositor at the expiration of the interest term.

756. To find the balance due a depositor when there are no withdrawals.

757. Example. The interest term of Wildey Savings Bank is 6 months. A deposited in this bank Dec. 20, 1903, $200; Feb. 10, 1904, $100; Apr. 1, 1904, $50; June 8, 1904, $50. No withdrawals having been made, what was due July 1, 1904, if interest at 4% per annum be reckoned on the deposits (*a*) from the first of each quarter? (*b*) from the first of each month?

(*a*)

DATES	DEPOSITS	DAILY BALANCES	INTEREST DAYS	SMALLEST QUARTERLY BALANCES	QUARTERLY INTERESTS
1903					
Dec. 20	200	200	Jan. 1		
1904					
Feb. 10	100	300			
Apr. 1	50	350	Apr. 1	200	2.00
June 8	50	400			
July 1		400	July 1	350	3.50
					5.50
					400.
					405.50

SOLUTION. If interest begins on the first of each quarter, only the smallest balance for any quarter will draw interest. The deposit of Feb. 10, being made after the beginning of the first quarter, will not begin to draw interest until the beginning of the second quarter. Hence, the only sum that draws interest for the first quarter is the deposit of Dec. 20, $200. The interest on $200 for one quarter is $2. The deposits of Feb. 10 and Apr. 1, together with the smallest balance for the first quarter, will draw interest for the second quarter. The deposit of June 8, being made after the beginning of the second quarter, will not draw interest until the beginning of the third quarter. Hence, the sum to draw interest for the second quarter is $350 ($200 + $100 + $50). The interest on $350 for one quarter is $3.50. $2 + $3.50 = $5.50, the interest to be added to the account July 1. $400 + $5.50 = $405.50, the balance due on the account July 1.

(*b*)

Dates	Deposits	Interest Days	Smallest Monthly Balances	Monthly Interests
1903				
Dec. 20	200	Jan. 1	200	
1904				
Feb. 10	100	Feb. 1	200	1.00
Apr. 1	50	Mar. 1	300	1.50
June 8	50	Apr. 1	350	1.75
		May 1	350	1.75
		June 1	350	1.75
		July 1	400	3)7.75
				2.58
				5.17
				400.
				405.17

Solution. Only the smallest balance on deposit each month will draw interest. The amount deposited Dec. 20 will not begin to draw interest until Jan. 1. The smallest balance for each month is as shown above. The aggregate interest on the smallest monthly balance is found to be $7.75 at 6%, or $5.17 at 4%. $400, the balance on deposit July 1, + $5.17 = $405.17, the balance due the depositor July 1, 1904.

WRITTEN EXERCISE

1. A made the following deposits in a savings bank: July 1, 1903, $50; July 30, $50; Aug. 20, $100; Oct. 5, $200; Nov. 8, $150; Dec. 15, $200. If the interest term is 6 months, what is the balance due A Jan. 1, 1904, interest being allowed on balances from the first day of each quarter at 4% per annum?

2. J. M. Carroll made the following deposits in the Security Savings Bank: Dec. 18, 1903, $ 400; Jan. 5, 1904, $ 200; Mar. 8, 1904, $ 100; May 20, 1904, $ 30; July 1, 1904, $ 40. If the interest term is 3 months, what is the balance due J. M. Carroll July 1, 1904, interest at 4% being computed from the first day of each quarter?

758. To find the amount due depositors when there are withdrawals.

759. Example. Find the balance due July 1, 1904, on the following account. Deposits: Dec. 10, 1903, $ 600; Apr. 10, 1904, $ 200; May 20, $ 150. Withdrawals: Mar. 10, $ 300; May 1, $ 50. The interest term is 6 months and interest at the rate of 4% per annum is allowed from the first day of each quarter.

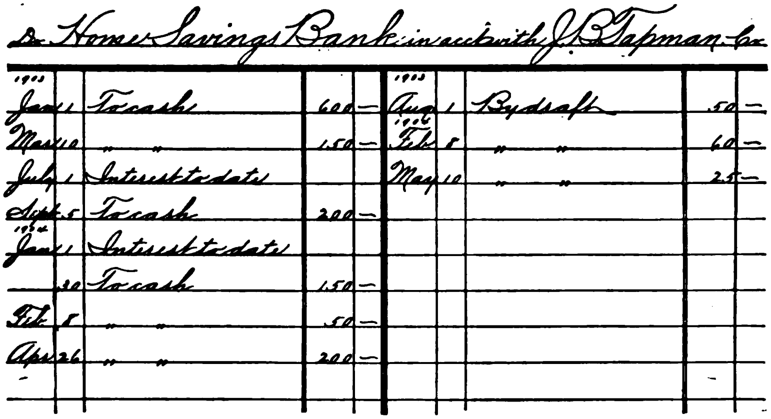
DATES	DEPOSITS	WITHDRAWALS	DAILY BALANCES	INTEREST DAYS	SMALLEST QUARTERLY BALANCES	QUARTERLY INTERESTS
1903						
Dec. 10	600		600	Jan. 1		
1904						
Mar. 10		300	300			
Apr. 1			300	Apr. 1	300	3.00
Apr. 10	200		500			
May 1		50	450			
May 20	150		600			
				July 1	300	3.00
						6.00
						600.
						606.00

SOLUTION. Interest begins quarterly and the interest days are Jan. 1, Apr. 1, and July 1. The smallest balance for the first quarter is $ 300, and the smallest balance for the second quarter is $ 300. The quarterly interest on these two balances aggregates $ 6.00. $ 600, the amount on deposit July 1, plus $ 6.00, the interest due on that date, equals $ 606.00, the balance of the account July 1, 1904.

WRITTEN EXERCISE

1. W. E. Small deposits in a savings bank as follows: Jan. 1, $ 400; Feb. 2, $ 200; Mar. 10, $ 150; Apr. 2, $ 60; May 18, $ 200; during the same time he withdrew as follows: Jan. 10, $ 50; Feb. 4, $ 50; Apr. 5, $ 50; June 30, $ 80. The interest term is 6 months. What interest at 4% per annum, to commence from the first of each quarter, should be added to the account July 1?

2. In the Home Savings Bank the interest term is 6 months, and the interest days are Jan. 1, Apr. 1, July 1, and Oct. 1. Find the balance due on the following account July 1, 1904, at 4% per annum.



Dr. Home Savings Bank in acc't with J. B. Tapman Cr.

1903 Jan.	1	To cash	600	—	1903 Aug.	1	By draft	50	—
Mar.	10	" "	150	—	1904 Feb.	8	" "	60	—
July	1	Interest to date			May	10	" "	25	—
Sept.	5	To cash	200	—					
1904 Jan.	1	Interest to date							
	30	To cash	150	—					
Feb.	8	" "	50	—					
Apr.	26	" "	200	—					

STOCKS

760. A **joint stock company** is a partnership in which the affairs of the business are conducted by officers chosen by the stockholders.

761. A **corporation** is a fictitious person. It consists of several natural persons who, in the name of the corporation, are authorized by law to transact business.

The instrument which defines the rights and duties of the corporation is called a *charter*. It is issued by government under seal.

762. **Stocks** is a general term applied to shares in the capital stock of banks, insurance, railroad, and other incorporated or joint stock companies.

763. A **stock certificate** is a written or printed instrument of a joint stock company or corporation issued to the stockholder, certifying the value of each share and the number of shares such certificate represents.

764. A **share** represents simply a certain component part of the capital stock. It is commonly $25, $50, or $100. The stock certificate represents the number of shares specified thereon.

765. The **capital stock** of a company is the sum of all the shares issued at their par value.

766. The **common stock** of a corporation is the stock which is ordinarily issued to the incorporators.

767. **Preferred stock** is stock on which dividends are paid before any allowance is made for dividends on the common stock.

Preferred stock is sometimes issued to take up the floating indebtedness of a corporation. Agreed dividends are declared on it at certain intervals out of the net earnings, and before any dividend can be declared on the common stock. Such stock is frequently issued upon the reorganization of railroads or the consolidation of joint stock companies.

768. The **par value** of stocks is their face value; their **market value** is the sum at which they are quoted in the market.

769. Stocks are **above par** or **at a premium** when they are worth *more* than their face value; **below par** or **at a discount** when they are worth *less* than their face value.

770. **Stock quotations** are published prices or rates per share that stocks sell for.

Thus, when stock is 3% above par it is quoted at 103; when it is 2% below par it is quoted at 98.

771. A **dividend** is a pro rata division of profits among the stockholders of a company or corporation.

The income from stocks is in the nature of dividends, and is dependent upon the prosperity of the company or corporation. Dividends are declared at a certain per cent on the par value of the capital stock of the company, either quarterly, semiannually, or annually. The dividend on preferred stock is often at a different rate from that on common stock.

772. An **assessment** is a sum levied pro **rata** upon the stockholders of a corporation to cover losses, etc.

773. **Stock brokers** are persons who act for others in buying and selling stocks at a stock exchange. For this service they charge a certain rate per cent commission, called **brokerage**, on the par value of the stocks dealt in.

774. Brokerage is usually $\frac{1}{8}\%$ of the par value of the stock dealt in. Occasionally it is as high as $\frac{1}{4}\%$ or $\frac{1}{2}\%$, or as low as $\frac{1}{16}\%$.

775. Stocks are generally bought and sold either "regular way," or "cash," or "buyer three," or "seller three." Stock sold "regular way" is to be paid for and delivered the next day; stock sold "cash" is deliverable the day sold. When stock is bought "seller three," the seller of the stock may deliver it on any one of the three days following the transaction, at his option, but cannot be required to deliver it till the third day. When stock is sold "buyer three," the buyer may demand delivery at any time within three days, but is obliged to take and pay for it by the third day. If a stock pays a dividend while a transaction is being executed, the dividend belongs to the purchaser of the stock.

776. A **margin** is a deposit made with a broker by a person who wishes to speculate in stocks, such deposit being used by the broker to protect himself against loss. The margin is usually 10% of the par value of the stock dealt in.

A wishes to speculate, and deposits with B, his broker, $1000 as a margin, directing B to buy 100 shares of a stock quoted at 90. B would pay for the stock $9000, $1000 of which is the margin furnished by A; B furnishes $8000, and charges the usual rate of interest on that sum for "carrying" the stock. In case the quoted value of the stock drops below 90, the margin must be made good by A's depositing an additional amount. If A fails to make good his margin, B may sell the stock to protect himself from losing any of the money he has furnished.

777. **Collateral** consists of stocks, notes, etc., given in pledge as security when money is borrowed.

778. If any one has sold stock he does not own, in the hope of realizing a profit by buying it in at a lower price, he is said to be "short."

779. If stock has been sold "short," and the seller buys it in to realize a profit or to protect himself against loss, he is said to "cover his short sales."

780. Stock sold **Ex. Div.** means that a recently declared dividend is received by the seller.

781. When a corporation increases the quantity of its stock without increasing the value of its property, which the stock is supposed to represent, the stock of such a corporation is said to be **watered** to the extent of the increase.

782. The following list, showing the highest, lowest, and closing quotations of certain stocks, and net changes from closing prices of the previous day, is reproduced from the Wall Street edition of the *New York Sun* under date of Jan 17, 1907.

THE STOCK MARKET

	Highest	*Lowest*	*Closing*	*Net Ch'ge*		*Highest*	*Lowest*	*Closing*	*Net Ch'ge*
Allis Chalm	15	15	15	− 1	Great Nor pf	179¼	178	179	
Amal Cop	117½	115¼	115⅞	− ⅜	Hock Val pf	91½	91½	91½	
Am Beet Su	21¾	21¾	21¾	+ ¼	Inter-Met	36⅜	35⅝	36¼	+ ⅝
Am Car & F	44½	43⅞	44	− ⅛	Inter-Met p	73¼	73	73	
Am Ice Secs	86	85	86	− 1	Iowa Cent	27	26½	27	− ½
Am Loco	73¼	72¾	72¾	− ½	Iowa Cen pf	48½	47	48½	− 1½
Am Loco pf	111½	111½	111½		Kan C So pf	60¼	60⅛	60⅛	− ¾
Am Smelt	151	150	150¼		Lou & Nash	142½	141¾	142½	+ ½
Am Sugar	133½	132¾	132¾	− ⅜	Manhattan	143	142⅛	143	
Am Tob pf	97	97	97		Mex Cent	26	25¼	25⅝	− ⅛
Anaconda	282½	280	281½	+ 1½	M & St L pf	89	89	89	+ 1
A T & S F	106¼	105¼	105½	− ⅜	M St P & S S M	130¼	130	130	− ½
A T & S F pf	101	100½	100½	− ¼	M K & Tex	39½	38½	39	− ¾
At C Line	129	128½	128½	+ 1	M K & T pf	70⅞	70⅜	70⅝	+ ½
Balt & Ohio	118¾	118⅛	118⅜	− ⅜	Mo Pacific	88¼	86¾	88	+ ½
Bklyn R T	80⅞	78½	79	− 1¼	Nat Bis Co	85½	84⅝	85	+ ½
Can Pac	190¼	188¾	190	+ ½	Nat Lead	72	71½	72	+ ½
Ches & Ohio	52⅜	51⅞	52⅜	+ ⅛	N Y Air Br	135¾	135	135¼	+ ¼
Chi & N W	189½	188	189	+ ½	N Y Central	130½	129⅛	129⅞	− ⅝
C & N W pf	230	230	230		Nor Pacific	160¼	158	158½	− ½
Chi Gt W	16⅞	16	16½	− ⅜	Ont & Wes	46¼	45¼	45½	+ ¼
C Gt W pf B	24¼	24¼	24¼		Ont Silver	7	6¾	7	
C M & St P	155½	153¾	154⅜	+ ½	Pac Coast	121½	121	121	− ½
CMSP sb 1 pd	33¾	32⅝	33		Pacific Mail	38	37½	37½	+ ¼
Chi U Tr pf	17½	17¼	17½	+ ¼	Penn RR	135⅞	135	135⅝	+ ⅛
Col Fuel & I	54⅝	53¼	54¼	+ ¼	Reading 1 pf	90	90	90	+ ⅜
Cons Gas	137½	137	137	− 1	Rock Island	27⅞	27	27¼	+ ⅛
Del & Hud	216	215½	215½	− ½	Rock Isl pf.	62	61	61⅞	+ 1⅞
Del L & W	480	480	480	− 32½	St P & Om pf	165	165	165	− 16
Den & R Gr	39⅛	37½	38	− 1	Sou Pacific	95¾	94½	95	+ ⅛
Den & R G pf	81½	80¾	80¾	− 1¾	Sou Pac pf	117¾	117¾	117¾	− ¼
Erie	40⅜	38⅜	39	− 1	Twin C R T	106¾	104	105	− 1¾
Erie 1st pf	73¾	71¾	72½	− 1½	Union Pac	180	179	179⅛	− ½
Erie 2d pf	63½	62¾	63¼	− ¾	U S Steel	49⅝	49⅛	49⅝	+ ⅝
Gen Elec Co	159	157½	157½	+ 1½	U S Steel pf	107⅛	106¾	107	+ ¼

783. To find the market value, cost, or selling price of stocks.

784. Examples. *1.* According to the closing quotation given in the list find the market value of 350 shares American Sugar Refining Co. stock.

Solution

$ 132.75 = market value of 1 share,
$ 132.75 × 350 = $ 46,462.50 = market value of 350 shares.

Note. Unless otherwise specified, the par value of one share is $ 100.

2. Find the cost of 500 shares of Atlantic Coast Line at the closing quotation; brokerage $\frac{1}{8}$%.

Solution

$ 100 = the par value of 1 share.
$ 128.50 = the market or quoted value of 1 share.
$\frac{1}{8}$% of $ 100 = $ .12$\frac{1}{2}$, brokerage on 1 share.
$ 128.62$\frac{1}{2}$ = cost of 1 share. $ 128.62$\frac{1}{2}$ × 500 = $ 64,312.50 = the cost of 500 shares.

3. A broker sells for me 350 shares Chicago & Northwestern at the highest quotation; usual brokerage. How much should I receive?

Solution

$ 189.50 × 350 = $ 66,325, the amount received by the broker.
$ 100 × 350 = $ 35,000, the par value.
$\frac{1}{8}$% of $ 35,000 = $ 43.75, the brokerage.
$ 66,325 − $ 43.75 = $ 66,281.25, the proceeds of the sale.

WRITTEN EXERCISE

Find the market value of the following stocks at the closing quotations in the list, p. 303:

1. 50 shares American Locomotive.
2. 75 shares American Tobacco preferred.
3. 150 shares Amalgamated Copper.
4. 56 shares General Electric Co.
5. 300 shares Missouri Pacific.

6. A broker sold for me at par 260 shares Atchison, Topeka, & Santa Fe; brokerage $\frac{1}{8}$%. How much should I receive?

7. Find the proceeds of 650 shares Manhattan Railroad stock at the closing quotation in the list, p. 303; brokerage $\frac{1}{8}$%.

8. How much must be sent to a broker in order that he may buy 60 shares Iowa Central preferred at 48½; usual brokerage?

9. What is the total par value of 300 shares Canadian Pacific; 150 shares Denver & Rio Grande common; 50 shares Denver & Rio Grande preferred; 75 shares National Biscuit; 80 shares Ontario & Western; 80 shares Rock Island common; and 70 shares Rock Island preferred? What is the total market value, taking each stock at the closing quotation for that stock in the list, p. 303?

10. The directors of a rapid transit company decide to increase their stock, by declaring a stock dividend of 5% (**781**); that is, each stockholder is presented with newly issued stock to the extent of 5% of his holdings. Just before this, the stock sold at **115¼**. What ought to be the market value afterwards?

785. To find the number of shares or the par value of stock.

786. Example. Sold Consolidated Gas stock at 137½ for **$44,000.** **Find the number of shares and the par value of the stock.**

Solution

$ 137.50 = the market value of 1 share of stock.
$ 44,000 = the market value of the whole number of shares.
$ 44,000 ÷ $ 137.50 = 320, the number of shares sold.
$ 100 × 320 = $ 32,000, the par value of the stock.

WRITTEN EXERCISE

1. How many shares of Chicago & Northwestern common can be bought for $ 75,850, at the highest quotation on page 303; brokerage $\frac{1}{8}$%? Upon what sum will dividends be declared?

2. How many shares of Brooklyn Rapid Transit must be sold at the lowest quotation, regular brokerage, to bring $ 39,187.50?

3. How many shares Twin City Rapid Transit bought at the lowest quotation and sold at the highest quotation will produce a gain of $ 1850; usual brokerage both ways?

4. A broker bought on his own account Southern Pacific at the highest quotation, for $ 28,725. How many shares did he get? What is the amount of a 4% dividend on them?

787. To find the amount of investment.

788. Example. What sum must be invested, regular brokerage, in Twin City Rapid Transit preferred, at the lowest quotation in the list, p. 303, to earn $1500 per annum if the annual dividend is 4 %?

SOLUTION

The dividend on 1 share = $4.
$1500 ÷ $4 = 375.
Hence, the dividend on 375 shares = $1500.
The cost of 1 share = $104.12½ ($104 + $.12½ brokerage).
$104.12½ × 375 = $39,046.88, the cost of 375 shares.

WRITTEN EXERCISE

1. What sum must be invested in Delaware & Hudson, at the highest quotation in the list, p. 303, including ⅛ % brokerage, to earn an income of $2700 per year, the average annual dividend being 9 %?

2. If Baltimore & Ohio stock pays an annual dividend of 6 %, how much must be invested in this stock at 118¾, regular brokerage, to produce an annual income of $1500?

3. What can one afford to pay, exclusive of brokerage, for stock that averages an annual dividend of 9 % in order to realize 5 % on the investment?

4. If New York Central pays an annual dividend of 6%, and you buy through a broker, at ⅛%, enough stock to give you an income of $2400 per year, what would it cost you at the quotation 129⅛?

789. To find the rate per cent of income.

790. Example. What is the per cent of income on an investment in Canadian Pacific at the closing quotation in the list, p. 303, regular brokerage, if the dividends are 6 % per annum?

SOLUTION

$190.125 = the cost of 1 share.
$6 = the income on 1 share.
$6 ÷ $190.125 = 3.155 + %, the rate of income.

WRITTEN EXERCISE

1. If American Sugar Refining Co. stock yields an annual dividend of 7 %, what is the rate per cent of income on the investment, for stock bought at the lowest quotation in the list, page 303, regular brokerage?

2. Which is the better investment — Reading first preferred, at 90, paying a dividend of 4 % annually, or New York Air Brake, at 135, paying 8 % annual dividends; no brokerage? What is the difference in the rate of income?

3. What per cent of income will you receive on an investment in New York Central, paying an annual dividend of 6 %, if purchased at 130½ through a broker at ⅛ %?

791. **To find the dividend on stocks.**

792. **Example.** The Atchison, Topeka & Santa Fe Railroad Co. reported a net income of $29,701,795 for one year. Their common stock was $102,707,000. The directors declared a 5 % dividend on this, and the remainder of the net income was carried to surplus fund. How much was the dividend? How much was the surplus fund amount? If you owned 25 shares of this stock, what would be your part of the dividend?

Solution

5% of $102,707,000 = $5,135,350, the amount of the dividend.
$29,701,795 – $5,135,350 = $24,566,445, the surplus fund amount.
Dividends are declared on the par value of the stock.
The par value of your stock is $2500.
5% of $2500 = $125, the dividend due you.

WRITTEN EXERCISE

1. You paid through a broker, at ⅛ %, $30,870 for Atlantic Coast Line stock bought at the closing quotation in the list, page 303. What annual income is derived if 3 % semiannual dividends are declared?

2. The common stock of a railroad company is $54,000,000, and the preferred stock (**767**) is $6,000,000. The directors declare a 4½ % dividend on the preferred stock and a 3 % dividend on the common stock. What is the surplus if the total net earnings are $1,965,475.50?

3. The Southern Pacific has a common stock capitalization of $197,849,200, and declares an annual dividend of 5%. What is the total dividend and how much is due C, who owns 22 shares?

4. The Twin City Rapid Transit Co., capital $20,100,000, declared a quarterly dividend of 1¼%. What is the total dividend, and how much of it is due F, who owns 75 shares?

5. The New York Air Brake Co. is capitalized at $10,000,000. If its net earnings for a year are $1,345,308.25, and if 3½% of the net earnings is set aside as a surplus fund, an 8% dividend is declared, and the balance is carried to undivided profits, what amount goes to surplus fund, dividend, and undivided profits respectively?

793. To find the rate per cent of dividend, the capital stock and net earnings being given.

794. Example. The Canada Southern Railway Co. has a capitalization of $15,000,000. If its net earnings are $383,762.50, how great an even per cent dividend may be declared if 2% of the net earnings are to go to surplus fund before any allowance is made for dividend?

SOLUTION

2% of $383,762.50 = $7675.25 surplus.

$383,762.50 – $7,675.25 = $376,087.25, the amount to go to dividend and undivided profits.

1% of $15,000,000 = $150,000.

$376,087.25 ÷ $150,000 = 2, with a remainder of $76,087.25.

Hence a 2% dividend may be declared, and $76,087.25 will be left as undivided profits.

WRITTEN EXERCISE

1. The Delaware & Hudson Co. has a capitalization of $42,250,500. If its net earnings are $3,802,545, how large an even rate per cent dividend may be declared?

2. The common stock of the Baltimore & Ohio Railroad Co. is $152,165,500. If its net earnings for a year are $9,141,065.60, what is the greatest even per cent of dividend that may be declared and what balance will there be for undivided profits?

3. Suppose the Canadian Pacific Railway Co. has gross earnings of $22,973,312, and expenses of $8,350,545. If its capitalization is $105,307,100, what even per cent dividend may be declared and what will be the amount of undivided profits if 2½% of the net earnings is first set aside as a surplus fund?

795. To find the profit or loss when a person buys and sells stocks on a margin.

796. Example. On Jan. 5, a speculator deposited with his broker $1500 as a margin. The broker purchased for him 150 shares Southern Pacific at 90. On Jan. 17 the broker sold the stock at the highest quotation in the list, p. 303. What was the customer's profit; interest 6%, brokerage ⅛%?

SOLUTION

Dr. Customer Cr.

Jan.	5	150 sh. So. Pac. 90	13500	—	Jan.	5	Margin deposited	1500	—
Jan.	5	Brok. ⅛ buying	18	75	Jan.	17	150 sh. So. Pac. at 95¾	14362	50
Jan.	17	Brok. ⅛ selling	18	75	Jan.	17	Int. on $1500 for 12 ds.	3	—
Jan.	17	Int. $13,518.75 for 12 ds.	27	04					
		Balance due	2300	96					
			15865	50				15865	50

$2300.96 − $1500 = $800.96, the amount gained.

WRITTEN EXERCISE

1. On Jan. 11 a speculator deposited with his broker $3000 as a margin, and through him bought 150 shares Chesapeake & Ohio at 48, and 150 shares Missouri Pacific at 83. On Jan. 17 the broker sold the stock at the highest quotation in the list, p. 303. How much did the speculator gain; interest at 6%, brokerage ⅛%?

2. On Jan. 7 W deposited $2000 as margin with his broker, who bought for him 100 shares Delaware & Hudson at 217, and 100 shares Baltimore & Ohio at 120. On Jan. 17 the stock was sold at the closing quotations in the list, p. 303. What was W's gain or loss; interest 6%, brokerage ⅛%?

3. On Jan. 17 a speculator deposited $4000 as a margin, and by his orders the broker bought 200 shares Atchison, Topeka & Santa Fe at 108¼, 100 shares Atlantic Coast Line at 133⅜, and 100 shares Chicago & Alton at 27½. On Jan. 25 (the margin being nearly exhausted) the broker "sold him out" at the following quotations: Atchison, Topeka & Santa Fe at 98⅜, Atlantic Coast Line at 119¾, and Chicago & Alton at 24½. How much did the speculator lose; interest 6%, brokerage ⅛%?

BONDS

797. A **bond** is a written or printed obligation under seal issued by a company or corporation, municipal or state government, or by the federal government. It is conditioned to pay a certain sum of money at a specified time and at a fixed rate of interest, payable at regular intervals.

Bonds of business corporations are usually secured by mortgages on their real estate. Municipal bonds are issued by vote of the people or their representatives, and for their payment a sinking fund is accumulated by a yearly rate per cent levied on all the real property within the limits of the municipality.

798. **Government bonds** are bonds issued by the federal government. Their names are usually derived from the interest they bear and the time when due.

Thus "U.S. 4's, 1912," is understood to mean "United States bonds bearing 4% interest, and due in 1912"; and "U.S. 3's, 1925," is understood to mean "United States 3% bonds due in 1925."

799. A **coupon bond** is a bond that has coupons or certificates of interest attached. When the interest becomes due, these coupons are detached and surrendered upon receipt of the interest represented by them.

The interest coupons on government coupon bonds are payable to the bearer, and will be cashed by any bank or banker in the United States. Coupon bonds may be converted into registered bonds of the same issue.

800. A **registered bond** is one which is payable to the owner as registered in the books of the corporation or government issuing it. Registered bonds can be transfered only by assignment and registry on the books.

The interest on registered bonds is paid by checks, payable to the order of the registered owner, and sent to him. The checks for interest on government bonds are readily cashed by banks and bankers.

801. As with stocks, the **par value** of **bonds is their face value**; the **market value** is the amount at which they are quoted in the market. Bonds are **above par** or **at a premium** when they are worth more than their face value; **below par** or **at a discount** when they are worth less than their face value.

802. Bond quotations are the market prices or rates that the bonds sell for.

The income from bonds, unlike that from stocks, is fixed; that is, it is in no way affected by the general conditions of the corporation, so long as the corporation is solvent.

Bonds are usually quoted flat; that is, the quoted price is for the bond as it is at the time of the quotation, including accrued interest, except that after the closing of the books registered bonds are quoted less the interest. The interest then due belongs to the holder of the bonds at the time the books are closed.

803. The list herewith shows part of one day's bond sales (so many dollars **par value of each**) on the floor of the New York Stock Exchange; **it is taken from the** ***New York Sun,*** Wall Street edition, **under date of Jan. 17, 1907:**

Adams Exp 4s
500......102½
3000......102½
Am Ice deb 6s
1000...... 89
Am Tobacco 6s
3000......110¼
39000......110
2000......110¼
Am Tobacco 4s
3000...... 77¾
6000...... 78
registered
500...... 75¾
Balt & Oh gold 4s
16000......101½
Ches & Ohio 5s
1000......115
Ches & Ohio 4½s
1000 .....104¼
Chi & Alton 3½s
4000...... 76
Chi Bur & Q 4s
2000...... 96½

Chi & E Ill s f 6's
1000......100⅞
Chi Mil & St P 4s
1000......106
Chi R I & Pac RR gold 5s
1000...... 90¼
3000...... 90⅝
2000...... 90½
5000...... 90
Erie cv 4s ser A
10000......100½
5000......100¼
15000......100
37000...... 99½
2000...... 99⅞
10000......100
69000...... 99½
Erie prior lien 4s
4000...... 97½
Green Bay & West deb ser B
2000...... 14½

Ill Central 4s 1953
2000......103½
Ill Cent L div 3½s
1000...... 89¾
3000...... 87½
Mo K & T s f 4½s
19000...... 87½
Mo Pacific 5s 1920
1000......105
N Y Central 3½s
4000...... 93½
58000...... 93¼
North Pacific 3s
4000...... 73⅜
5000...... 73¼
5000...... 73⅛
2000...... 73
Or S L fdg 4s
4000...... 94
5000...... 94¼
Penna 4½s 1921
1000......107
3000......106½
3000......106¼

Penna cv 3½s
20000...... 96⅝
1000...... 96⅞
Penna 3½s 1915
20000...... 92⅞
22000...... 93
3000...... 92⅞
24000...... 93
20000...... 92⅞
2000...... 93
7000...... 93⅛
Union Pacific 4s
2000......101½
4000s 15...101⅝
U S 3s cpn
3500......103
U S Steel s f 5s
6000...... 98⅛
17000...... 98¼
4000...... 98⅛
11000...... 98¼
11000...... 98⅛
1000...... 98¼
19000...... 98⅛

Note. Brokers usually charge ⅛% brokerage for transactions in bonds. That rate is to be understood if none is specified.

804. Example. What rate per cent per annum interest will Chicago & Alton 3½'s yield on the investment, if bought through a broker at the price quoted in the list, p. 311?

SOLUTION

$76 + .125 = cost of $100 par value of the bonds.
$3.50 = the income on $100 worth of the bonds.
$3.50 ÷ $76.125 (cost of bonds) = 4.5977+ %, the rate of income.

WRITTEN EXERCISE

1. If a broker invested on his own account in Chesapeake & Ohio 5's as quoted in the list, p. 311, what per cent of income would he receive?

2. Find the proceeds of the United States 3's coupon sold through a broker.

3. How much must be invested in Chesapeake & Ohio 4½'s to produce a semiannual income of $1350; regular brokerage?

4. What per cent income will be produced by $358,750 invested in Adams Express Company's 4's at the market quotation, allowing the regular brokerage?

5. How much must you invest through a broker in Pennsylvania 4½'s, 1921, at the last quotation, so that you may have an income of $2700 per year? This income is what per cent of the investment?

6. A sells through a broker one $5000 Pennsylvania 3½, 1915, at the last quotation, and loans the proceeds at 5%. How much will his yearly income thereby increase?

7. How many $1000 Illinois Central 4's, 1953, bought at 98½ and sold at the list quotation, will yield $2375 gain, usual brokerage both ways?

8. A has an annual income of $880 on an investment in Baltimore & Ohio $500 gold 4's. How many does he own? If they were bought at the quotation in the list, through a broker, what rate per cent per annum does he receive on his investment?

9. Which would be the better investment, Erie convertible 4's, series A, at the last quotation in the list, or Missouri Pacific 5's, 1920, if both were purchased through a broker? How much better?

WRITTEN REVIEW

1. What will be the cost, including $\frac{1}{8}$% brokerage, of 250 shares Denver & Rio Grande, 300 shares Atchison, Topeka & Santa Fe, 50 shares New York Central, 40 shares Ontario & Western, 125 shares Louisville & Nashville, 150 shares American Tobacco preferred, all at the closing quotations on the list, p. 303?

2. What annual income is derived from investing $27,500, exclusive of brokerage, in Consolidated Gas at 137$\frac{1}{2}$, if it averages 5% annual dividends?

3. How much must be invested, exclusive of brokerage, in Amalgamated Copper at 115$\frac{1}{4}$, so that an annual income of $2500 may be realized if a 4% yearly dividend is declared?

4. A bought through his broker, at $\frac{1}{8}$%, 500 shares Pacific Coast, for which he paid the broker $60,812.50. With what market quotation does the price he paid agree? How much was the brokerage?

5. In example 1 if the average annual dividend was 4$\frac{1}{2}$%, what was the rate per cent of interest on the investment?

6. In example 2 what is the rate per cent interest on the investment?

7. On Jan. 2 I deposited with my broker $6000 as a margin, and he bought for me 250 shares Erie at 40$\frac{3}{8}$, 200 shares Missouri Pacific at 88$\frac{1}{4}$, and 150 shares Brooklyn Rapid Transit at 82. On Jan. 17 the stocks were quoted late in the day at the closing figures in the list, p. 303. How much must I deposit to make my margin good? If the broker had "sold me out" because I could not make my margin good, how much would I have lost?

8. A sold 500 shares Louisville & Nashville stock at 135$\frac{7}{8}$, through a broker, and bought with the proceeds of the sale Chicago, Milwaukee & St. Paul 4% bonds at 106, through a broker. How many $500 bonds did he get, and how much unexpended balance was there due him?

9. Sold three $1000 American Tobacco 6's at 110¼, and with the proceeds bought Northern Pacific at 158. Later in the day I sold the stock at 160¼. How much did I gain, allowing the usual brokerage on all the transactions? How much did the broker have belonging to me?

10. American Ice Securities pays an annual dividend of 7%; Delaware & Hudson, 9%; Baltimore & Ohio, 6%; Erie, 2d preferred, 4%. If bought at the closing quotations in the list, p. 303, with no brokerage, what is the rate per cent of income on each?

11. The American Smelting Refining Co. had reported net earnings, during the fiscal year 1906, of $10,161,358. Its common stock was $50,000,000. If 5% of the net earnings is set aside as surplus fund, a 7% dividend is declared, and the balance carried to undivided profits, what sums go respectively to surplus fund, to dividend, to undivided profits?

12. What sum invested in Chicago & Eastern Illinois sinking fund 6's at 100⅞ will produce a yearly income of $3000, no brokerage?

13. X owns 200 shares Reading, 1st preferred, which cost him $18,000. He realizes annually 5% on his investment. What rate of dividend was declared?

14. The net earnings of the Canadian Pacific for the fiscal year 1906 were $22,973,312, and the capital stock was $105,307,100. If 50% of the net earnings is carried to surplus fund, what even per cent of dividend may be declared, and how much will be left as undivided profits?

15. Y owned 300 shares of the Canadian Pacific stock. Z owned 250 shares. If Y bought his stock at 188¾, and Z bought his stock at 190, what is the rate per cent of income on each man's investment, making no allowance for brokerage? What does each receive as dividend?

INSURANCE

805. **Insurance** treats of those computations arising from contracts guaranteeing security against loss or damage.

806. The **parties** to insurance are the *insured* or *assured* and the *insurer* or *underwriter*.

807. The **insured** or **assured** is the person protected, or insured, against loss or damage.

808. The **insurer** or **underwriter** is the party that guarantees security against loss or damage.

Insurers or underwriters are usually incorporated companies.

809. A **policy** is a written contract between the insured and the insurer. It sets forth the conditions under which the risk is taken, the liability of the insurance company, the time the insurance is to continue, the premium.

810. A **valued** or **closed** policy is one in which a fixed value is given to the thing insured.

A valued or closed policy is the ordinary form used in general fire insurance.

811. An **open** policy is one in which no fixed value is given to the thing insured. In an open policy additional insurance may be entered at any time at rates and under conditions agreed upon.

812. The **premium** is the sum paid for insurance.

813. The **term of insurance** is the period of time for which the risk is taken or the property insured.

814. **Premium rates** are sometimes given as a specified number of cents per $100, and sometimes as a certain per cent of the sum insured. They depend upon the nature of the risk and the length of time for which the policy is issued.

Insurance is usually effected for a year or a term of years.

815. **Short rates** are certain rates of premium charged by insurance companies for terms less than one year. Short rates are proportionately higher than yearly rates.

816. An **insurance agent** is one who acts for an insurance company in obtaining insurance, collecting premiums, adjusting losses, reinsuring, etc.

817. An **insurance broker** is a person who negotiates insurance for others, for which he receives a brokerage from the company taking the risk; he is considered, however, an agent of the insured, not of the company.

818. **Insurance companies** are distinguished by the way in which they are organized; as *stock companies, mutual companies*, and *mixed companies*.

819. A **stock insurance company** is one whose capital has been contributed and is owned by the stockholders, who share the gains and are liable for the losses.

820. A **mutual insurance company** is one in which the gains and losses are shared by the insured parties.

821. A **mixed insurance company** is one which combines the features of both stock and mutual companies.

In mixed companies all gains above a limited dividend to the stockholders are divided among the policy holders.

PROPERTY INSURANCE

822. **Property insurance** is the insurance of property against any specified casualty.

823. Property insurance includes:

1. **Fire insurance**, or indemnity for loss of, or damage to, property by fire.

2. **Marine insurance**, or indemnity for loss of, or damage to, a ship or its cargo by any specified casualty at sea or on inland waters.

3. **Live stock insurance**, or indemnity for loss of, or damage to, horses, cattle, etc., and from lightning or other casualty.

4. **Transit insurance**, or indemnity for loss of, or damage to, goods transported from one place to another by land or by both land and water.

824. **Insurance policies** are sometimes classified as *ordinary policies* and *average clause policies*.

825. Under an **ordinary policy** the company will pay the full amount of any loss or damage that does not exceed the sum covered by the policy.

Thus, if a house worth $12,000 is insured for $9000, and a fire occurs by which a loss of $7000 is sustained, the company is bound to pay the full loss, or

$7000; but if the loss should be $10,000, or any sum in excess of $9000, the company will pay only the $9000 specified in the policy.

826. Under an **average clause policy** the company will pay only such a proportion of the loss as the policy is of the entire value of the thing insured.

Thus, if a vessel valued at $12,000 is insured for $8000, and a fire occurs by which a loss of $6000 is sustained, the company will pay two thirds ($\frac{8000}{12000}$) of $6000, or $4000; but if the loss is total, the company will pay the full $8000, which is two thirds of the entire valuation, $12,000.

827. Marine insurance policies usually contain the average clause.

828. Almost all insurance companies will not issue a policy above a certain fixed sum; and they will issue only one policy covering the same property. Therefore, if a person owns a valuable building, he must ordinarily have it insured in several different companies, in order to protect his interests.

829. If property that is insured in several companies is damaged by fire to the extent of the total amount of the insurance, each company must pay the full amount of its policy. If the loss is less than the total amount of the insurance, each company must pay such a portion of the loss as its policy is a part of the entire insurance.

830. **To cancel a policy** is to annul the contract between the insurer and insured.

In case a policy is terminated at the request of the insured, he is charged the short rate premium. If, however, it be terminated at the option of the company, the lower long rate will be charged, and the company will refund the premium for the unexpired time of the policy.

831. **Salvage** is an allowance made to those rendering voluntary aid in saving vessels or cargoes from marine casualties.

Insurance companies usually reserve the privilege of rebuilding, replacing, or repairing damaged property.

832. Computations in property insurance are made in accordance with the general principles of abstract percentage, the *amount insured* corresponding to the *base*; the *rate of the premium* to the *rate*; and the *premium* to the *percentage*.

DRILL EXERCISE

1. Find the cost of insuring a barn and contents for $4000 at 1½%.

2. At 2%, what amount of insurance can I procure for $74?

3. If $25 is paid for insuring property worth $1000, what is the rate?

4. State a formula for finding the premium when the amount insured and the rate of premium are given.

5. Given the premium and rate of premium, how may the amount of insurance be found?

6. Given the premium and the amount insured, how may the rate of premium be found?

7. A dealer paid $125 for insuring a cargo of grain at 1½% on ⅚ of its value. Find the value of the grain.

833. To find the cost of insurance.

834. Example. How much will it cost to insure a store and contents for $42,000 at 1½%?

Solution

$42,000 = the amount insured.
1½% of $42,000 = $630, the premium charged.

WRITTEN EXERCISE

1. A store is valued at $12,000 and the contents at $18,000. Find the cost of insuring ¾ of the value of the store at ⅘%, and ⅔ of the value of the contents at ⅜%.

2. An insurance company, having insured a block of buildings for $200,000 at 75¢ per $100, reinsured $60,000 with another company at ⅗%, and $80,000 with another at ⅜%. What amount of premium did it receive more than it paid?

835. To find the rate of insurance.

836. Example. I paid $30 for insuring a house worth $6400 at ¾ valuation. What was the rate?

Solution

¾ of $6400 = $4800, the face of the policy.
$30 ÷ 4800 = .00625, or ⅝, the rate of insurance.

WRITTEN EXERCISE

1. The cost of insuring $\frac{3}{4}$ of a cargo of wheat worth $24,000 was $240. What was the rate of insurance?

2. I insured $\frac{3}{5}$ of a stock of goods worth $4500, and paid $18 premium. What was the rate of insurance?

837. To find the amount insured.

838. Example. A man paid $280 to insure a stock of goods for 3 months. If the rate of premium was $\frac{7}{8}\%$, for what amount was the policy issued?

SOLUTION

Let 100 % represent the amount of the policy.
$280 = the premium paid.
$\frac{7}{8}\%$ = the rate of premium.
Therefore $\frac{7}{8}\%$ = $280.
$\frac{1}{8}\%$ = $40; $\frac{8}{8}\%$ or 1 % = $320.
100 % = $32,000, the amount of the policy.

WRITTEN EXERCISE

1. A gentleman paid $35.60 per annum for insuring his house at 2% on $\frac{2}{5}$ of its value. What was the value of the house?

2. A ranchman paid a premium of $76.00 for insuring $\frac{2}{3}$ of his herd of cattle at 60¢ per $100. If the cattle were valued at $40 per head, how many had he?

3. The contents of a factory were insured for a certain sum at $1\frac{1}{2}\%$. Later the goods were damaged by fire and losses paid by the company to the amount of $18,750, which was $\frac{2}{3}$ of the amount insured. If the amount insured was $\frac{3}{4}$ of the value inventoried, what was the total value of the goods?

WRITTEN REVIEW

1. Find the cost of insuring a cargo of wheat valued at $24,000 at $1\frac{1}{4}\%$.

2. How much insurance, at $1\frac{1}{8}\%$, can be procured for $90?

3. If it cost $663 to insure a certain block for $44,200, what will be the cost, at the same rate, to insure a block valued at $105,000 if $1.50 extra be charged for the policy in the latter case?

4. How much will it cost to insure a factory for $ 42,000 at ¾%, and its machinery for $ 16,500 at 1¼%, the charge for policy and survey being $ 2.50?

5. The premium on a cargo of 3000 tons of coal valued at $ 3.50 per ton, and insured at ⅗ of its valuation, is $ 47.25. Find the rate of insurance.

6. If a store and its contents are valued at $ 27,000, for how much must it be insured at 1½%, to cover loss and premium in case of total destruction?

7. A cargo of teas, valued at $ 33,000, was insured for $ 18,000 in a policy containing an "average clause." In case of damage to the amount of $ 21,000, how much should the company pay?

8. The steamer *Norseman*, valued at $ 90,000, is insured for $ 75,000, at 2½%. What will be the actual loss to the insurance company in case the steamer is damaged to the amount of $ 20,000?

9. A speculator bought 2000 barrels of flour, and had it insured for 80% of its cost, at 3½%, paying a premium of $ 429. At what price per barrel must he sell the flour, to make a net profit of 10%?

10. I insured my grocery store, valued at $ 13,500, and its contents, valued at $ 33,000, and paid $ 350 for premium and policy. If the policy cost $ 1.25, what was the rate per cent of premium?

11. A canal boat load of 8400 bushels of wheat, worth 90¢ per bushel, is insured for ¾ of its value, at 1⅜% premium. In case of the total destruction of the wheat, how much will the owner lose?

12. A stock of goods valued at $ 30,000 was insured for 18 months, at 1½%; at the end of 12 months the owner surrendered the policy. If the "short rate" for 6 months was 65¢ per $ 100, what should be the return premium?

13. The German Insurance Company insured the Field block for $ 105,000, at 60¢ per $ 100; but thinking the risk too great, it reinsured $ 40,000 in the Home, at ⅜%, and $ 45,000 more in the Mutual, at ¾%. How much premium did each company receive? What was the gain or loss of the German? What per cent of premium did it receive for the part of the risk not reinsured?

14. A block of stores and contents was insured for $ 220,000 and became damaged by fire and water to the amount of $150,000. Of the risk, $ 40,000 was taken by the Hartford Co., $ 65,000 by the

Manhattan, $ 35,000 by the Ætna, and the remainder was divided equally between the Phœnix and the Provident. What was the net loss of each company, if the premium paid was $1\frac{1}{8}$ % ?

15. A factory worth $45,000 is insured, with its contents, for $ 62,500; $ 30,000 of the insurance is on the building, $ 12,500 on machinery worth $ 20,000, and $ 20,000 on stock worth $ 35,000. A fire occurs by which the building and the machinery are both damaged, each to the amount of $ 15,000, and the stock is entirely destroyed. How much is the claim against the company, if the risk is covered by an "ordinary" policy? How much if the policy contains the "average clause"?

PERSONAL INSURANCE

839. Personal insurance is the insurance of person. It includes:

1. **Life insurance**, or indemnity for loss of life.

2. **Accident insurance**, or indemnity for loss from disability occasioned by accident.

3. **Health insurance**, or indemnity for loss occasioned by sickness.

840. Life insurance policies are usually either *life policies* or *endowment policies.*

841. A **life policy** agrees to pay to the beneficiary named in it a fixed sum of money on the death of the insured.

The *beneficiary* is the one to whom the insurance is guaranteed to be paid.

842. An **endowment policy** guarantees the payment of a fixed sum of money at a specified time, or at death, if the death occurs before the specified time.

843. Life insurance companies are known as *stock, mutual, mixed,* and *coöperative.* Losses sustained by stock and mixed companies are paid either from reserve funds, or by assessment on the stockholders; those sustained by mutual and coöperative companies are paid by pro rata or fixed contributions of the policy holders.

844. Life insurance may be made payable to any one named by the insured. If made payable to himself, at his death it becomes a part of his estate and is liable for his debts; if payable to another, that other cannot be deprived of the benefit of the insurance, either by the will of the person taking out the insurance, or by his creditors.

845. Any one having an insurable interest in the life of another may take out, hold, and be benefited by, a policy of insurance upon the life of that person; and any one may take out a policy in his own name and then assign it to any creditor or to any one having an insurable interest.

846. The following table shows the rates of one of the leading life insurance companies:

ANNUAL PREMIUMS FOR AN INSURANCE OF $1000

LIFE POLICIES, PAYABLE AT DEATH ONLY					ENDOWMENT POLICIES, PAYABLE AS INDICATED OR AT DEATH, IF PRIOR						
Age	Continuous Premiums	10 Premiums	15 Premiums	20 Premiums	Age	In 10 Yrs.	In 15 Yrs.	In 20 Yrs.	In 25 Yrs.	In 30 Yrs.	In 35 Yrs.
21	$18 40	$46 30	$34 19	$28 25	21	$101 53	$65 43	$47 75	$37 45	$30 86	$26 41
22	18 80	47 00	34 71	28 69	22	101 60	65 51	47 84	37 55	30 97	26 55
23	19 23	47 73	35 26	29 15	23	101 68	65 60	47 94	37 66	31 10	26 71
24	19 67	48 47	35 82	29 63	24	101 76	65 69	48 04	37 78	31 24	26 88
25	20 14	49 24	36 40	30 12	25	101 85	65 79	48 15	37 90	31 39	27 06
26	20 63	50 04	37 00	30 63	26	101 94	65 89	48 26	38 04	31 56	27 26
27	21 15	50 87	37 63	31 16	27	102 04	66 00	48 39	38 19	31 73	27 49
28	21 69	51 72	38 27	31 71	28	102 14	66 11	48 52	38 35	31 93	27 73
29	22 26	52 61	38 94	32 28	29	102 25	66 24	48 67	38 52	32 14	28 00
30	22 85	53 52	39 64	32 87	30	102 37	66 37	48 83	38 71	32 38	28 29
31	23 48	54 46	40 36	33 49	31	102 49	66 52	49 00	38 92	32 63	28 61
32	24 14	55 44	41 10	34 13	32	102 63	66 68	49 18	39 14	32 92	28 96
33	24 84	56 45	41 88	34 80	33	102 77	66 85	49 38	39 39	33 23	29 35
34	25 58	57 50	42 68	35 49	34	102 93	67 03	49 60	39 67	33 57	29 78
35	26 35	58 58	43 51	36 22	35	103 10	67 23	49 85	39 97	33 95	30 24
36	27 17	59 70	44 38	36 98	36	103 28	67 45	50 11	40 30	34 36	30 76
37	28 04	60 86	45 28	37 77	37	103 48	67 68	50 41	40 67	34 82	31 33
38	28 95	62 06	46 22	38 60	38	103 69	67 94	50 73	41 07	35 33	31 95
39	29 92	63 30	47 20	39 47	39	103 93	68 23	51 09	41 52	35 89	32 63
40	30 94	64 59	48 22	40 38	40	104 18	68 55	51 48	42 02	36 50	33 36
41	32 03	65 93	49 28	41 34	41	104 46	68 90	51 92	42 57	37 18	34 20
42	33 18	67 31	50 39	42 35	42	104 77	69 28	52 41	43 17	37 93	35 10
43	34 40	68 76	51 56	43 41	43	105 11	69 71	52 95	43 85	38 76	36 08
44	35 70	70 25	52 78	44 54	44	105 49	70 19	53 55	44 59	39 67	37 15
45	37 08	71 81	54 06	45 73	45	105 92	70 73	54 22	45 42	40 67	38 32
46	38 55	73 44	55 40	46 99	46	106 39	71 32	54 96	46 33	41 78	. . .
47	40 12	75 13	56 82	48 33	47	106 91	71 98	55 78	47 34	42 99	. . .
48	41 78	76 90	58 31	49 75	48	107 50	72 71	56 69	48 46	44 31	. . .
49	43 56	78 74	59 88	51 26	49	108 15	73 53	57 70	49 69	45 76	. . .
50	45 45	80 66	61 54	52 87	50	108 87	74 43	58 81	51 05	47 35	. . .

847. The following tables illustrate the three options of the insured if he ceases to pay premiums before the maturity of the policy: (1) to receive a certain amount of cash at once; or (2) to be insured for the amount of the policy in case of death within a limited time; or (3) to be insured for a certain smaller amount in case of death at any time.

SPECIMEN TABLE INDORSED ON ORDINARY LIFE POLICIES FOR $1000

AGE, 35 PREMIUM, $26.35

At End of Year	Cash Surrender Value Loan Value	Automatic Extended Insurance		Paid-up Policy
		Years	Days	
2d	$16.13	1	297	$37.00
3d	29.76	3	122	67.00
4th	43.77	4	313	97.00
5th	58.16	6	182	127.00
6th	72.94	7	292	156.00
7th	88.11	9	47	185.00
8th	103.68	10	115	213.00
9th	119.65	11	128	242.00
10th	136.01	12	86	270.00
11th	152.76	12	357	297.00
12th	169.87	13	215	324.00
13th	187.35	14	82	351.00
14th	205.16	14	175	377.00
15th	223.28	14	284	402.00
20th	317.58	15	86	521.00
25th	415.49	14	215	623.00
30th	512.92	13	149	709.00
35th	605.14	11	326	779.00
40th	688.21	10	48	834.00

SPECIMEN TABLE INDORSED ON 20-YEAR ENDOWMENT POLICIES FOR $10,000

AGE, 35 PREMIUM, $498.50

At End of Year	Cash Surrender Value Loan Value	Automatic Extended Insurance			Paid-up Policy
		Years	Days	Pure Endowment	
2d	$604.00	7	46		$980.00
3d	975.00	11	185		1540.00
4th	1359.10	15	203		2090.00
5th	1757.10	15		$750.00	2640.00
6th	2169.80	14		1550.00	3170.00
7th	2596.60	13		2820.00	3710.00
8th	3039.40	12		3060.00	4230.00
9th	3498.50	11		3760.00	4740.00
10th	3974.50	10		4450.00	5250.00
11th	4468.40	9		5100.00	5750.00
12th	4980.80	8		5730.00	6240.00
13th	5512.80	7		6330.00	6720.00
14th	6065.50	6		6910.00	7200.00
15th	6640.00	5		7460.00	7660.00
16th	7287.70	4		7990.00	8120.00
17th	7860.50	3		8500.00	8580.00
18th	8510.10	2		8990.00	9020.00
19th	9189.10	1		9460.00	9460.00
20th	10000.00			10000.00	10000.00

848. Two important special kinds of insurance are somewhat similar in character to personal insurance:

1. **Guaranty and fidelity insurance** is indemnity for loss because of fraud, dishonesty, or negligence on the part of agents or employees.

2. **Bonding companies** guarantee the payment of bonds given by an agent, contractor, treasurer, secretary, or employee, for the proper performance of some specific or general line of duty.

For example, when the New York subway was contracted for, the contractor was obliged to give the city of New York bonds for some $35,000,000. A number of bonding companies agreed that if the contractor failed entirely or in part to execute his contract, these bonding companies would indemnify the city of New York for the loss caused by his failure.

Bonding companies have certain rates for the risk they assume. For building tunnels the charge is usually 1% on the amount of contract up to $50,000, and ½% for any excess. For instance, if George L. Benton makes a contract to build a tunnel for $65,000, the cost of a bond would be 1% on $50,000, or $500, and ½% on $15,000, or $75; a total of $575. The usual charge for bonds for agents working on a salary is 60¢ per hundred dollars up to $2500, and 50¢ per hundred for any excess over $2500; minimum charge, $7.50. For instance, if an agent were obliged to give a bond of $4000, the charge would be $22.50 (60¢ per hundred on $2500, or $15, and 50¢ per hundred on $1500, or $7.50). Should the bond be for either $500 or $1000, the minimum charge of $7.50 would apply.

849. Examples in life insurance. *1.* What continuous premium should be paid annually to secure $7500 at death, if the insured is 39 years of age at the time the policy is issued?

SOLUTION

The table, p. 322, shows for age 39, continuous premiums of $29.92 per $1000.
$29.92 × 7.5 = $224.40, the annual premium for $7500.

2. A, 42 years of age, wishes to secure a policy of $15,000 payable at his death. How much premium would he be obliged to pay annually if he chose to have a paid-up policy after making ten premium payments?

SOLUTION

In the 10-premiums column, opposite age 42, we find $67.31.
$67.31 × 15 = $1009.65, the amount of each of the ten annual premiums.

3. B is 27 years of age and secures a 20-year endowment policy for $20,000. How much is he obliged to pay at the beginning of each year?

SOLUTION

In the table, p. 322, opposite age 27, we find in the 20-years column, $48.39.
$48.39 × 20 = $967.80, one of the twenty premiums.

4. If B had lived to be 47 years of age and had put the amount of the premium in example 3 in a savings bank at the beginning of each year, at 3% compound interest, how much would be due him? On this basis, what was the amount of the total cost of the pure insurance (insurance for the difference between cash surrender value and face of policy) in example 3?

SOLUTION

$1 placed at 3% compound interest at the beginning of each year amounts in 20 years to $27.677. (See table, p. 398.)

$27.677 × 967.80 = $26,785.80, the amount due from the savings bank.

$26,785.80 − $20,000 (cash value of policy at age 47) = $6785.80, the amount of the cost of the pure insurance.

WRITTEN EXERCISE

1. What premium must be paid annually upon a life policy to give my beneficiary $6000 upon my decease, if I am now 35 years of age?

2. X takes out an ordinary life policy for $10,000 in favor of his wife. What are his annual premiums if he is now 37 years of age?

3. Y is 32 years old and takes out a 15-year endowment policy for $5000. What premium must he pay at the beginning of each year?

4. In example 3, if Y had lived to be 47 years of age and had put the amount of the yearly premium in a bank each year, how much would be due him at 3 % compound interest? How much more would the bank then pay than the insurance company?

5. A life insurance company issued a 10-year endowment policy for $12,000 to a man 32 years of age. The insured died at 38 years of age. How much more than the face of the premiums did the company have to pay? If the company invested each year as a reserve fund 60 % of the annual premium at 4 % compound interest (see p. 398), how much more or less than the premiums and the interest they earned did the company pay?

6. A man 35 years of age takes out an ordinary life policy for $5000. At the end of the tenth year he exercises his loan privilege and borrows from the company the cash surrender value of the policy. How much does he get? If he had deposited the annual premiums in a savings bank paying 3 % compound interest, how much could he have withdrawn from the bank after ten years?

7. Two persons, each 38 years of age, insured for $30,000 each. One secured a life policy and the other a 10-year endowment policy. How much had each paid in premiums in eight years?

8. A man 32 years of age secured a 15-year endowment policy for $8000. If he died at the end of the seventh year, how much more than the premiums paid in did the insurance company have to pay his beneficiary? If the company loaned 50 % of the premium at the beginning of each year at 3½ % compound interest, how much did it lose on the policy?

9. Y obtained a 20-year endowment policy at the age of 28 years. How much more or less than the face value of the policy will he pay in per thousand dollars in actual premiums if he survives the twenty years? At 3 % simple interest what would these premiums have yielded in principal and interest in twenty years?

10. M insured his life at the age of 41 years for $ 25,000 and paid the annual ordinary premiums till his death at 76 years of age. How much did he pay in premiums?

11. A, aged 35 years, and O, aged 33 years, two partners, each insured his life in favor of the other by an ordinary policy for $ 5000. At the end of five years O died. A now receives the face of O's policy, and also exercises his right to take the cash surrender value of his own policy. How much more than the premiums paid in by both men does A get? If the amount of the two premiums had been placed at compound interest each year at 3 %, how much more or less than the sum received by A would they have amounted to?

12. What is the cash surrender or loan value for an ordinary $ 5000 policy issued to a man aged 35 years, after he has paid seven years' premiums? If he takes extended insurance instead of cash surrender value, how long will it extend? What would be the amount of a paid-up policy if he chooses that?

13. What would be the answer to the three questions in Ex. 12 if the policy were a 20-year endowment policy for $ 10,000?

TAXES

850. A **tax** is a sum of money levied on the person, property, business, or income of an individual for the support of the government or for any public purpose.

851. The taxes levied by the national government are **indirect taxes**; they consist of *customs duties* and *internal revenue.*

852. The taxes levied by the state and local governments are mostly **direct taxes**; they consist of *poll tax* and *property tax.*

853. A **poll tax** is a tax levied on a person without regard to the property he owns. It is a certain amount per each adult male citizen.

854. A **property tax** is a tax assessed upon property at a given rate per cent of the valuation.

855. **Property** is of two kinds: *personal* and *real.*

856. **Personal property** is any movable property, such as merchandise, ships, cattle, money, stocks, mortgages.

857. **Real property**, or **real estate**, is any fixed or immovable property, such as houses, lands, mines, quarries.

858. An **assessor** is a public or government officer, appointed to estimate the value of property to be taxed, and to apportion the taxes in proportion to the value of each man's property.

859. An **assessment roll** is a descriptive list of taxable property. It shows in some detail the names of the owners of the property in the district assessed, its location, and assessed valuation.

860. A **collector** is a public officer appointed to receive and collect taxes.

Taxes are generally assessed and made payable in money, but road taxes are sometimes made payable in day's work.

The methods of collecting taxes vary in different states. In some states all the taxes are collected in the several counties, while in others they are all collected in the several towns. In almost all the states the different taxes, — state, county, town, school, etc., — are aggregated; that is, paid in one amount.

In certain states the common schools are supported by a tax or a rate bill, made out on a basis of the total attendance.

861. The **rate of taxation** is the sum charged on each dollar of the assessed valuation to raise the required amount of taxes.

862. Computations in taxes are made in accordance with the general principles of abstract percentage, the assessed valuation of the property *corresponding* to the *base;* the *rate of taxation,* to the *rate;* and the *tax* to the *percentage.*

DRILL EXERCISE

1. If the rate of taxation is 70 cents on each $ 100, how much tax must I pay on property, the assessed valuation of which is $ 9000?

2. If a man pays $ 600 tax on property worth $ 120,000, what is the rate of taxation?

3. I have property assessed at $ 12,000 and pay for 2 polls at $ 2.50 each. If my total tax is $ 65, how many cents on the dollar is the tax rate?

4. Given the valuation and rate of taxation, how may the tax be found?

5. Given the tax and the rate of taxation, how may the assessed valuation be found?

6. Given the assessed valuation and the tax to be raised, how may the rate of taxation be found?

863. **To find a property tax.**

WRITTEN EXERCISE

1. Henry Wilson is assessed $4000 on his real estate and $3500 on his personal property. If the rate of taxation is 2.5 mills on $1, what is the amount of his tax?

2. The taxable property of a town is $472,500, and the rate of taxation is 2.4 mills on $1. What is the amount of tax to be raised, and how much should B pay, who is assessed $4000 on his real estate and $1600 on his personal property?

864. **To find a general tax.**

865. **Example.** A tax of $2505 is to be assessed upon a certain village. The valuation of the taxable property is $600,000 and there are 324 polls to be assessed at $1.25 each. What will be the tax on $1, and how much will be the tax of Mr. Scott whose property is valued at $12,500 and who pays for 2 polls?

SOLUTION

$1.25 × 324 = $405, the amount of poll tax.
$2505 − $405 = $2100, the amount of property tax.
$2100 ÷ $600,000 = .0035, the rate of taxation.
$12,500 × .0035 = $43.75, Mr. Scott's property tax.
$1.25 × 2 = $2.50, Mr. Scott's poll tax.
$43.75 + $2.50 = $46.25, Mr. Scott's total tax.

866. **To find the rate of taxation.**

867. **Example.** A tax of $3750 is to be levied upon a valuation of $1,250,000. Find the rate of taxation.

SOLUTION

$3750 ÷ $1,250,000 = .003.
Therefore, the rate of taxation is $.003 on $1.

WRITTEN EXERCISE

1. A tax of $37,500 is levied on a city, the assessed valuation of which is $2,500,000. What is the rate of taxation, and what amount of tax will a person have to pay who is assessed $4500 on his real estate and $3500 on his personal property?

2. The cost of a new schoolhouse was $3500. If the taxable property of the district is assessed at $700,000, what is the tax rate on $100? What is B's tax if he is assessed $250 on his real estate and $2000 on his personal property?

WRITTEN REVIEW

1. A tax of $125,000 is levied on a city, the assessed valuation of which is $15,000,000. What is the rate of taxation, and what amount of tax will a person have to pay whose property is valued at $7500?

2. If a tax of $120 is assessed on a mill valued at $24,000, what is the valuation of a residence that is taxed $17.75 at the same rate?

3. The tax assessed upon a town is $20,914.80; the town contains 2580 polls, taxed $.62½ each, and has a real estate valuation of $4,062,000, and a valuation of personal property to the amount of $227,400. Find the rate of taxation, and C's tax, who pays for 4 polls, and whose property is assessed at $15,000.

4. My son and daughter each attended school 214 days, and the expense, including teacher's wages and incidentals, was paid by a rate bill. How much must I pay if the teacher's wages amounted to $440, fuel and repairs $101.50, and janitor's fees $74.75, the total number of days' attendance being 7460?

5. A tax of $24,000 is levied upon a town which has taxable property with the assessed valuation of $1,500,200, and which contains 520 polls, assessed at $1.25 each. The town receives from the state $4000 as its share of the corporation taxes. Find the rate of taxation, and the amount of tax to be paid by James Brown, who is assessed $4200 on his real estate and $1800 on his personal property, and who pays for 4 polls.

6. The cost of maintaining the public schools of a city during the year 1903 was $ 112,000, and the taxable property of the city was $ 44,800,000. How many mills on a dollar must be assessed for school purposes? If 10% of the tax cannot be collected, how many mills on a dollar must then be assessed?

7. A tax of $ 13,943.20 is assessed upon a town containing 860 taxable polls; the real estate is valued at $ 2,708,000, and the personal property at $ 151,600. If the polls be taxed $ 1.25 each, what will be the rate of property taxation, and what will be the tax of Frederick Benton, who pays for 3 polls, and has real and personal estate valued at $ 23,750?

8. In a school district the valuation of the taxable property is $ 752,400, and it is proposed to repair the schoolhouse and ornament the grounds at an expense of $ 5000. If old materials sell for $ 673.70, what will be the rate per cent of taxation, and what will be A's tax, whose property was valued at $ 9400?

9. The assessed valuation of the real estate of a county is $1,910,887, of the personal property, $ 921,073, and it has 4564 inhabitants subject to a poll tax. The year's expenses are: for schools, $ 8400; interest, $ 6850; highways, $ 7560; salaries, $ 5150; and contingent expenses, $ 13,675. If the poll tax was $ 1.50, and the revenue from fairs and licenses $ 6200, what must be levied on a dollar to meet expenses and provide a sinking fund of $ 7000?

10. In a certain town there are 680 polls. The assessed valuation of the real estate is $ 850,000, and of the personal property $ 750,000. The poll tax is $ 1.50 per poll, and the tax on the property is $1\frac{1}{2}$%. Only 98% of the whole tax can be collected, and the collector is paid $2\frac{1}{2}$% of the amount collected. How much does the town receive from the tax, and how much does the collector receive for his services?

NOTE. It is suggested that the teacher give additional examples in taxes according to the regulations of his own town or city. These regulations can generally be obtained with little trouble from the local tax collector or other officers of the local government. The work on the next three pages has been selected to give practice, if desired, in examples under regulations of more than usual intricacy.

868. The following regulations as to assessments and taxes are those for Philadelphia under the laws of the state of Pennsylvania:

TAX RATES

1907

Real Estate, full city rate, $1.50 ℔	$100
Real Estate, suburban, $1.00 ℔	100
Real Estate, farm, $.75 ℔	100
Horses, Mules, and Cattle, $1.50 ℔	100
Vehicles for Hire, State Tax,	4 Mills
Money at Interest, State Tax,	4 Mills

APPORTIONED AS FOLLOWS

Poor,	5
School,	23
Lighting City,	9
Loan,	30
Highways,	7
Water,	10
Police,	18
Markets and City Property,	5
Fire,	7
Prisons,	2
City Commissioners,	6
Health,	2
Expense of Municipality,	26
	$1.50

DISCOUNT ON CITY TAXES

One per cent discount on Bills paid during January, February, and March.

After the last day of March a discount **at the rate of 1 per cent per annum** on all bills paid **ON OR BEFORE JUNE 30.** Beginning April 1 with ¾ of one per cent, and decreasing daily until June 30, when discount will be ½ of one per cent. No discount or penalties on bills paid during July and August.

PENALTIES ON CITY TAXES

After August 31, 1 per cent.
After September 30, 2 per cent.
After October 31, 3 per cent.
After November 30, 4 per cent.

After December 31, Delinquent.

No Discount is allowed on State Tax, but a Penalty of 5 per cent is added after July 31st.

Every person twenty-one years of age and upwards, being a resident of or domiciled within this State, and every corporation not specially exempted, and every co-partnership or unincorporated association, joint stock association or company, limited partnership, and co-partnership, located or doing business within this Commonwealth, owning or holding any personal property of the classes enumerated in Section 1 of the Act of June 8, 1891 [consisting of horses, cattle, vehicles to hire, and money at interest], whether the same be held in his, her, or its own right or as Trustee, Executor, Administrator, Guardian, Assignee, Committee, Receiver, or in any other Fiduciary capacity for the use and benefit of some other person or corporation, is required each year to make return, under oath, of the amount of such property, to the Assessor.

In case no return is made within ten days the assessors are required to make one, to which estimated return 50 per cent is to be added subject to appeal as provided by law.

HOUSE No.	NAME OF OWNER AND DESCRIPTION OF PROPERTY	REAL ESTATE VALUE DOLLARS	MONEY AT INTEREST DOLLARS	HORSES		CATTLE		CARRIAGES TO HIRE		REMARKS
Chestnut Street				No.	Value Dollars	No.	Value Dollars	No.	Value Dollars	
1420–1422	John H. Carr, 13 story brick building, offices and stores	840,000								
1520–1522	H. R. F. Baker, 4 story marble front building	575,000								
1524–1526	Wm. Weightman est., 10 story iron and stone building	535,000								
1620	William H. Embeck, stone and brick building	95,000								
1910–1922	Joshua B. Lippincott estate, "Aldine Hotel"	900,000								
1400–1408	The Land Title Trust Co., 15 story building	1,620,000	4,207,221							
2002	Mary B. Evans and Rebecca L. Bringhurst, residence	25,000	58,768	1	100					
2116	Mary A. McKeehan, 4 story brick house	40,000	20,000							
1511	William Blanchard estate	110,000								
	Anna Blanchard		472,400	2	200					
1601	George A. Huhn		100,000							
	4 story stone house	190,000								
1607–1609	Ger. Life & Tr. Co. & M. P. Harris, trustees, 4 sto. house	123,000								
	Mary P. Harris		1,246	1	100	2	100			
1809	Thomas Dolan		1,816,140	4	400	4	200			
1423	William Ellis and L. D. C. Ellis			14	1,600			7	1,400	
	Brick and iron stable	40,000								
City	Corporation of Haverford College	12,300								
Suburban	2 sto. stone house, 2 sto. frame house, about 59 acres	29,000								
Farm	About 56 acres	280,000								
59th st.	John N. McGarvey, lot 179¼ x 480 to Haddington st.									
City	Landsdowne avenue front 179 x 100	4,500								
Farm	Balance in rear	11,000								
City ave.	Joseph R. Rhoads		10,000	3	200	2	50			
Suburban	3 sto. stone house, 2 sto. stable and shop, irreg. lot	50,000								
		5,479,800	6,685,775		2,600		350		1,400	

869. In the assessment book, p. 332, the full city rate of $ 1.50 per $ 100 applies to the improved real estate when not otherwise specified. The suburban rate (see table, p. 331) is levied on "unimproved property" — chiefly areas relatively small fronting on unimproved streets. The farm rate is levied on tracts of relatively large area used for farming and usually lying in the rear of lands fronting upon some improved street. The state tax of 4 mills per dollar on vehicles to hire and money at interest is collected by the city and sent to the state; but the state then returns ¾ of it to the city.

WRITTEN EXERCISE

1. From the table of rates and the assessment book, pp. 331, 332, how much tax is levied on the property 1400–1408 Chestnut St.?

2. What is Thomas Dolan's total tax? If he had failed to make return for the value of his property, how much would his tax have been? (See last paragraph on p. 331.)

3. What is the total tax of the persons listed on p. 332?

4. How much of this total tax would the state finally keep?

5. If the trustees of the William Blanchard estate did not pay the real estate tax till Dec. 5, how much would the penalty be?

6. What is the penalty if Anna Blanchard does not pay her tax till Aug. 5?

7. If the Land Title Trust Co. paid their real estate tax in March, what was the discount? How much did they have to pay?

8. If the Corporation of Haverford College paid their city tax March 20, their suburban tax March 30, and their farm tax July 5, what was the total discount?

NOTE. The teacher can easily vary the number of examples as desired, by replacing example 3 with a few specific examples, and by giving additional examples based on the regulations for discounts, penalties, etc.

CUSTOMHOUSE BUSINESS

870. A **customhouse** is an office established by the national government for the transaction of business relating to duties, or customs, and for the entry and clearance of vessels.

871. A **port of entry** is a port at which a customhouse is established for the legal entrance of vessels and merchandise.

The waters and shores of the United States are divided into collection districts in each of which there is a port of entry which is also a port of delivery; other ports than those of entry may be specified as ports of delivery. Duties are paid (or secured to be paid), and clearances made, at ports of entry only, but after vessels have been properly entered, their cargoes may be discharged at any port of delivery.

872. **Duties**, or **customs**, are taxes levied by the national government upon imported goods. They are of two kinds, *ad valorem* and *specific*.

873. An **ad valorem duty** is a certain per cent levied on the appraised value of the goods, which is the market value in the country from which they are imported.

Ad valorem duties are not computed on fractions of a dollar; if the cents in an invoice are less than 50, they are rejected; if 50 or more, they are counted as another dollar. On pages 338–340 it is assumed (unless otherwise stated) that the appraised value corresponds to the invoiced cost.

874. A **specific duty** is a tax levied upon the number, weight, or measure of goods, regardless of their value; as, a fixed sum per bale, ton, barrel, etc.

Upon some goods both specific and ad valorem duties are levied. Before specific duties are finally determined, allowances are made for tare, leakage, etc.

875. An **invoice**, or **manifest**, is a written account of the particular goods sent to the purchaser or consignee, showing the quantity and the actual cost or value of the goods.

All invoices must be made out in the weights and measures of the place or country from which the goods are imported, and in the currency of that country or in the currency actually paid for them.

When the value of foreign currency is fixed by law, such value must be taken in estimating the duties.

876. The value in United States money of the foreign currency of the different nations of the world is proclaimed by the Secretary of the Treasury every three months. The following values of foreign coins were proclaimed Apr. 1, 1907:

VALUES OF FOREIGN COINS

COUNTRY	STANDARD	MONETARY UNIT	VALUE IN U. S. GOLD
Argentina	Gold	Peso	$.965
Austria-Hungary	Gold	Crown	.203
Bolivia	Gold	Peso	.9733
Brazil	Gold	Milreis	.546
Chile	Gold	Peso	.365
Denmark, Norway, Sweden	Gold	Crown	.268
Egypt	Gold	Pound	4.943
France, Belgium, Switzerland	Gold	Franc	.193
German Empire	Gold	Mark	.238
Great Britain, India	Gold	Pound sterling	4.8665
Japan	Gold	Yen	.498
Mexico	Gold	Peso	.498
Netherlands	Gold	Florin	.402
Newfoundland	Gold	Dollar	1.014
Philippine Islands	Gold	Peso	.50
Portugal	Gold	Milreis	1.08
Russia	Gold	Ruble	.515
Turkey	Gold	Piaster	.044
Uruguay	Gold	Peso	1.034

The *drachma* of Greece, the *lira* of Italy, the *peseta* of Spain, the *bolivar* of Venezuela are of the same value as the franc. The *dollar*, of the same value as our own, is the standard of the British possessions of North America (except Newfoundland), of Liberia, and of Colombia. The *libra* of Peru has the same value as the British *pound sterling*. The *gourde* of Haiti has the same value as the *peso* of Argentina.

877. A **tariff** is a schedule of goods, and the legal rates of import duties imposed by law on the same.

878. A **free list** is a list of such articles as are exempt from duty.

879. **Tonnage** is a tax levied upon a vessel, independent of its cargo, for the privilege of coming into a port of entry.

880. Duties are collected at the port of entry by a customs officer appointed by the United States government, and known as the **Collector of the Port**. Under him are deputy collectors, surveyors, and appraisers, and many inspectors, weighers, gaugers, etc.

881. A **naval officer**, appointed only at the more important ports, receives copies of all manifests, countersigns all documents issued by the collector, and certifies his estimates and accounts.

882. The **surveyor** is the outdoor executive officer of the port. He supervises the inspectors, controls the unlading of foreign merchandise, etc.

883. The **appraiser** examines imported merchandise and determines its dutiable value; that is, the foreign market value at the time of exportation.

884. The **public store** is a place provided for the examination of imported merchandise.

One package of every invoice of merchandise, and at least one package out of every ten similar packages, must be sent to the public store for examination.

Bulky and heavy articles are examined at the wharf where they are unloaded.

Weighable and gaugeable goods paying only specific duties are seldom sent to the public store for examination.

885. **Warehousing** is the depositing of imported goods in a government or bonded warehouse.

886. A **bonded warehouse** is a place provided by law for the storage of dutiable merchandise.

Goods may be withdrawn from a bonded warehouse for export without the payment of the duties. If goods on which the duty, amounting to $50 or more, has been paid are exported, the amount of duty, less 1 %, is refunded; the sum so refunded is called a *drawback*.

887. **Smuggling** is the act of bringing foreign goods into a country illegally without paying the required duty.

Smuggling is a crime, for the prosecution and punishment of which stringent laws are enacted.

888. A **customs broker** is a person familiar with customs law and practice, who makes entries and transacts similar business for importers. He frequently acts as agent or attorney for his principal.

889. **Tare** is an allowance made for the box, bag, crate, or other covering of the goods. **Leakage** is the allowance made for waste of liquids imported in barrels or casks.

890. The **gross weight** is the weight before any allowances for tare, etc., are made.

891. **Net weight** is the weight after all allowances have been made.

The ton by law consists of 2240 avoirdupois pounds in all cases where it is used in the customs.

892. To find a specific duty.

893. **Example.** Find the specific duty on 160 gallons of wine at $2 per gallon; leakage, 10%.

SOLUTION

10% of 160 gallons = 16 gallons leakage.
160 gallons − 16 gallons = 144 gallons, the net quantity.
$2 × 144 = $288, the specific duty.

WRITTEN EXERCISE

1. Find the total specific duty on 1250 bushels barley at 30¢ per bushel; 400 bushels onions at 40¢ per bushel; 1260 pounds cheese at 6¢ per pound; 2500 bushels wheat at 25¢ per bushel.

2. Find the total duty on the following: 900 pounds unground cayenne pepper at 2½¢ per pound; 1200 bushels malt at 45¢ per bushel; 800 pounds butter at 6¢ per pound.

3. If the duty on plate glass is 8¢ per square foot, how much will be the charge on an importation of 175 boxes, each containing 25 plates 16 by 24 inches in size?

4. Find the specific duty on 1656 pounds macaroni, at 1½¢ per pound; 900 pounds hops at 12¢ per pound; 3150 pounds filler tobacco, unstemmed, at 35¢ per pound; and 165 pounds hemp cordage at 2¢ per pound.

5. After being allowed 10% for leakage, a wine merchant paid $864 duty at $2 per gallon, on 12 casks of wine. How many gallons did each cask originally contain?

894. To find an ad valorem duty.

895. Example. What is the duty on an invoice of leather goods from Vienna, the dutiable value being 15,240 crowns and the rate of duty 35% ad valorem?

SOLUTION

$.203 = the value on 1 crown in United States money.
$.203 × 15,240 = $3093.72, the dutiable value in United States money.
35% of $3094 = $1082.90, the ad valorem duty.

WRITTEN EXERCISE

1. Find an ad valorem duty of 35% on an importation invoiced at 17,450 francs.

2. What is the duty at 50% ad valorem on a consignment of 650 dozen cotton gloves invoiced at 90 francs per dozen?

3. Find the duty at 60% ad valorem on 3 cases of silk goods from Berlin, invoiced at 4692 marks each.

4. I imported from England 20 cases woolen goods, weighing 390 pounds each; tare 10%; invoiced at £410 per case. What was the total duty at 44¢ per pound and 60% ad valorem?

5. I received by steamer *Raglan* from Liverpool the following invoice of goods: 768 yards velvet, invoiced at £1, 12*s.* per yard; 2150 yards lace, invoiced at 3*s.* 4*d.* per yard; 1200 yards broadcloth, invoiced at 15*s.* per yard; 3520 yards carpet, invoiced at 11*s.* 6*d.* per yard. If the duty on the velvet was 60%, on the lace and broadcloth 35%, and on the carpet 50%, how much was the total duty to be paid?

WRITTEN REVIEW

1. What is the duty on 1000 yards Brussels carpet 27 inches wide, invoiced at 6*s.* 9*d.* per yard; duty 28¢ per square yard specific and 40% ad valorem?

2. If the duty on flannel is 22¢ per pound specific and 30% ad valorem, how much must be paid on an invoice of 2150 yards, weighing 420 pounds, and valued in Canada, whence it was imported, at 75¢ per yard?

3. Find the duty at 40% ad valorem on 3 dozen clocks, invoiced at $21.50 each, and 6 dozen watches, invoiced at $35 each.

4. Find 35% ad valorem duty on 250 cases German toys invoiced at 175 marks per case.

5. I imported from Belgium 300 meters Brussels carpet, ¾ of a yard wide, at 5 francs per meter. I paid a specific duty of 28¢ per square yard and an ad valorem duty of 30%. What was the total duty?

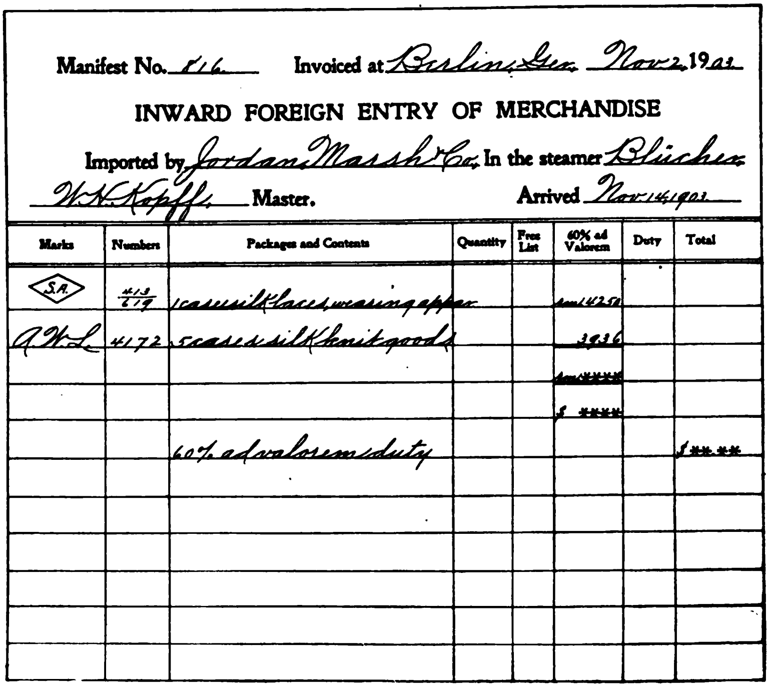
Note. A meter is equivalent to 1.0936 yards.

6. What is the amount of duty chargeable on 4000 pounds worsted yarn invoiced at £490, when the rate of duty is 38½¢ per pound and 40% ad valorem?

7. A merchant imported 10 gross table knives costing 15*s.* per dozen in Sheffield, England. What was the duty at $2.40 per dozen specific and 40% ad valorem?

Find the dutiable value and compute the duty on the following entries of merchandise:

8



Manifest No. 816 Invoiced at Berlin, Ger. Nov. 2, 1901

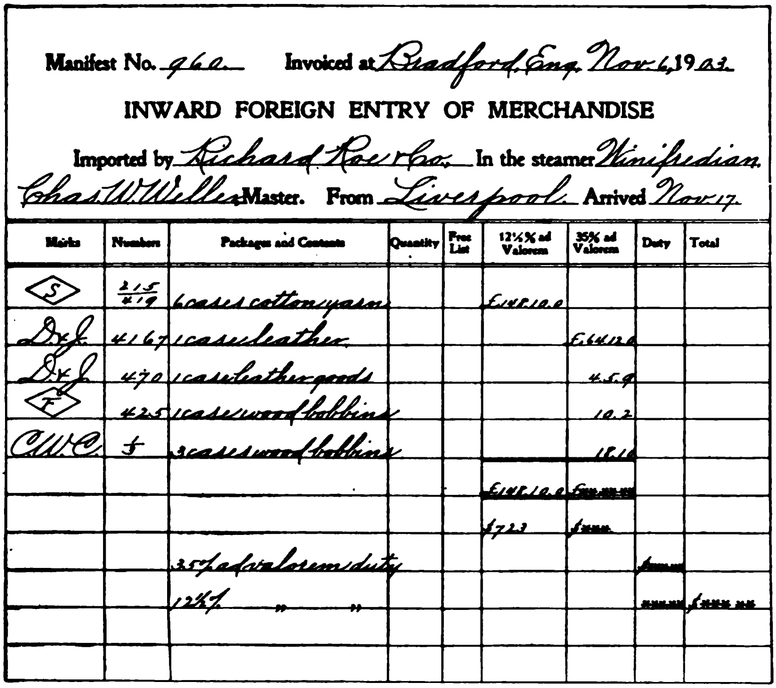
INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by Jordan, Marsh & Co. In the steamer Blücher

W. H. Kopff, Master. Arrived Nov. 14, 1901

Marks	Numbers	Packages and Contents	Quantity	Free List	60% ad Valorem	Duty	Total
◇ S.A.	413/619	1 case silk laces, wearing apparel			M 14250		
A.W.L.	4172	5 cases silk knit goods			3936		
					M ****		
					$ ****		
		60% ad valorem duty					$**.**

9.



Manifest No. 960 Invoiced at Bradford, Eng. Nov. 6, 1902.

INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by Richard Roe & Co. In the steamer Winifredian.
Chas. W. Welles Master. From Liverpool. Arrived Nov. 17.

Marks	Numbers	Packages and Contents	Quantity	Free List	12½% ad Valorem	35% ad Valorem	Duty	Total
◇S	215/419	6 cases cotton yarn			£148.10.0			
D&J	4167	1 case leather				£64.12.0		
D&J	470	1 case leather goods				4.5.9		
◇F	425	1 case wood bobbins				10.2		
C.W.C.	5	3 cases wood bobbins				18.16		
					£148.10.0	£**.**.**		
					$723	$****		
		35% ad valorem duty					$****	
		12½% " "					****.**	$****.**

10. A merchant imports 1200 yards Brussels carpet, ¾ of a yard wide, invoiced at £200. Compute a duty of 28 ¢ per square yard and an ad valorem duty of 40%. If freight charges and losses aggregated $185.50, at what price per yard must the carpet be sold to gain 20%?

11. A Boston merchant imported mandolins invoiced in Germany at 40 marks each. If he paid an ad valorem duty of 45%, what price must he sell them for to gain 20% on the cost?

EXCHANGE

896. **Exchange** treats of methods of making payments at distant places without the transmission of money.

Settlements are effected by means of written orders called *bills of exchange*, *express money orders*, *telegraphic money orders*, *letters of credit*, etc., and the risk and expense of sending the money itself is avoided.

897. An **exchange center** is some recognized money center.

The principal exchange centers of the United States are New York, Boston, Philadelphia, Chicago, St. Louis, Baltimore, Cincinnati, and San Francisco; of Europe, London, Paris, Antwerp, Geneva, Amsterdam, Hamburg, Frankfort, Berlin, and Vienna.

898. Exchange is of two kinds: *domestic,* or *inland,* and *foreign.*

DOMESTIC EXCHANGE

899. **Domestic exchange** is exchange payable in the country in which it is drawn.

Domestic bills of exchange are commonly called *drafts.*

The business of making payments by means of drafts and bills of exchange is usually conducted through the medium of banks and bankers.

900. Funds may be remitted from one place to another place in the same country in six different ways without the transmission of money, as follows:

1. By a postal money order.
2. By an express money order.
3. By a telegraphic money order.
4. By a bank draft.
5. By a check.
6. By a sight draft of a creditor on a debtor.

901. A **postal money order** is an order drawn by the postmaster, or his clerk, at one office, directing the postmaster of another office to pay to the person named in his private letter of advice the sum specified in the order.

Applications for postal money orders must be in writing, and must state the amount of each order wanted, the name and address of the person to whom the order is to be paid, and the name and address of the remitter.

At the present time the maximum amount for which a single postal money order may be issued is $100, and the rates charged are as follows:

$2.50 or less	3¢.	$30.00 to $40.00 . . .	15¢.
$2.50 to $5.00 . . .	5¢.	$40.00 to $50.00 . . .	18¢.
$5.00 to $10.00 . . .	8¢.	$50.00 to $60.00 . . .	20¢.
$10.00 to $20.00 . .	10¢.	$60.00 to $75.00 . . .	25¢.
$20.00 to $30.00 . .	12¢.	$75.00 to $100.00 . . .	30¢.

The payee who desires a money order to be paid to another person must fill out and sign the form of transfer which appears on the face of the order. More than one transfer is prohibited by law.

If a money order is lost, a certificate should be obtained from both the paying and issuing postmasters stating that it has not been paid and will not be paid. The Post Office Department at Washington will then issue another order upon application.

902. An **express money order** is an order drawn by the agent of the express company at any given office directing another agent of the company at some designated place to pay to the person named therein a certain sum of money.

Express money orders may be obtained for any number of dollars, and the rates at the present time are the same as for postal money orders.

Express money orders are transferable by indorsement, the same as notes, checks, etc.

903. A **telegraphic money order** is an order drawn by a telegraph agent at any given office instructing the agent at some designated office to pay to the person named in the telegraphic message the sum specified, upon his personal application and proper identification.

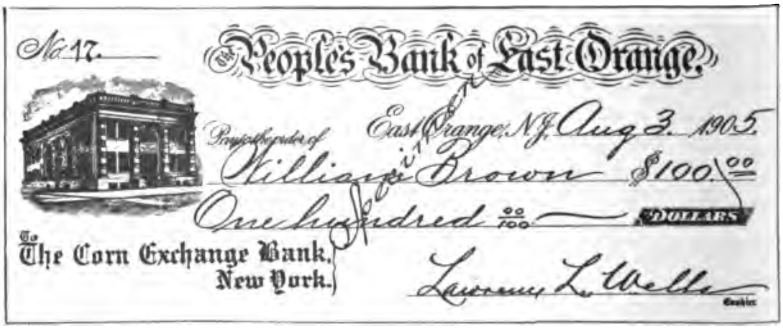
At the present time telegraphic transfer rates are as follows:

$50 or less . . .	50¢.	$200 to $300 . .	$1.50.
$50 to $100 . . .	1%.	$300 to $400 . .	$1.75.
$100 to $200 . . .	$1.25.	$400 to $500 . .	$2.00.

Over $500, special rates.

The rates in the above table are entirely apart from the cost of telegraphic service, which is based upon distance and the number of words contained in the message.

904. A **bank draft** is an order written by one bank directing another bank to pay a specified sum of money to a third party, or to his order.



No 17.

The People's Bank of East Orange.

East Orange, N.J. Aug 3. 1905

Pay to the order of William Brown $100.00

One hundred 00/100 DOLLARS

To The Corn Exchange Bank,
New York.

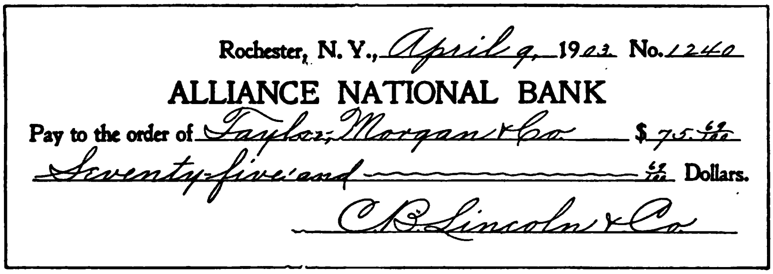
Lawrence L. Wells
Cashier

905. Nearly all banks keep money deposited with some other bank, called a **correspondent**, at one or more commercial centers against which they draw drafts to sell to their customers for remittance purposes. These drafts pass as cash in the section tributary to the commercial centers upon which they are drawn.

Banks usually make a charge called **exchange** for the trouble of keeping funds on deposit at commercial centers and drawing drafts against these funds. These charges range from $\frac{1}{10}\%$ to $\frac{1}{4}\%$. On many small drafts a definite charge ranging from 10¢ to 50¢ is frequently made. Some banks make no charge for drafts sold to regular depositors.

906. Instead of making remittances by bank drafts merchants frequently send their personal checks in payment of bills.

907. A **check** is an order on a bank by a depositor for the payment of money; except that it is drawn by a person, it is very much like a bank draft.



Rochester, N. Y., April 9, 1903 No. 1240

ALLIANCE NATIONAL BANK

Pay to the order of Taylor, Morgan & Co. $ 75.$\frac{69}{100}$

Seventy-five and ——————— $\frac{69}{100}$ Dollars.

C. B. Lincoln & Co.

908. **Commercial drafts** play a prominent part in facilitating the payment of bills at distant places.

Commercial drafts, which include sight and time drafts, were discussed on pages 260 and 261.

Exercises in discounting time drafts were given in connection with bank discount, pages 269 and 270.

909. Formerly domestic exchange was at a premium or discount in the city where purchased according as the balance of trade between that city and the one on which the draft was drawn was in favor of or against the former city. If the drawer city owed the drawee city, exchange on the latter would be at a premium at the former place; if the balance of trade was in favor of the drawer city, the condition of exchange would be reversed in the two places. For a number of years past, however, domestic exchange has been practically at par.

910. Bankers usually make a charge called **collection** for collecting out-of-town drafts deposited with them.

911. Sometimes unaccepted time drafts are left with a bank for collection, and sometimes they are offered for discount.

912. Banks are usually willing to accept for discount the unaccepted drafts of responsible parties when they are properly indorsed.

WRITTEN EXERCISE

1. W. J. Boone & Co., of San Francisco, Cal., have bills to pay as follows: T. W. Brooke, Dayton, O., $650; E. L. Greyson & Son, Cedar Rapids, Ia., $46.53; Barnes & Snyder, Bolton, Mo., $48.50; and their traveling salesman, W. H. Post, is wanting $100 for expenses at Denver, Col. They pay the amounts by remitting as follows: T. W. Brooke and E. L. Greyson & Son, express money orders; Barnes & Snyder, postal money order; and W. H. Post, by telegraphic transfer in a ten-word message. If the cost of the telegram was 50¢, what was the total amount required?

2. Barnum & Co., of St. Paul, drew a sight draft of $1400 on Martin & Cole, 415 High St., Boston, on account of an invoice of hides shipped to them valued at $3000, as per bill of lading attached to the draft. They sold the draft at the bank at $\frac{1}{2}\%$ discount. What were the proceeds?

3. A commission merchant of Charleston, S.C., bought a ninety-day commercial draft at $\frac{1}{2}\%$ discount for $800 drawn on a Boston firm.. If money be worth 6%, what did the draft cost him?

SOLUTION

$.015 = the bank discount on $1 for 90 da.
$.005 = the commercial discount on $1.
$.015 + $.005 = $.02, the total discount on $1.
$1 – $.02 = $.98, the proceeds of $1.
$.98 × 800 = $784, the cost of the draft.

4. A commission merchant holds, subject to the order of his principal, $5005. His principal directs him to remit the amount by New York draft after deducting the cost of the draft. If the bank charges exchange at the rate of $\frac{1}{10}$%, what will be the face of the draft?

5. Gates & Son, of Memphis, drew a sight draft on Perrin & Boon, Portland, Me., for $8750.85, which they sold at the Cotton Exchange Bank at $\frac{5}{8}$% discount. How much were the proceeds?

6. I drew a 60-day draft on one of my customers and sold it to a broker at $\frac{3}{4}$% discount, receiving $1354.18 as proceeds. What was the face of the draft, money being worth 6%?

7. Jno. W. Williams, of Boston, Mass., remitted Janis Bros. & Co., of Milwaukee, $1750 by draft on New York, exchange 15¢ per each $100; Martin & Co., of Allentown, Pa., by American Express money order, $ 89.75; and Theodore Emens, $28.50, by post office money order. Find the total cost of exchange.

8. Thomas, Bailey & Co., of St. Louis, drew a sight draft for $1900 on Slocum, Wilde & Co., 291 Milk St., Boston, Mass., on account of an invoice of molasses shipped them, valued at $3500, as per bill of lading attached to draft. They sold the draft at a bank at $\frac{1}{2}$% discount. What were the proceeds?

9. A wholesale grocer owed for an invoice of $5425.40, purchased in New York, subject to a discount of 6% if paid within 10 days. Within the required time he discounted the bill and remitted for balance as follows: A sight draft which he bought of E. M. Brooks on Gunn & Baker for $4000, at $\frac{1}{2}$% discount, and a bank draft for the remainder, the exchange being 10¢ for each $100. How much was required to settle the bill, and how much was gained by discounting it?

10. Hedman & Son drew a 60-day draft on Johnson Manufacturing Co. for $2500, and had it discounted at a bank at 6%. If the rate of collection was $\frac{1}{8}$ %, what were the proceeds of the draft?

FOREIGN EXCHANGE

913. Foreign exchange is exchange payable in another country than that in which it is drawn. It is by means of the system of foreign exchange that the people of the various nations pay their debts to one another.

914. The business of foreign exchange was brought about by the fact that goods are exported and imported by the nations of the earth, and the fact that investors put money in the enterprises and securities of nations foreign to their own.

During the year 1906 the goods imported by the United States amounted to $ 1,226,563,843, and the goods exported to $ 1,717,953,382 ; hence that year foreign countries owed us $ 491,389,539 for goods we exported in excess of what we owed them for goods imported. The imports and exports were paid for chiefly through the medium of foreign exchange. This same means is used in paying the sums invested by Americans in foreign countries, the sums invested by foreigners in this country, the amounts spent by Americans abroad and by foreigners traveling in this country, and the sums of money sent by foreigners in this country to their families in Europe. The grand total of our foreign exchange is thus extremely large.

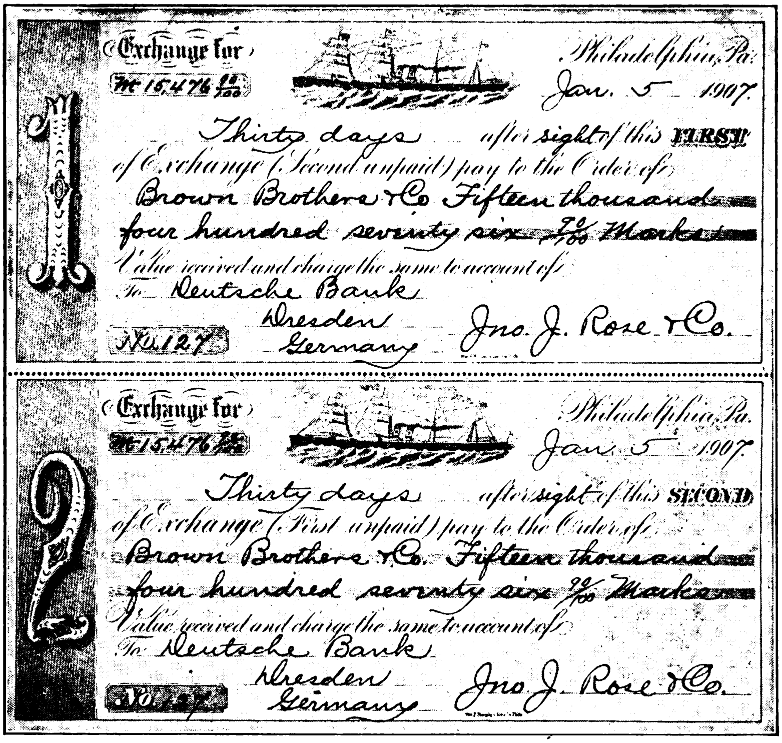
915. The following are some of the more common forms of foreign exchange: a draft, check, bill of exchange, money order, circular letter of credit, traveler's cheque, an order (either written or cabled) to pay certain persons money in some foreign country. The transportation of gold or specie from one country to another is also an important part of the system of foreign exchange.

Exchange of any kind whatever may be made payable in the money of the country in which payment is to be received, or in the money of a country in which a great financial city is located. London is the greatest financial center of the world, and many drafts on Germany, France, Norway, Sweden, Russia, China, India, and other countries are payable in sterling exchange. New York is the financial center of all America, and many drafts or bills of exchange drawn on Canada, Mexico, and the countries of South America are payable in New York funds.

916. A **bill of exchange** is a draft drawn payable in a foreign country. If it is drawn to cover the value of goods exported, a bill of lading and an insurance certificate usually accompany it; such a bill is known as a **documentary bill of exchange.** If the bill of lading and the insurance certificate are not attached, the bill of exchange is known as a **clean bill of exchange.**

917. Frequently bills of exchange are drawn in sets of two or sometimes three, called *first, second,* and *third of exchange.* When bills are drawn in sets of two they are sent by different mails, so that if one is lost the other may be presented. If three bills are drawn, the third one is kept by the purchaser as a memorandum.

A SET OF EXCHANGE



Exchange for

M 15,476 $\frac{90}{100}$

Philadelphia, Pa.

Jan. 5 1907

Thirty days after sight of this FIRST of Exchange (Second unpaid) pay to the Order of Brown Brothers & Co Fifteen thousand four hundred seventy six $\frac{90}{100}$ Marks

Value received and charge the same to account of

To Deutsche Bank

Dresden

Germany

No. 127

Jno. J. Rose & Co.

Exchange for

M 15,476 $\frac{90}{100}$

Philadelphia, Pa.

Jan. 5 1907

Thirty days after sight of this SECOND of Exchange (First unpaid) pay to the Order of Brown Brothers & Co. Fifteen thousand four hundred seventy six $\frac{90}{100}$ Marks

Value received and charge the same to account of

To Deutsche Bank

Dresden

Germany

No. 127

Jno. J. Rose & Co.

918. Bills of exchange are many times used as a means of collecting debts due in foreign countries. The method employed is similar to that used in collecting debts by means of commercial drafts.

919. The **par of exchange** is the established value of the standard unit of money of one country expressed in that of another. It is of two kinds: *intrinsic* and *commercial.*

920. The **intrinsic par of exchange** is the real or intrinsic value of the coins of one country as compared with those of another.

Thus, the pound sterling of Great Britain contains 113 grains of pure gold, and the dollar of the United States contains 23.22 grains of pure gold. Since 113 grains are 4.8665 times 23.22 grains, the pound sterling is worth 4.86\frac{65}{100}$.

921. The **commercial par of exchange**, commonly called the *course of exchange*, is the market value of the standard unit of money of one country in the currency of another.

922. The course of exchange is usually governed by the relative state of indebtedness of merchants of different countries and the supply of gold and silver; hence it may be at a **premium** or at a **discount**.

If the merchants of England owe the merchants of the United States more than is usual, exchange in London on New York quickly advances to a high rate, while exchange in New York on London declines.

923. Normally foreign exchange rates fluctuate between gold-exporting and gold-importing points. If it costs less to export gold than to buy a bill of exchange, the gold itself is shipped. However, gold is seldom actually shipped in quantities of less than $20,000.

If the rate paid for demand sterling in New York drops below $4.84½ per pound, it is cheaper to import gold in large quantities than to have remittances made from abroad by means of draft. If exchange in New York goes above $4.88½ per pound sterling, then gold can be exported at less cost than to buy a draft on London.

924. In the exportation and importation of gold there must be considered the risk of loss by shipwreck, the loss because of abrasion, the cost of transportation, and the charge for insurance.

The cost of sending gold from New York to London is usually as follows: Insurance and freight, each ⅛%; abrasion on $5 gold pieces, ¼%; on $10 gold pieces, ⅛%; on $20 gold pieces, $\frac{1}{64}$% to ⅛%.

925. Quotations of foreign exchange are given by means of market value equivalents, no reference being made to the intrinsic par value.

926. Exchange on Great Britain is quoted by giving the exchange value of £1 in dollars and cents.

Thus, when exchange on London is quoted at 4.86¾, a bill for £1 will cost $4.86¾.

927. Exchange on France, Belgium, and Switzerland is usually quoted by giving the exchange value of $1 in francs.

Thus, when exchange on Paris is quoted at 5.18, $1 will buy 5.18 francs. Notice that such quotations differ from all others in character; a change from 5.18 to 5.19 is a *fall* in the rate.

928. Exchange on the Netherlands is quoted by giving the exchange value of 1 guilder, or florin, in cents.

When exchange on Amsterdam is quoted at 41, 1 guilder is equal to 41 cents.

929. Exchange on Germany is quoted by giving the exchange value of 4 reichsmarks in cents.

When exchange on Berlin is quoted at $96\frac{1}{2}$, 4 reichsmarks are equal to $96\frac{1}{2}$ cents.

930. Dealers in foreign exchange refer to changes in the quotations as being of so many **points**. A point is one hundredth of a cent, or one unit in the fourth decimal place.

If the quotation for London exchange is 4.8255, the addition of 75 points would make it 4.833.

931. Quotations are sometimes made in the form $93\frac{1}{2}+\frac{1}{32}$ or $93\frac{1}{2}-\frac{1}{32}$, or $5.17-\frac{1}{16}$, or the like. The fraction means the fraction of 1% of the rate. A + fraction always makes the rate higher; a − fraction, lower.

Thus $93\frac{1}{2}-\frac{1}{32}$ means \$.935 − $\frac{1}{32}$% of \$.935, or \$.93471; but $5.17-\frac{1}{16}$ means Fr. 5.17 + $\frac{1}{16}$% of Fr. 5.17, or Fr. 5.17323. See **927**.

932. The law of England requires that **revenue stamps** be affixed to all drafts drawn for more than five days' sight. Revenue stamps are required also on drafts for more than three days' sight on Holland, and on drafts at one or more days' sight on France or Germany; but the three days' sight "letters of delegation" on Germany are exempt. The required amount of stamps is $\frac{1}{20}$% of the face of the draft; their cost may therefore be included by adding $\frac{1}{20}$% to the quoted rate.

933. A **cable transfer** is a telegraphic order to pay a certain person in a foreign country a certain sum of money. Such transfers are made by cipher codes. The rate charged for a cable transfer is the quoted or market rate of exchange, plus a commission of about $\frac{1}{4}$%, plus the telegraph and cable charges.

By cable transfer, a merchant who desires to ship wheat to London can complete the transaction in a few hours. He can ship the wheat, telegraph the fact to the consignee at London, obtain particulars concerning the condition of the market, and, if he thinks best, have the wheat sold at once, "to arrive," and the proceeds remitted through a London banker. A bill does not appear in the transaction.

A very large amount of business in foreign exchange is done by means of the cable. Operators in this line conduct their business upon a very close margin, and calculate the outcome of their transactions to a nicety. This is possible because there is little delay during which the rates might materially change so as to cause a loss. During a part of each day the cable enables persons in New York, Philadelphia, Chicago, London, Berlin, and other money centers, to conclude transactions without material delay.

934. **Arbitrage of exchange** is the calculation of the relative values of exchange at the same time at two or more places with the purpose of taking advantage of the difference in price. It is conducted (largely and most profitably by cable) by buying simultaneously in the cheaper and selling in the dearer market.

Suppose that a merchant owes a debt of £1500 in London, and that direct exchange on London is 4.87½, and direct exchange on Paris is Fr. 5.24¼ to the dollar, and that Paris exchange on London is Fr. 25 to the pound. Then Fr. 37,500 in Paris will purchase £1500. Fr. 37,500 will cost, in U. S. money at 5.24¼, $7149.67. Direct exchange on London at $4.87½ for £1500 = $7312.50; 7312.50 − $7149.67 = $162.83, amount gained by arbitrage or indirect exchange.

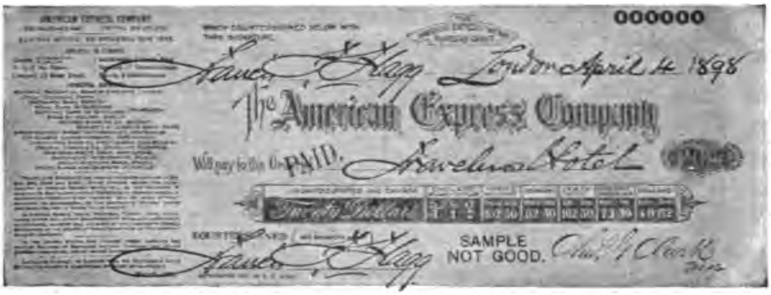
If the German mark could, because of some condition, purchase more sterling in proportion in Berlin than could the franc in Paris, then the merchant would gain still more by buying marks at the lowest rates. But if the mark could be had for less by remitting via Paris than by direct exchange on Berlin, the payment of the £1500 would be made in London through both Paris and Berlin.

935. A **letter of credit** is a circular letter issued by a banking house to a person who desires to travel abroad. The letter is usually addressed to the foreign correspondents of the bank issuing it, requesting them to furnish the traveler such funds as he may require up to the aggregate amount named in the letter.

When the traveler desires funds, he goes to any correspondent mentioned in the letter of credit, and draws a draft on that correspondent for the amount desired. The draft is signed in the presence of the correspondent, who carefully compares the signature with the one on the letter, and, if they are found to agree, the draft is cashed and the amount inscribed on the back of the letter. The last draft drawn is attached to the letter itself.

The difference between a bill of exchange and a letter of credit is that the former is payable at a certain designated place, at a specified time, and in one amount, while the latter is payable at several places, at different times, and in variable amounts.

936. **Travelers' cheques** are a substitute for letters of credit and bills of exchange. They are similar in form to bank bills. They are issued for fixed printed amounts, with the equivalent of each denomination in the money of the principal European countries, and are payable to order, after being signed and countersigned by the purchaser or holder. They are cashed without discount or commission by an extended list of banks and bankers, and are received in settlement of hotel bills by the principal hotels in Europe.



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London April 4 1898

The American Express Company

Will pay to the Order of PAID. Traveler Hotel

Twenty Dollars

SAMPLE NOT GOOD.

Travelers' Cheque

937. To find the cost of a foreign bill of exchange.

938. Examples. *1.* Find the cost of a draft on London for £380 10*s.* 6*d.* sterling, exchange being quoted at $\$4.86\frac{5}{8}$.

SOLUTION

£380 10*s.* 6*d.* = £380.525.
$4.86\frac{5}{8}$ = the market quotation of £1.
$\$4.86\frac{5}{8} \times 380.525 = \$1851.73$, the cost of the draft.

2. Find the cost of a sight draft on Paris for 3108 francs, exchange being quoted at 5.18.

SOLUTION

$1 = 5.18 francs.
3108 francs ÷ 5.18 francs = 600.
Therefore a bill for 3108 francs will cost $600.

939. To find the face of a foreign bill of exchange.

940. Example. The cost of a bill of exchange on London was $3654.47. When exchange was quoted at $\$4.86\frac{1}{4}$, what was the face of the bill?

SOLUTION

$4.86½ = the market value of £1.
$3654.47 ÷ $4.86½ = 751.175.
Therefore $3654.47 = £751.175.
£.175 = 3*s.* 6*d.*
Hence the face of the bill was £751 3*s.* 6*d.*

WRITTEN EXERCISE

1. Find the cost of a sight draft on London for £400 when exchange is quoted at 4.865.

2. I bought a bill of exchange on Paris and paid $2156. What was the face of the bill, exchange being quoted at 5.17⅝?

3. A New York importer who owed a Dresden manufacturer 21,320 reichsmarks, bought a bill of exchange on Berlin at 95¾, and paid for the same by check. What was the face of the check?

4. What is the face of a bill of exchange on London which can be bought for $5807.25 if quoted at $4.85, brokerage ⅛%?

5. An exporter sold through a broker a bill of exchange on Hamburg at 95⅜, and received $5953.49 as net proceeds. What was the face of the bill, brokerage ⅛%?

6. Hibbard & Co., of Brooklyn, purchased a bill of exchange on London at 3 days' sight for £342 12*s.* 6*d.* at 4.86½. How much did the bill cost?

7. An importer purchased a sixty-day bill of exchange on Bremen at 95¾ for $446.20. What was the face of the bill?

8. A New York diamond merchant purchased a bill of exchange on Amsterdam at 3 days' sight for 63,892 guilders at 40½. What did the bill cost?

9. I purchased a bill of exchange on Paris for 33,250 francs and paid $6412.72. What was the course of exchange?

10. An importer purchased a bill of exchange on Amsterdam for 3575 guilders and paid $1443.41 for it. What was the course of exchange?

11. A Manchester, England, manufacturer drew a bill of exchange at 3 days' sight for £450 10*s.* 8*d.* on a Rochester, N.Y., merchant. The draft was presented to the drawee by a local bank, and paid by check. What was the face of the check, exchange being $4.85\frac{1}{4}$?

12. Langdon & Perry, of New York, owed on foreign invoices as follows: T. C. Shepherd Sons, London, £1800 8*s.*; J. L. Von Buesche, Berlin, 1600 marks; Perrie, Buzzell & Co., Paris, 4016 francs; F. Gonzalez, Mexico, 816 dollars. They bought at their bank: exchange on London at $4.86\frac{1}{8}$; on Berlin, $96\frac{1}{4}$; on Paris, $5.19\frac{1}{2}$; on Mexico, $79\frac{1}{2}$, and issued one check to cover the total purchase. What was the amount of the check?

941. **To find the cost of exporting gold.**

942. **Example.** Strawbridge & Clothier, Philadelphia, are quoted $4.89\frac{1}{2}$ on London for a sight bill of £6540. They export the gold instead of buying the exchange, paying $\frac{1}{8}\%$ insurance, $\frac{1}{8}\%$ freight, $\frac{3}{16}\%$ abrasion. How much did they save by shipping the gold?

SOLUTION

$\$4.89\frac{1}{2} \times 6540 = \$32,013.30$, the cost of the draft.

$4.8665 =$ intrinsic par of exchange.

$\frac{1}{8}\% + \frac{1}{8}\% + \frac{3}{16}\% = \frac{7}{16}\%$.

$\frac{7}{16}\%$ of $4.8665 = .02129$.

$4.8665 + .02129 = 4.88779$, the rate for exporting gold.

$\$4.88779 \times 6540 = \$31,966.15$, the cost of gold shipment.

$\$32,013.30 - \$31,966.15 = \$47.15$, saved by gold shipment.

943. **To find the proceeds of a bill of exchange.**

944. **Examples.** *1.* John J. Rose & Co. sell to Brown Bros. & Co. a sight draft for Mk. $1852\frac{80}{100}$. What did they receive at $93\frac{5}{16}$?

SOLUTION

$\$.93\frac{5}{16} = \$.933125 =$ the rate for 4 marks.

$\$.933125 \times 1852\frac{80}{100} \div 4 = \$432.22$, the proceeds.

2. J. Griffith & Co. sold to Brown Bros. & Co. at $5.22\frac{1}{8} - \frac{1}{16}$ a sight draft for Fr. $14,645\frac{50}{100}$. What are the proceeds?

SOLUTION

$\$1 =$ Fr. $5.22\frac{1}{8} - \frac{1}{16} =$ Fr. 5.22451. See **931**.

Fr. $14,645\frac{50}{100} \div$ Fr. $5.22451 = 2803.23$. Therefore the proceeds are \$2803.23

NOTE. The following examples show how a banker determines what rate to quote to a customer.

3. On Jan. 3, 1907, there is drawn by the Baldwin Locomotive Works, Philadelphia, on the London & Brazilian Bank, London, a commercial bill of exchange at 90 days after sight for £617 8*s.* 3*d.* At 4.865 market quotation what should a banker pay for it, allowing for interest at 3%, revenue stamps, and a commission of $\frac{1}{2}$% ?

NOTE. Interest on London exchange is reckoned on a basis of $4.85 per pound 365 days to the year. English law allows 3 days grace.

SOLUTION

Market rate on London =		$4.865
Disct. rate 93 ds. (3 ds. grace) $= 4.85 \times .03 \times \frac{93}{365} =$	.03707	
Com. $\frac{1}{2}$% per pound =	.02433	
Cost per pound revenue stamps $\frac{1}{20}$% of rate =	.00243	.06383
The banker's quotation		$4.80117

£617 8*s.* 3*d.* = £617.4125.

$4.80117 × 617.4125 = $2964.30, the sum the banker would pay.

4. A bill of exchange on Paris at 30 days after sight is drawn in favor of Lazard Frères for Fr. 16,764$\frac{70}{100}$ with documents. Interest rate, 3%; market quotation, Fr. 5.165; commission, $\frac{1}{8}$% of rate. What are the proceeds?

SOLUTION

Market quotation on Paris =	Fr. 5.165
Discount, 30 ds. at 3% = $\frac{1}{4}$% of rate =	.01291
Com. $\frac{1}{8}$% of rate =	.00646
Revenue stamps $\frac{1}{20}$% =	.00258
The banker's quotation	$1 = Fr. 5.18695

Fr. 16,764.70 ÷ Fr. 5.18695 = 3232.09. Hence the proceeds are $3232.09.

5. A documentary bill of exchange is drawn on Amsterdam at 60 days after sight. What rate per guilder can Brown Bros. & Co. offer for it, allowing for discount rate, 3$\frac{1}{2}$%; market quotation, 40$\frac{1}{8}$ + $\frac{1}{32}$; commission, $\frac{1}{8}$%; revenue stamps, $\frac{1}{20}$% ?

SOLUTION

Market quotation (40$\frac{1}{8}$ + $\frac{1}{32}$) =		$.401375
Revenue stamps $\frac{1}{20}$% of rate =	.0002	
Discount, 60 ds. at 3$\frac{1}{2}$% of rate =	.00234	
Commission $\frac{1}{8}$% of rate	.0005	.00304
The banker's quotation		$.398335

WRITTEN EXERCISE

1. The Baldwin Locomotive Works of Philadelphia sell a sight draft for £416 9*s.* 4*d.* on the London & Brazilian Bank to Brown Bros. & Co. at $4.81\frac{1}{8}$. What amount of money does it bring?

2. J. S. Griffith & Co. sell to Brown Bros. & Co. at $5.20\frac{5}{8} - \frac{1}{16}$ a draft on Credit Lyonnais, Havre, for Fr. $19{,}745\frac{60}{100}$. What amount do they receive for it?

3. The Baldwin Locomotive Works draw a draft on the London & Brazilian Bank at 90 days sight for £314 3*s.* 4*d.* How much will Brown Bros. & Co. pay for it; market quotation, 4.826; commission, $\frac{1}{8}$ %; interest rate, 3 %; revenue stamps, $\frac{1}{20}$ %?

4. John J. Rose & Co. sell to Brown Bros. & Co. the draft shown on page 347. Market rate, $91\frac{3}{16}$; interest rate, $3\frac{1}{2}$ %; commission, $\frac{1}{4}$ %; revenue stamps, $\frac{1}{20}$ %. What amount of money do they receive?

Note. Interest on German exchange is computed at 95 cents per 4 marks, 360 ds. to a year.

5. John Wanamaker sells in New York a documentary bill of exchange for £640 12*s.* 6*d*, at 90 days sight, on a London bank. What should he receive, allowing for 3 % interest, revenue stamps, and a commission $\frac{1}{2}$ %, if the quotation is $4.835?

6. The Rogers Locomotive Works of Paterson, N.J., sell to Brown Bros. & Co. a documentary bill of exchange on a Paris bank for Fr. $18{,}765\frac{70}{100}$, drawn 90 days after sight. What are the proceeds, if market rate is $5.17\frac{1}{2} + \frac{1}{32}$; interest rate, 3 %; commission, $\frac{1}{8}$ %; revenue stamp, $\frac{1}{20}$ %?

7. The Wm. H. Hostman Co. bought from John Heckemann, Bremen, a bill of goods amounting to $4695.45 on 60 days time, $2\frac{1}{2}$ %, 10 days. They send him a draft to-day at sight, so as to take advantage of the discount. What is the face of the draft, in marks, if demand exchange is $94\frac{1}{4}$, which includes broker's commission?

8. James McCreery & Co. sell a documentary bill of exchange on a Geneva banker for Fr $14{,}764\frac{50}{100}$ at 90 days after sight, to Lazard Frères. What are the proceeds, the rate of exchange being $5.16\frac{1}{2}$; rate of interest, 3 %; commission, $\frac{1}{8}$ %; revenue stamps, $\frac{1}{20}$ %; and cost of collecting and sending funds to Paris, $\frac{3}{16}$ %?

9. Tiffany & Co. import an invoice of statuettes and bronzes from Florence, Italy, amounting to $14{,}725\frac{35}{100}$ lire. They buy a three days after sight draft of Brown Bros. & Co. on a Florence banker to pay the bill. What will they pay Brown Bros. & Co. for it if the market rate is $5.17\frac{1}{2}$; commission, $\frac{1}{8}$ %; revenue stamps, $\frac{1}{20}$ %; discount rate, 3 %?

10. B. Altman & Co., New York, are quoted $4.89\frac{3}{4}$, which includes commission, for a sight bill on London for £7560. They buy the gold and ship it, instead of buying the draft. How much is saved if they pay $\frac{1}{8}$ % insurance, $\frac{1}{8}$ % freight, and $\frac{1}{16}$ % abrasion?

11. You are a clerk in the office of Brown Bros. & Co., and a merchant hands you a bill on London for £312 3*s.* 4*d.* at 60 days after sight; a bill at 90 days after sight on Havre, France, for Fr. $15{,}612\frac{25}{100}$; a bill on Hamburg at 30 days after sight, for Mk. $3412\frac{52}{100}$; a sight bill on Amsterdam for $1345\frac{40}{100}$ guilders. The posted rates are $4.83\frac{1}{2}$ on London; $5.16-\frac{1}{64}$ on France; $92\frac{1}{4}$ on Hamburg; $40\frac{1}{2}+\frac{1}{64}$ on Amsterdam. The foreign rate of interest is 3 %. You are to allow for a commission of $\frac{1}{8}$ %, and for revenue stamps on time paper, $\frac{1}{20}$ %. How much should you pay the merchant?

12. Altman & Co., New York, are quoted a rate of $4.89\frac{3}{4}$, which includes commission, on London, for a sight bill amounting to £8460 10*s.* Instead of buying the bill they export the gold. If they pay $\frac{1}{8}$ % insurance, $\frac{1}{8}$ % freight, $\frac{3}{16}$ % for abrasion, how much do they gain or lose by shipping the gold instead of buying the exchange?

13. New York rates on Amsterdam are $41\frac{1}{2}$; on Paris are $5.15\frac{1}{4}$. A merchant owes a debt of Fr. 13,450 in Paris. He pays it by remitting via Amsterdam at Fr. $2.14\frac{1}{2}$ to the guilder. What is saved by the indirect exchange (**934**)?

14. Mr. W. of Philadelphia goes to London for a visit and directs his Philadelphia broker to remit him $10,000. Brown Bros. & Co. quote London exchange at 4.89 and Berlin exchange at $95\frac{1}{2}$, both rates including the commission. In Berlin, exchange on London is quoted at Mk. 20.3 to the pound. How much more or less in English money does Mr. W. receive by indirect exchange than by direct exchange, if there is a charge of $\frac{1}{8}$ % for remitting from Berlin to London?

SHARING

PROPORTIONAL PARTS

945. **Sharing** is the process of dividing a number into shares proportional to other given numbers.

DRILL EXERCISE

1. A and B agree to perform a certain piece of work for $160. If B can earn $10 while A earns $6, how much should each receive as his share of the $160?

Solution

$160 is to be divided into shares proportional to 6 and 10.
The numbers 6 and 10 may be regarded as shares.
The whole will then be represented by 6 + 10, or 16 shares.
16 shares = $160.
1 share = $10.
6 shares = $60, or A's part.
10 shares = $100, or B's part.

2. Divide 240 into shares proportional to 8 and 16.

3. Divide $600 into shares proportional to 1, 3, and 6.

4. Divide a profit of $1200 among three partners in a business in which A invests $2 as often as B invests $4, and C $6.

5. Divide $150 into shares proportional to $\frac{1}{4}$ and $\frac{1}{6}$.

Solution

Fractions must be similar to be compared.
$\frac{1}{4}$ and $\frac{1}{6}$ = $\frac{3}{12}$ and $\frac{2}{12}$, respectively.
$\frac{3}{12}$ and $\frac{2}{12}$ stand in the same relation to each other as 3 and 2, respectively.
Hence the whole may be represented by 5 shares.
5 shares = $150.
1 share = $30.
3 shares = $90, or the first part.
2 shares = $60, or the second part.

6. Divide $780 among three persons, whose shares are to be in proportion to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

7. Three men engage in business. A puts in $2000; B, $3000; and C, $4000. They gain $900. What should be each man's share of the gain?

8. A bankrupt owes $500 to A, $2000 to B, and $1500 to C. If his net resources are $2000, what will each of his creditors receive?

9. Divide $2100 among A, B, and C so that A's part will be twice B's part and one half of C's part.

10. Three boys bought a watermelon for 24¢, of which price Charles paid 9¢, John 8¢, and Walter 7¢. Ralph offered 24¢ for one fourth of the melon, which offer was accepted and the melon divided. How should the 24¢ received from Ralph be divided among the other three boys?

WRITTEN EXERCISE

1. A will provided that an estate be divided among three persons in proportion to their ages, which are 15, 18, and 20 years, respectively. If the amount received by the youngest person was $3000, what was the value of the estate?

2. A, B, and C engage in trade for one year. A puts in $6000, B $3000, and C $2000. If their gain for the year is $4400, what is each man's share?

3. A, B, and C rent a pasture for $740. A put in 3 cows for 4 months; B, 5 for 6 months; and C, 8 for 4 months. How much should each pay?

4. An estate was so divided between A and B that A's part was to B's part as $\frac{1}{6}$ is to $\frac{1}{4}$. If B received $2400 more than A, what was the value of the estate?

5. A man bequeathed his property to his wife and two daughters. The wife received $5 for every $3 received by the elder daughter and for every $2 received by the younger daughter. What was the value of the estate, the younger daughter having received $1500 less than her sister?

6. Divide the simple interest on $2600 for 6 years 3 months at 4% among A, B, and C in proportion to $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$, respectively.

7. A man left his property to be divided among his three sons and two daughters in proportion to their ages. The sons are aged 20, 16, and 10 years, respectively; the daughters 18 and 8 years, respectively. If the share of the youngest child was $ 7200, what was the value of the property?

8. So divide $30,000 among A, B, C, and D, that their portions shall be to each other as 1½, 3, 4½, and 6.

9. Coe, Hall, Tell, and Lee have a contract to dig a ditch which Coe can dig in 35 days, Hall in 45 days, Tell in 50 days, and Lee in 60 days. How long will it take all together to do the work? If $ 100 be paid for the work and all join till it is completed, how much should each get?

PARTNERSHIP

946. **Partnership** is an association resulting from an agreement between two or more persons to place their capital or services, or both, in some enterprise or business, and to share the gains and bear the losses in certain proportions.

947. Partnerships may be formed by: 1. Oral agreement. 2. Written agreement, (*a*) under seal, or (*b*) not under seal. 3. Implication.

All important partnership agreements should be in writing, and all of the conditions relating to the partnership should be definitely stated.

948. The **partners** are the persons associated in any business. Collectively they are called a *firm*, a *house*, or a *company*.

949. Partners are of four classes: 1. *Real* or *ostensible.* 2. *Dormant, silent,* or *concealed.* 3. *Limited.* 4. *Nominal.*

950. A **real** or **ostensible partner** is one who appears to the public to be, and who actually is, a partner.

951. A **dormant** or **concealed partner** is practically a real partner whose connection with the partnership is concealed from the public.

A concealed partner is responsible for the debts of a firm if his connection with the partnership becomes known.

952. A **limited partner** is one whose responsibility is limited instead of general.

In case of failure of the firm, the general partner is individually responsible for all the debts of a firm, while the limited partner is responsible only for the amount named in the partnership agreement.

Limited partnerships are forbidden by the laws of some states. In the states where they are permitted, it is generally provided that at least one member of a firm must be a general partner.

953. A **nominal partner** is one whose name appears to the public as a partner, but who has no investment and receives no share of the gains.

A nominal partner is responsible to third parties for the debts of a firm.

954. The **capital** or **stock** is the money or other equivalent property invested in the business.

Capital is frequently real estate, personal property, time, skill, etc.

955. **Resources** or **assets** are the entire property of a firm, including accounts receivable, bills receivable, etc.

956. **Liabilities** are the entire debts of a firm.

957. The **net capital** is the excess of the resources over the liabilities.

958. The **net insolvency** is the excess of the liabilities over the resources.

When the resources of a firm exceed the liabilities, the business is said to be *solvent;* when the liabilities exceed the resources, the business is said to be *insolvent*, or *bankrupt*.

959. The **net investment** of a person is his investment minus all withdrawals for personal use.

960. A **business statement** contains an itemized list of all resources and liabilities, of losses and gains, the present worth of the business, and the net gain or net loss for any given period.

961. The **net gain** is the excess of total gains over the total losses for any given period.

962. The **net loss** is the excess of total losses over the total gains for any given period.

DRILL EXERCISE

1. A commenced business with a cash investment of $6000. At the end of the year his net capital is $3500. What is his net gain or loss, no withdrawals having been made?

2. B began business with an investment of $7500. At the end of one year his net capital is found to be $9500. Find the net gain or loss.

3. C began business Jan. 1 with a cash investment of $2500. At the close of the year his net insolvency is $1700. Required the net gain or loss.

4. At the beginning of a year D's net insolvency was $600; at the close of the year his net capital was $150. Required his net gain or loss for the year.

5. E's insolvency at the beginning of the year was $4000, and at the close of the year is $3500. What has been his net gain or loss for the year?

6. F's loss for one year is $700. His insolvency at the end of the year is $300. Required the net capital at the beginning.

7. G's net gain for one year is $2000; his net capital at the end of the year is $1500. What was the net capital or net insolvency at the beginning of the year?

8. Jan. 1, 1903, H's resources were $3000, and his liabilities $2000; one year later his resources were $2000 and his liabilities $3000. What was the net gain or loss for the year?

9. I's insolvency at the end of one year is $3000. If he gained $1200 during the year, what was his net capital or net insolvency at the beginning of the year?

10. J's capital at the end of one year was $5200. If his gain for the year was $6900, what was his net capital or net insolvency at the beginning of the year?

963. To divide the gain or loss when each partner's investment has been employed for the same period of time.

964. Example. A and B enter into partnership to carry on a commission business for one year, A investing $7000 and B $4000,

During the year they gain $3300. What should be each man's share of the gain?

SOLUTION

The total investment = $7000 + $4000, or $11,000.

A's investment = $\frac{7000}{11000}$, or $\frac{7}{11}$ of the total investment.

B's investment = $\frac{4000}{11000}$, or $\frac{4}{11}$ of the total investment.

Each partner receives such a part of the gain as his investment is a part of the total investment. Therefore,

A's share of the gain = $\frac{7}{11}$ of $3300, or $2100.

B's share of the gain = $\frac{4}{11}$ of $3300, or $1200.

ORAL EXERCISE

Find each man's gain or loss in each of the following problems, the gains being shared or losses borne in proportion to investments.

1. A invested $300 and B $200; they gained $150.

2. C invested $800 and D $300; they gained $330.

3. E invested $1000 and F $800; they lost $360.

4. G invested $1200 and H $900; they gained $560.

5. I invested $900 and J $700; they lost $320.

6. K invested $1500 and L $1200; they gained $540.

7. M invested $2000 and N $800; they lost $560.

8. O invested $1000 and P $500; they gained $186.

WRITTEN EXERCISE

1. A and B unite in the purchase of a house costing $4200; A pays $1800 and B $2400. The property rents for $294 per annum. What share of the rent ought each to receive?

2. A, B, and C enter into partnership for the purpose of carrying on a manufacturing business. A joint capital of $65,000 is formed of which A furnishes $\frac{2}{5}$, B $\frac{2}{3}$ of the remainder, and C what still remains. Their net gain for one year is equivalent to 25% of the net capital invested. Required each man's share of the net gain.

3. Two men bought a mine for $ 20,000, of which sum A paid $ 12,500 and B the remainder. They sold the mine for $ 42,000. How much of the gain should each man receive? What part of the selling price should each receive?

4. A, B, and C engage in business, A investing $ 1250; B, $ 750; and C, $ 1000. They gain $ 957.30. How much of the gain should each receive? How much is each man worth after receiving his share of the gain?

5. B, C, and D unite their capital amounting to $ 12,000 in a business venture and realize a gain of $ 1500, of which B received $ 750; C, $ 500; and D, $ 250. What was the investment of each?

6. In a partnership A invested $ 5000 and received $\frac{2}{3}$ of the gain; B invested $ 2400 and received $\frac{1}{3}$ of the gain. The gains and losses were as follows: merchandise, gain, $ 840.30; expense, loss, $ 310.40; real estate, gain, $ 265.61. What was the net gain? What was the gain of each partner? After receiving his share of the gain, what was each partner worth?

7. In a partnership C invested $ 4000 and D $ 2000. It is agreed that if gains are realized they are to be shared according to the investment, each partner receiving such part of the gain as his investment is a part of the total investment; that if losses occur they are to be borne equally. At the end of one year the gains and losses were as follows: merchandise, gain, $ 534.20; expense, loss, $ 325.60; real estate, loss, $ 675; interest, gain, $ 34.25; discount, loss, $ 56.35. What was the net gain or loss? What was each partner worth after his share of the net results of the business was carried to his account?

8. In a partnership E invested $ 7654, F $ 8000, and G $ 7000; the gains and losses were to be shared equally. During a month the gains and losses were as follows: merchandise, gain, $ 2318; stocks and bonds, gain, $ 735; expense, loss, $ 1140; interest, gain, $ 342. What was the net gain? What was each partner's interest at closing?

9. H's net loss for one year was $ 17,290. His insolvency at the close of the year was $ 10,000. Find the net capital at the beginning.

965. To divide the gain or loss according to the amount invested and the time the investment is employed.

966. The best method of solving partnership problems is to treat them from the accountant's standpoint. In connection with the solution of each problem a ledger page may be used, an account opened with each partner, the net gain or loss properly divided, and the ledger accounts with the partners closed.

967. Examples. *1.* In a partnership, A invested $2000 for 8 months, B, $3000 for 6 months, and C, $4000 for 5 months. A gain of $1350 was realized. Find the gain of each partner.

Solution

Dr. A Cr.

		Present worth	*2400*				Net Gain	2000	
								400	
			2400					2400	

Dr. B Cr.

		Present worth	*3450*				Net Gain	3000	
								450	
			3450					3450	

Dr. C Cr.

		Present worth	*4500*				Net Gain	4000	
								500	
			4500					4500	

A's investment, $2000 for 8 mo. = $16,000 for 1 mo.
B's investment, $3000 for 6 mo. = 18,000 for 1 mo.
C's investment, $4000 for 5 mo. = 20,000 for 1 mo.
Firm's investment = $54,000 for 1 mo.

Each partner should receive such a part of the gain as his investment for 1 mo. is a part of the total investment for 1 mo. Therefore,

A's gain = $\frac{8}{27}$ ($\frac{16000}{54000}$) of $1350, or $400.
B's gain = $\frac{9}{27}$ or $\frac{1}{3}$ ($\frac{18000}{54000}$) of $1350, or $450.
C's gain = $\frac{10}{27}$ ($\frac{20000}{54000}$) of $1350, or $500.

2. A and B were partners in a manufacturing business. Their investments and withdrawals for one year were as follows: Jan. 1, A invested $2000; Apr. 1 he withdrew $500; July 1 he invested $1000; Oct. 1 he withdrew $700. Jan. 1, B invested $3000; May 1 he withdrew $1000; Sept. 1 he invested $500. During the year the firm gained $1780. What was each man's share of the gain?

SOLUTION

Dr. A Cr.

Apr.	1		500		Jan.	1		2000	
Oct.	1		700		July	1		1000	
Jan.	*1*	*Present worth*	*2580*		Jan.	1	Net Gain	780	
			3780					3780	

Dr. B Cr.

May	1		1000		Jan.	1		3000	
Jan.	*1*	*Present worth*	*3500*		Sept.	1		500	
					Jan.	1	Net Gain	1000	
			4500					4500	

A's Investment

$2,000 for 3 mo. = $6,000 for 1 mo.
1,500 for 3 mo. = 4,500 for 1 mo.
2,500 for 3 mo. = 7,500 for 1 mo.
1,800 for 3 mo. = 5,400 for 1 mo.
A's total investment = $23,400 for 1 mo.

B's Investment

$3,000 for 4 mo. = $12,000 for 1 mo.
2,000 for 4 mo. = 8,000 for 1 mo.
2,500 for 4 mo. = 10,000 for 1 mo.
B's total investment = $30,000 for 1 mo.

$23,400 + $30,000 = $53,400, the firm's investment for 1 mo.
A's gain = $\frac{39}{89}$ of $1780, or $780.
B's gain = $\frac{50}{89}$ of $1780, or $1000.

WRITTEN EXERCISE

1. Three persons traded together and gained $900. A invested in the business $1000 for 6 months; B invested $750 for 10 months; and C invested $1200 for 5 months. How should the gain be divided?

2. A, B, and C were partners. A had $ 800 in the business for 1 year, B had $ 1000 in for 9 months, and C had $ 2000 in for 8 months. How should a gain of $ 2150 be divided?

3. A, B, and C hired a pasture for 6 months for $ 95.10; A put in 75 sheep, and 2 months later took out 40; B put in 60 sheep, and at the end of 3 months put in 45 more; C put in 200 sheep, and after 4 months took them out. What part of the rent should each pay?

4. A commenced digging a ditch, and after working 6 days was joined by B, after which the two worked together 9 days, when they were joined by C. The three then worked 12 days, and at the end of that time A left the job and D worked with the other two 3 days, when the work was completed. If $ 92 was paid for the work, how much should each receive?

5. Martin and Eaton were partners one year, Martin investing at first $ 5000 and Eaton $ 3000; after 6 months Martin drew out $ 3000 and Eaton invested $ 1500; they gained $ 3600. What was the gain of each and the present worth of each, at the time of the dissolution of the partnership?

6. A and B engaged in a grocery business for 3 years from March 1, 1901. On that date each invested $ 1600; June 1, of the same year, A increased his investment $ 400, and B withdrew $ 300; Jan. 1, 1902, each withdrew $ 1000; Jan. 1, 1903, each invested $ 1500. How should a gain of $ 7500 be divided at the expiration of the partnership contract?

PARTNERSHIP SETTLEMENTS

968. **A partnership settlement** is an adjustment of the net value of a business among the partners when a partnership is dissolved, either by mutual consent or by limitation of contract.

969. In most partnership affairs a business statement would be required in connection with the finding of the condition of the business at the close of any given period.

WRITTEN EXERCISE

1. Copy the following statement of resources and liabilities, filling out the missing terms:

Statement of Resources & Liabilities

	Resources						
	Mdse on hand			7265	35		
	Cash " "			4327	50		
	Accounts Receivable			5132	75		
	Real Estate			6000	—		
	Bills Receivable on hand			3250	—		
	U S Bonds par value			2400	—		
	Total Resources					*****	**
	Liabilities						
	Accounts Payable			4725	25		
	Bills Payable outstanding			2100	—		
	Total Liabilities					****	**
	† Present Worth of firm					*****	**
	Investments and Withdrawals						
	A's investment	12000	—				
	A's withdrawals	2000	—				
	A's net investment			10000	—		
	B's investment	10000	—				
	B's withdrawals	2500	—				
	B's net investment			7500	—		
	Net investment of firm					*****	**
	Net Gain					****	**
	Verification						
	A's net investment	10000	—				
	A's one-half net gain	****	**				
	A's present worth			*****	**		
	B's net investment	****	**				
	B's one-half net gain	****	**				
	B's present worth			****	**		
	† Present Worth of firm					*****	**

† To be written in red ink.

Using the foregoing statement form as a model, make statements to solve the questions embodied in each of the following problems:

2. At the time of closing business, the resources of a firm were: cash, $931.50; merchandise, per inventory, $13,196.25; notes and accounts due it, $8154; interest on same, $211.50; real estate, $11,150. The firm owed, on its notes, acceptances, and bills outstanding, $7142, and interest on the same, $348.50; and there was an unpaid mortgage on the real estate of $2500, with interest accrued thereon of $88.50. If the invested capital was $22,500, what was the net solvency or insolvency of the firm at closing, and how much has been the net gain or net loss?

3. Burke, Brace, and Baldwin became partners, each investing $15,000, and each to have one third of the gains or sustain one third of the losses. Burke withdrew $2100 during the time of the partnership, Brace $1800, and Baldwin $2000. At the close of business their resources were: cash, $3540; merchandise, $14,785; notes, acceptances, and accounts receivable, $16,250; real estate, $28,500. They owed on their outstanding notes $8125, and on sundry personal accounts $1950. Find the present worth of each partner at closing.

4. Parsons and Briggs became partners Apr. 1, 1901, under an agreement that each should be allowed 6% simple interest on all investments, and that, on final settlement, Briggs should be allowed 10% of the net gains, before other division, for superintending the business, but that otherwise the gains and losses be divided in proportion to average investment. Apr. 1, 1901, Parsons invested $18,000, and Briggs $4000; Jan. 1, 1902, Parsons withdrew $5000, and Briggs invested $3000; Aug. 1, 1902, Briggs withdrew $1500; Dec. 1, 1902, the partners agreed upon a dissolution of the partnership, having resources and liabilities as follows:

Resources		Liabilities	
Cash on hand and in bank,	$1,101.05	Notes and acceptances,	$6,520.00
Accounts receivable,	16,405.50	Outstanding accounts,	21,246.50
Bills receivable,	2,550.00	Rent due,	1,200.00
Interest accumulated on same,	287.41		
Mdse. per inventory,	9,716.55		

If only 80% of the accounts receivable prove collectible, what has been the net gain or net loss? What has been the net gain or net loss of each partner? What is the firm's net insolvency at dissolution? What is the net insolvency of each partner?

5. A and B became partners for one year, A investing ⅗ of the capital, and B ⅖, the agreement being that the gains or losses shall be apportioned according to average net investment, and that each partner be allowed 6% interest per annum on all investments, and be charged interest at the rate of 6% on all sums withdrawn. At the end of the year the firm had resources and liabilities as follows:

Resources		Liabilities	
Mdse., per inventory,	$21,460.00	Mortgage on real estate,	$7,000.00
Real estate,	15,000.00	Interest accrued on same,	210.00
Cash,	1,950.00	Notes outstanding,	26,950.00
Bills receivable,	13,146.50	Interest accrued on same,	811.75
Interest accrued on same,	519.25	Due Barnes, Clay & Co.,	33,560.00
Accounts due the business,	11,218.50		
Furniture,	1,320.00		
Delivery wagons and horses,	2,100.00		

It is found that 33⅓% of the accounts due the firm are uncollectible. If the firm's losses during the year have been $12,000, how much was invested by each partner? What is the present worth or net insolvency of the firm, and of each partner, at closing?

6. Mason and Rivers were joint partners, each investing an equal part of $9000. At the end of the year the resources and liabilities were as follows: cash on hand, $2212.45; merchandise on hand, $7278.54; bills receivable, $943.50; interest accrued on bills receivable, $22.70; office safe, $160; accounts receivable, $2956.20; 5% of the accounts receivable were estimated as not collectible; accounts payable, $1147; unpaid freight bill, $64.50; bills payable $560; interest accrued on bills payable, $17.25. What was the net gain? What was each partner's present worth at the close of the year?

7. D and E are partners, each investing $9000. The losses and gains, respectively, are to be borne or shared equally. At the end of one year the following is a list of their resources and liabilities: merchandise on hand, $6235.42; personal accounts due the firm, $4785.15; cash on deposit, $4756.20; cash in safe, $543.82; bills receivable on hand, $2658.90; N. Y. C. & H. R. R. R. stock, $3600; First National Bank stock, $2000; bills payable outstanding, $4298.75; due sundry persons on account, $3215.60. During the year D withdrew $1800 and E withdrew $2000. What is the net gain and the present worth of each partner?

8. A and B are equal partners in a business, the losses and gains of which are to be borne and shared equally. The following is the condition of the business at the close of one year. Resources: cash, $4275; merchandise, per inventory, $5476.20; accounts receivable, $2356.75. Liabilities: bills payable, $2240; Davis & Weller, $1400. The net gain for the year is $4267.95. What was the investment of each partner at the beginning of business?

9. The following is the trial balance of the firm of Austin & Leland, at the close of one year's business:

	Dr.	Cr.
Charles Austin, Proprietor,	$350.00	$6,385.24
William Leland, Proprietor,	327.00	6,385.24
Cash,	5,647.27	
Merchandise,	3,187.56	
Bills receivable,	9,000.00	
Bills payable,		5,909.00
Expense,	475.00	
Interest,	76.45	
Discount,		54.65
Accts. receivable,	3,427.50	
Accts. payable,		3,756.65

Inventories:

Mdse. (goods on hand),	$8,764.50
Mdse. (unpaid freight bill),	82.25
Expense (coal on hand),	$28.45
Expense (unpaid gas bill),	10.50

It is estimated that 10% of the accounts receivable cannot be collected. Make a business statement. What is the present worth of the business? What is the net gain? What is the present worth of each proprietor?

10. E. H. Hill was associated in business with N. P. Pond for one year, each investing $5600. At the end of the year their books show the following: Resources: mdse., $4500; notes, $2500; accounts receivable, $13,000; cash, $1500. Liabilities: notes, $5000; accounts payable, $1250; due Pond for special services, $400. Find the net loss or net gain, which divide equally between the partners, and then find each partner's present worth.

BUILDING AND LOAN ASSOCIATIONS

970. Building and loan associations were organized originally for the purpose of assisting members to build homes with their savings. The funds of such an association are loaned from time to time, on satisfactory real estate security, to that member who will pay the highest premium for the loan in addition to the regular interest.

971. The Department of Labor outlines a building and loan association as follows: "The stockholder pays a stipulated minimum sum, say one dollar, when he takes his membership and buys his **share** of stock. He then continues to pay a like sum each month until the aggregate sums paid, augmented by the profits, amount to the maturing value of the stock, usually $200; and at this time the stockholder is entitled to the full maturing value of the share and surrenders the same.

"It is seen clearly, then, that the capital of a building loan association consists of the continued savings of its members paid to the association upon shares of stock, increased by the interest and premiums which the association has received from loans made by it from the savings of its members thus paid to the association, and from all other sources of income. The amount of the capital of the association increases from month to month.

972. "Shares are usually issued in **series.** When a second series is issued, the issues of a previous series cease.

"The term during which a series is open for subscriptions is usually either three months, six months, or twelve months.

973. "Before a share matures it has two values: the **book value** is ascertained by adding all the dues that have been paid to the profits that have accrued; that is, the book value is the actual value. The **withdrawal value** is that amount which an association is willing to pay a stockholder who desires to sever his connection with the association prior to the date upon which his share matures."

974. Every building and loan association adopts its own **by-laws.** A few by-laws of one association may be summarized as follows:

(1) Provision is made for monthly dues of $1 and maturing value of $200 per share, as suggested in 971.

(2) Every stockholder may receive a loan of $200 for each share of stock held by him or her, to be secured by mortgage on satisfactory real estate; but every loan shall be awarded to the highest bidder at stated meetings, and the bid shall be at

so many cents per share, which premium, together with interest at the rate of six per cent per annum, shall be paid monthly during the continuance of the series. (When the series matures, the loan is canceled by the maturing value of the stock.)

(3) During the first year of any series the withdrawal value of the stock thereof shall be the amount paid in as dues, less all fines and charges and a tax of fifteen cents per share; after the first year of any series the withdrawal value of the stock thereof shall be the amount paid in as dues with such sum in addition, not exceeding two thirds of the actual profits, as may be ordered from time to time by the Board of Directors, less all fines and charges.

975. When a member fails to pay his dues on any meeting night, a **fine** is charged, which he must pay in addition to the dues.

Sometimes the fine is a certain sum (as 10 cents) per share; sometimes 2% per month interest on the amounts unpaid. Under the latter rule, for example, if the owner of 10 shares neglected to pay his $10 monthly dues for 5 successive meetings, he would owe, at the sixth meeting, $60 in dues and also $3 in fines; the fines having been charged against him as follows; $.20 (2% of $10) at the second meeting; $.40 additional (2% of $20) at the third: $.60 more at the fourth; $.80 more at the fifth; and $1 more at the sixth.

976. The following shows a form of **roll book** for one month. The other months of the fiscal year are extended across the page:

No. Shares	Book Number	Name	March					
			Loans	Dues	Interest and Premiums	Fines	Total	Amount Paid
15	467	George Brown	$1200 00	$15 00	$6 00 60		$21 60	
10	463	Amos Burnett		20 00		$1 00	21 00	
15	395	Charles Cook		15 00		1 50	16 50	
10	465	John Dercum	2000 00	10 00	10 00 3 00		23 00	
5	466	Edward Enstice		10 00		50	10 50	
12	442	Frank French		12 00			12 00	

In the illustration George Brown has borrowed $1200, for which he pays interest at 6% per annum, and a premium of 10 cents per share per month on the 6 shares covered by the loan. This makes, with his dues, a total of $21.60 due from him for March. Amos Burnett owes dues for February and March and a fine of 10 cents per share on unpaid February dues. John Dercum's premium is 30 cents a share.

977. To find the withdrawal value.

978. Example. B owns 10 shares in a first series and has paid dues for 60 months, during which he has been a member. What amount can he now withdraw, if he is allowed annual profits of 3 % ?

SOLUTION

$10 × 60 = $600, total dues paid in.

The first $10 draws profits for 60 months, the second $10 for 59 months, and so on; but the custom is to allow an average of one half the total time, or 30 months, in this case, on the total amount.

$600 × .03 × $\frac{30}{12}$ = $45, profit allowed.

$600 + $45 = withdrawal value.

WRITTEN EXERCISE

1. K has 25 shares in a first series and has paid in dues for $8\frac{1}{2}$ years. If there are no fines or charges against him, how much can he withdraw, if allowed 6 % annual profit?

2. L owns 15 shares in a series, having been a member 9 years. He has paid his dues, but owes $1.20 for fines. How much can he withdraw, if allowed 4 % annual profit?

3. M holds 20 shares in a series and has paid in his dues for 6 years, but there are fines of $ 4.60 charged against him. How much can he withdraw, if allowed profits at the rate of 3 % per annum?

979. Under the by-laws in **974**, the following table shows how the money from dues will probably come in and be paid out for twenty-four months, in a series of 300 shares, where all dues are paid promptly.

The association receives, at the first meeting, $300 dues; one month later, $300 dues and $1.50 accrued interest on the first loan of $300, which was made at the first meeting; at the third meeting, $300 dues and $3 interest for one month on total loans of $600, and so on. At the twenty-first meeting, the interest received has accumulated to the amount of $315, so that month the association makes a loan of twice the usual amount. Notice the effect of this extra loan in the other columns for the following month.

The profit per share is determined by dividing the interest by the number of shares. The book value of a share at any meeting is found by adding to the previous book value the dues and profit for one month. The table takes no account of premiums or fines, which in practice will sometimes greatly increase the receipts, profits, and book values.

	Dues rec'd each Month		Interest rec'd each Month		Amount loaned each Month		Profit per Share		Book Value of each Share	
1st meeting	300				300				1	00
2d meeting	300		1	50	300			005	2	005
3d meeting	300		3	00	300			01	3	015
4th meeting	300		4	50	300			015	4	03
5th meeting	300		6	00	300			02	5	05
6th meeting	300		7	50	300			025	6	075
7th meeting . . .	300		9	00	300			03	7	105
8th meeting	300		10	50	300			035	8	14
9th meeting	300		12	00	300			04	9	18
10th meeting	300		13	50	300			045	10	225
11th meeting	300		15	00	300			05	11	275
12th meeting	300		16	50	300			055	12	33
13th meeting	300		18	00	300			06	13	39
14th meeting . . .	300		19	50	300			065	14	455
15th meeting	300		21	00	300			07	15	525
16th meeting	300		22	50	300			075	16	60
17th meeting . . .	300		24	00	300			08	17	68
18th meeting	300		25	50	300			085	18	765
19th meeting	300		27	00	300			09	19	855
20th meeting . . .	300		28	50	300			095	20	95
21st meeting	300		30	00	600			10	22	05
22d meeting	300		33	00	300			11	23	16
23d meeting . . .	300		34	50	300			115	24	275
24th meeting	300		36	00	300			12	25	395
25th meeting	300		37	50	300			125	26	52

In this table, the owner of 6 shares pays $6 a month dues. Suppose now that he borrows, to the limit of his stock, the sum of $1200 (taking from the association $300 a month for four months), paying a premium of 25 cents a share, what would he pay monthly thereafter? In addition to the $6 dues each month he will have to pay $1.50 premium and $6 monthly interest on the money borrowed — a total of $13.50. These payments will continue till his 6 shares accumulate the full value, $1200, when both the shares and the loan are canceled.

WRITTEN EXERCISE

1. Make out a table like that above, covering a period of six months (7 meetings), for a new series made up of 550 shares.

2. Make out a table for twelve months for a third series of 400 shares, reckoning interest (including premiums) at 6.6 %.

3. Formulate a table for a fourth series of 800 shares for twenty-four months, reckoning interest at 9 %.

4. B owns 8 shares of the series in the table on page 374. At the end of the twenty-fourth month (*i.e.* at the twenty-fifth meeting), what is the total profit on his holdings? What is their book value?

5. How much would B have to pay each month in dues, premium, and interest if he owns 8 shares and borrows to the limit of his stock, having bid 30 cents per share premium?

6. Suppose Y bids 20 cents premium for a loan to the limit of his 10 shares. What are his monthly payments thereafter if he secures the loan?

980. To find the profit per share in several series. In the table on page 374 but one series is shown, and the finding of profits is relatively a simple matter. When a new series is started every three months or every six months or every year, the profits must be distributed equitably among the stockholders of the different series.

For example, suppose that a series was started each year, and that we are now at the end of the fourth year. In the first series, the first dollar paid in on each share has run for 48 months; the second, 47 months, etc., to the last dollar, which has run but 1 month; it is the custom to take the average time, therefore, as 24 months. In the second series, the average time will be considered 18 months; in the third, 12 months; and in the fourth, 6 months.

Suppose A holds 110 shares and has been a member 48 months; B holds 88 shares and has been a member 36 months; C holds 66 shares and has been a member 24 months; and D holds 300 shares and has been a member 12 months; and that at the end of the fourth year there is a total profit of $1246.40 to be divided between them. Each one's portion of the profit would be adjusted as follows:

	Payments		Shares		Totals			Average Time in Months		For one Month
A	$48	×	110	=	$5280.	$5280	×	24	=	$126720
B	36	×	88	=	3168.	3168	×	18	=	57024
C	24	×	66	=	1584.	1584	×	12	=	19008
D	12	×	300	=	3600.	3600	×	6	=	21600
					Total invested one month			. . . .		$224352

		Shares		Gain per Share
A's part is $\frac{126720}{224352}$ of $1246.40 = $704.00.	$704.00 ÷	110	=	$6.40
B's part is $\frac{57024}{224352}$ of $1246.40 = 316.80.	316.80 ÷	88	=	3.60
C's part is $\frac{19008}{224352}$ of $1246.40 = 105.60.	105.60 ÷	66	=	1.60
D's part is $\frac{21600}{224352}$ of $1246.40 = 120.00.	120.00 ÷	300	=	40
Total gain . $1246.40.				

The preceding work shows clearly the principles involved in the distribution of profits. In practice, however, the work is simplified by using what are known as "earning powers" and "lowest terms," as follows:

	Payments	Average Time in Months	Earning Power per Share	Lowest Terms per Share	Shares	Lowest Terms	Total Powers
1st ser.	$48 ×	24 =	$1152.	1152 ÷ 72 = 16.	110 ×	16 =	1760
2d ser.	36 ×	18 =	648.	648 ÷ 72 = 9.	88 ×	9 =	792
3d ser.	24 ×	12 =	288.	288 ÷ 72 = 4.	66 ×	4 =	264
4th ser.	12 ×	6 =	72.	72 ÷ 72 = 1.	300 ×	1 =	300
					Grand total		3116

Total gain, $1246.40 ÷ 3116 = 40 cents, the gain per power.

The gain per power multiplied by the lowest term for a series gives the gain per share in that series. That is:

$.40 × 16 = $6.40, gain per share, 1st series
$.40 × 9 = 3.60, gain per share, 2d series
$.40 × 4 = 1.60, gain per share, 3d series
$.40 × 1 = .40, gain per share, 4th series

A's gain is therefore $6.40 × 110, etc.

WRITTEN EXERCISE

1. L has owned 120 shares of stock for 36 months; M has owned 75 shares for 30 months; N has owned 90 shares for 24 months; O has owned 250 shares for 12 months; P has owned 150 shares for 6 months. What will be each one's share of a gain of $966.35? What are the lowest terms of the earning powers per share? What is each one's profit per share?

2. Q owns 275 shares, R owns 150 shares, S owns 55 shares, T owns 44 shares, U owns 33 shares, and V owns 25 shares in a loan association. R has paid dues for 6 months, S 24 months, T 18 months, and Q, U, and V 12 months. How shall they share a gain of $262.01? What are the lowest terms of the earning powers per share? What is each one's profit per share?

3. K owns 50 shares, upon which he has paid dues for 60 months; W owns 75 shares and has paid dues for 48 months; X owns 90 shares and has paid dues for 36 months; Y owns 200 shares and has paid dues for 24 months; Z owns 40 shares and has paid dues for 12 months. If there is a gain of $1390.48, what part of it belongs to X?

RATIO AND PROPORTION

RATIO

981. The **ratio** of two numbers is their relative greatness as expressed by the quotient of the first divided by the second.

Thus, the ratio of 8 to 4, commonly written 8 : 4, is 8 ÷ 4, or 2 ; the ratio of 7 : 9 is 7 ÷ 9, or $\frac{7}{9}$; etc.

982. Since ratio is simply relative greatness, no ratio can exist between unlike numbers; and quantities that may be expressed in different denominations of the same unit value must be reduced to the same denomination before a ratio can exist.

983. The **terms of a ratio** are the numbers compared. Taken together the terms of a ratio are sometimes called a *couplet*.

984. The first term of a ratio is sometimes called the **antecedent,** and the second term the **consequent.**

985. General Principles. Since the antecedent corresponds to the dividend in a question in division or the numerator of a fractional form, and the consequent to the divisor or the denominator of a fractional form it follows :

1. That any change in the antecedent produces a like change in the value of the ratio.
2. That any change in the consequent produces an opposite change in the value of the ratio.
3. That a like change in both antecedent and consequent does not affect the value of the ratio.

986. Ratios may be either *direct* or *inverse.*

987. A **direct ratio** is the quotient of the antecedent divided by the consequent. An **inverse ratio** is the reciprocal of a direct ratio, or the quotient of the consequent divided by the antecedent.

Thus, the direct ratio of 8 : 4 is 8 ÷ 4, or 2 ; and the inverse ratio of 8 : 4 is 4 ÷ 8, or $\frac{4}{8}$, equal to $\frac{1}{2}$, the reciprocal of 2.

988. A **simple ratio** is the ratio of two numbers. A **compound ratio** is the ratio of the products of the corresponding terms of two or more ratios.

PROPORTION

989. **Proportion** is an equality of ratios. It is indicated by placing the sign of equality =, or a double colon :: between the ratios.

990. The *first* and *fourth* terms of a proportion are called the **extremes**, and the *second* and *third* terms, the **means**.

991. The **test of proportion** is that the two integers or fractions representing the ratios are equal, or that the product of the extremes is equal to the product of the means.

992. Hence, a missing extreme may be found by dividing the product of the means by the given extreme; and a missing mean may be found by dividing the product of the extremes by the given mean.

ORAL EXERCISE

Find the unknown term in each of the following proportions:

1. 16 : 4 :: ? : 12. *3.* ? : 8 :: 20 : 4. *5.* 12 : 2 :: 18 : ?

2. 30 : ? :: 25 : 5. *4.* 12 : ? :: 75 : 25. *6.* 6 : 8 :: 24 : ?

SIMPLE PROPORTION

993. A **simple proportion** is an equality of two simple ratios.

994. In problems in simple proportion two ratios are given, one having both terms and the other having only one term. In order to find the missing term two of the given terms must be of one kind, and the answer and the other term of another kind.

995. **Example.** If 8 hats cost $32, what will 11 hats cost at the same rate?

Solution

For convenience write ? for the required or fourth term.
The third and fourth terms must be of the same kind. **Therefore** $32 is the third term.
11 hats will cost more than 8 hats. Therefore
8 hats is the first term and 11 hats the second term.

The proportion may then be stated as follows:

8 hats : 11 hats : : $32 : ?

8 hats : 11 hats is equivalent to 8 : 11. Therefore

The required cost equals $\frac{\$32 \times 11}{8}$ or $44.

996. From the foregoing explanation the following rule may be derived:

For the third term of the proportion write the given number that is of the same kind as the required fourth term.

From the nature of the question determine whether the answer is to be greater or less than the third term. If greater, write the larger of the two remaining numbers for the second term and the smaller for the first term; if less, write the smaller of the two remaining numbers for the second term and the larger for the first term.

Divide the product of the means by the given extreme, and the result is the required answer.

WRITTEN EXERCISE

1. If a post $7\frac{1}{8}$ ft. high casts a shadow $1\frac{1}{2}$ ft. long, what is the height of a tower that casts a shadow 150 ft. at the same time?

2. If 15 bushels of wheat can be bought for $13.50, how many bushels can be bought for $430.20?

3. An insolvent debtor owes $14,400, and has an estate valued at $10,800. How much will A receive on a claim of $3750?

4. A friend loaned me $750 for 3 yr. 4 mo. 15 da. For what period of time should I loan him $900 to fully repay his favor?

5. If 17 acres of pasture land are sufficient for 51 sheep, how many acres will be sufficient for 420 sheep?

COMPOUND PROPORTION

997. **A compound proportion** is a proportion in which one or both the ratios are compound.

998. A compound ratio may be reduced to a simple ratio by finding the product of all the antecedents for a new antecedent, and the product of all the consequents for a new consequent. Hence a statement in compound proportion may be simplified in practically the same manner as a statement in simple proportion.

999. Problems in compound proportion may be stated in the same manner as outlined for simple proportion, or a statement involving the principle of **cause** and **effect** may be used.

1000. Every question in proportion involves the principle of cause and effect; that is, work done for pay, cash given for goods, wood cut for labor performed, etc.

Hence questions in proportion may be stated as follows:

1st cause : 2d cause : : 1st effect : 2d effect.

1001. Example. If 10 men working 12 days of 8 hours each can cut 200 cords of wood, how many cords should be cut by 12 men in 15 days, if they work 6 hours per day?

Solution

The 1st cause = 10 men working for 12 days of 8 hours per day.
The first effect produced = 200 cords.
The 2d cause = 12 men working for 15 days of 6 hours per day.
The 2d effect produced = what number of cords?

Hence we have the following statement:

10 : 12
12 : 15 : : 200 cords : what number of cords?
8 : 6

Or,

$$\frac{12 \times 15 \times 6 \times 200 \text{ cords}}{10 \times 12 \times 8} = 225 \text{ cords.}$$

WRITTEN EXERCISE

1. If 15 men earn $ 607.50 in 18 days, how much should 21 men earn in 12 days?

2. If 6 men, working for 12 days, dig a ditch 80 rods long, how many rods of such ditch should 15 men dig in 21 days?

3. If 5 men, working 6 days of 12 hours per day, can cut 24 acres of corn, how many acres of corn should 8 men cut in 5 days, if they work 10 hours per day?

4. If $ 145.35 interest accrue on $ 510, at 6%, in 4 yr. 9 mo., how much interest will accrue at the same rate and time on $ 1350?

5. If $ 760, put at interest at 10%, accrue $ 9.50 interest in 45 days, in how many days will $ 1140 accrue $ 17.67 interest at 6%?

STORAGE

1002. **Storage** is a charge made for storing movable property in a warehouse.

1003. The **rates of storage** may be fixed by an agreement between the parties to a contract, but they are often regulated by boards of trade, chambers of commerce, associations of warehousemen, and by legislative enactment.

1004. The **term of storage** is the period of time for which storage charges are made.

1005. Storage may be charged at a fixed rate per package, etc., or at a fixed sum per term; but it is usually charged by the day, week, or month, and a fractional term is counted as a full term.

CASH STORAGE

1006. **Cash storage** is the term applied to cases in which the storage is paid or estimated at the time of the withdrawal of goods from the warehouse.

1007. In cash storage all goods delivered are deducted from the oldest receipt on hand.

In private bonded warehouses of the United States goods may be taken out at any time in quantities not less than an entire package, or if in bulk, of not less than one ton, by the payment of duties, storage, and labor charges. The storage charges are computed for periods of one month each, a fractional part of a month being counted the same as a full month.

1008. **Example.** At 5¢ per barrel per month, or fraction thereof, how much should be paid for the following storage of apples, storage being charged at each delivery?

Receipts	Deliveries
Sept. 1, 2000 bbl.	Oct. 20, 1000 bbl.
Nov. 20, 300 bbl.	Nov. 2, 300 bbl.
	Dec. 20, 1000 bbl.

SOLUTION

Date	Receipts and Deliveries	Rate	Storage
Sept. 1 received	2000 bbl.		
Oct. 20 delivered	1000 bbl., which were in storage 49 da.	10¢	$100
	1000 bbl., balance in storage		
Nov. 2 delivered	300 bbl., which were in storage 62 da.	15¢	45
	700 bbl., balance in storage		
Nov. 20 received	300 bbl.		
	1000 bbl., balance in storage		
Dec. 20 delivered	1000 bbl., 700 of which were in storage 110 da.	20¢	140
	300 of which were in storage 30 da.	5¢	15
	Total storage		$300

WRITTEN EXERCISE

1. Find the storage of the following at 2¢ per month, or fraction thereof, storage being calculated at each delivery.

Receipts	Deliveries
Sept. 5, 200 cases	Oct. 1, 100 cases
Sept. 30, 400 cases	Nov. 2, 200 cases
Nov. 1, 100 cases	Dec. 1, 200 cases
Nov. 8, 200 cases	Dec. 10, 400 cases

2. At a warehouse there was received and delivered flour as follows:

Receipts	Deliveries
Jan. 3, 150 bbl.	Jan. 23, 250 bbl.
Jan. 20, 200 bbl.	Mar. 1, 400 bbl.
Feb. 1, 300 bbl.	

The storage charge on the above was 5¢ per barrel for the first 10 days, or fraction thereof, and 3¢ per barrel for each subsequent period of 10 days, or fraction thereof. What sum must be paid in settlement?

3. The receipts and deliveries of goods at a storage warehouse were as follows:

Receipts	Deliveries
Sept. 2, 100 bbl.	Sept. 20, 100 bbl.
25, 200 bbl.	30, 100 bbl.
Oct. 19, 350 bbl.	Oct. 10, 100 bbl.
31, 150 bbl.	20, 100 bbl.
Nov. 7, 200 bbl.	30, 100 bbl.
	Nov. 20, 500 bbl.

The contract required the payment of 6¢ per barrel for the present term of 30 days, or fraction thereof, and 3¢ per barrel for each subsequent term of 30 days, or fraction thereof. Find the storage bill.

CREDIT OR AVERAGE STORAGE

1009. Credit or average storage is the term applied to cases in which the storage is not paid or estimated until the last withdrawal is made.

1010. Example. The storage charges being 2¢ per barrel per month of 30 days, what will be the bill in the following transaction?

Receipts	Deliveries
July 19, 200 bbl.	Aug. 15, 100 bbl.
31, 300 bbl.	Sept. 17, 400 bbl.

Solution

From July 19 to July 31 = 12 da., 200 bbl. stored for 12 da.
= 1 bbl. stored for 2,400 da.
July 31 300 bbl. received.
From July 31 to Aug. 15 = 15 da., 500 bbl. stored for 15 da.
= 1 bbl. stored for 7,500 da.
Aug. 15 100 bbl. delivered.
From Aug. 15 to Sept. 17 = 33 da., 400 bbl. stored for 33 da.
= 1 bbl. stored for 13,200 da.
Sept. 17 400 bbl. delivered.
Total, 1 bbl. stored for 23,100 da.

23,100 ÷ 30 = 770 terms of 30 da. each.
$.02 × 770 = $15.40, the total storage bill.

WRITTEN EXERCISE

1. There was received at a warehouse: May 30, 4000 bu. wheat; June 5, 2600 bu. oats; June 24, 3500 bu. barley; July 18, 5000 bu. corn. If all of this was shipped July 20, what was the storage bill, the charge being 1½¢ per bushel per term of 30 days' average storage?

2. What will be the storage charge, at 4½¢ per barrel, for a term of 30 days' average, in the following transaction?

Receipts	Deliveries
Feb. 8, 180 bbl. flour	Mar. 1, 100 bbl. apples
27, 100 bbl. apples	28, 190 bbl. flour
Mar. 8, 60 bbl. potatoes	Apr. 15, 60 bbl. potatoes
13, 300 bbl. flour	15, 60 bbl. flour
	29, 230 bbl. flour

3. A farmer received for pasture: Apr. 30, 12 head of cattle; May 15, 14 head of cattle; May 23, 27 head of cattle; June 9, 5 head of cattle; June 30, 8 head of cattle; July 16, 40 head of cattle. All were delivered July 25, and the charges were 75¢ per head for each week of 7 days' average pasturage. How much was his bill?

4. A drover hired a pasture of a farmer, agreeing to pay $4.20 per head of stock pastured for each average term of 30 days. What was the amount of the bill, the receipts and deliveries being as follows?

Receipts	Deliveries
June 15, 21 head of cattle	July 1, 30 head of cattle
27, 20 head of cattle	20, 15 head of cattle
July 5, 15 head of cattle	30, 15 head of cattle
29, 40 head of cattle	Aug. 21, the remainder
31, 40 head of cattle	

5. Find the storage charges, at 3¢ per barrel, for a term of 30 days' average, on the following:

Receipts	Deliveries
Sept. 2, 1620 bbl.	Sept. 13, 520 bbl.
16, 2920 bbl.	26, 966 bbl.
Oct. 25, 1470 bbl.	Dec. 2, 4524 bbl.

APPENDIX

METRIC SYSTEM OF MEASURES

1011. The **metric system** is a decimal system of denominate numbers. It is in use in nearly all the European states, in South America, Mexico, and Egypt. It is also used in parts of Asia, is authorized by law in the United States, and is almost universally used in scientific treatises.

1012. The **fundamental unit** of the metric system is the **meter,** a measure of length, which is equal to about one ten-millionth of the distance from the equator to the pole. It is defined by law as being the length of the bar of platinum which is carefully preserved at Paris.

Accurate copies of the meter have been procured by the governments of all civilized nations.

1013. Among the advantages claimed for the metric system are:

1. It employs only five unit words and seven prefixes.
2. Every word used suggests its measure.
3. It is consistent, uniform, simple, and complete, and would do away with the present inconsistent system of compound numbers.
4. Being a decimal system it makes arithmetical operations relating to measure much more simple.
5. It gives to the nations a uniform system of measures and thus materially facilitates trade and exchange.

1014. The **principal units** of the metric system are:

1. The **meter** for lengths.
2. The **square meter** for small surfaces such as floors, ceilings, etc.
3. The **are,** of 100 square meters, for large surfaces such as land measurements.
4. The **cubic meter** for solids.
5. The **liter** for capacities.
6. The **gram** for weights.

1015. Each of the metric units is divided and multiplied decimally. The higher orders are indicated by four Greek prefixes, as follows: *deka*, meaning 10; *hecto*, meaning 100; *kilo*, meaning 1000; *myria*, meaning 10,000. The lower orders are indicated by three Latin prefixes, as follows: *deci*, meaning .1; *centi*, meaning .01; *milli*, meaning .001.

Metric Long Measure

1016. The **unit** of long measure is the **meter.**

TABLE

10 **millimeters** (mm.)	= 1 **centimeter** (cm.) =	.01	meter.
10 **centimeters**	= 1 decimeter (dm.) =	.1	meter.
10 decimeters	= 1 **meter** (m.) =	1.	meter.
10 **meters**	= 1 dekameter (Dm.) =	10.	meters.
10 dekameters	= 1 hektometer (Hm.) =	100.	meters.
10 hektometers	= 1 **kilometer** (Km.) =	1000.	meters.
10 **kilometers**	= 1 myriameter (Mm.) =	10,000.	meters.

In the above and each of the following tables the units in common use are indicated by the blackfaced type.

In the metric system all abbreviations which indicate a fractional part of a standard unit begin with a small letter, while all those which indicate a multiple of a standard unit begin with a capital letter.

Metric Square Measure

1017. The **units** of square measure are the **square meter** for small surfaces and the **are** for land measurements.

1018. The units of square measure are the square of the units of long measure; hence 100 units of any given denomination are required for 1 of the next higher.

TABLE

100 sq. millimeters	= 1 **sq. centimeter** (sq. cm.) =	.0001	**sq. meter.**
100 **sq. centimeters**	= 1 sq. decimeter (sq. dm.) =	.01	"
100 sq. decimeters	= 1 **sq. meter** (sq. m.) =	1.	meter = 1 **centare.**
100 **sq. meters**	= 1 sq. dekameter (sq. Dm.) =	100.	meters = 1 are.
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.) =	10,000.	" = 1 **hectare.**
100 sq. hektometers	= 1 **sq. kilometer** (sq. Km.) =	1,000,000.	"

1019. The **centare**, the **are**, and the **hectare**, are used **only** in measuring land.

Metric Cubic Measure

1020. The **units** of cubic measure are the **cubic meter**, for ordinary solids, and the **stere**, for wood measurements.

1021. The units of cubic measure are the cube of the units of long measure; hence 1000 units of any given denomination are required for 1 of the next higher.

Table

1000 cu. millimeters (cu. mm.)	= 1 cu. centimeter (cu. cm.)	= .000001 cu. meter.
1000 cu. centimeters	= 1 cu. decimeter (cu. dm.)	= .001 " "
1000 cu. decimeters	= 1 cu. meter (cu. m.)	= 1. " "

Metric Measures of Capacity

1022. The **unit** of capacity, for both liquid and dry measures, is the **liter**.

1023. The liter is a cube whose side is 1 decimeter; hence a liter is a cubic decimeter.

Table

10 milliliters (ml.)	= 1 centiliter (cl.)	= .01 liter.
10 centiliters	= 1 deciliter (dl.)	= .1 "
10 deciliters	= 1 liter (l.)	= 1. "
10 liters	= 1 dekaliter (Dl.)	= 10. liters
10 dekaliters	= 1 hektoliter (Hl.)	= 100. "
10 hektoliters	= 1 kiloliter (Kl.)	= 1,000. "

Measures of Weight

1024. The **unit** of weight is the **gram**, which is equal to 1 cubic centimeter of pure water at its greatest density; that is, at a temperature just above freezing.

Table

10 milligrams (mg.)	= 1 centigram (cg.)	= .01 gram.
10 centigrams	= 1 decigram (dg.)	= .1 "
10 decigrams	= 1 gram (g.)	= 1. "
10 grams	= 1 dekagram (Dg.)	= 10. grams.
10 dekagrams	= 1 hektogram (Hg.)	= 100. "
10 hektograms	= 1 kilogram (Kg.)	= 1,000. "
10 kilograms	= 1 myriagram (Mg.)	= 10,000. "
10 myriagrams	= 1 quintal (Q.)	= 100,000. "
10 quintals	= 1 tonneau (T.)	= 100,0000. "

1025. An act of Congress requires all reductions from the metric to the common system, or the reverse, to be made according to the following

TABLE OF EQUIVALENTS

Long Measure

1 inch = 2.54 centimeters.	1 centimeter = .3937 of an inch.
1 foot = .3048 of a meter.	1 decimeter = .328 of a foot.
1 yard = .9144 of a meter.	1 meter = 1.0936 yards.
1 rod = 5.029 meters.	1 dekameter = 1.9884 rods.
1 mile = 1.6093 kilometers.	1 kilometer = .62137 of a mile.

Square Measure

1 sq. inch = 6.452 sq. centimeters.	1 sq. centimeter = .155 of a sq. inch.
1 sq. foot = .0929 of a sq. meter.	1 sq. decimeter = .1076 of a sq. foot.
1 sq. yard = .8361 of a sq. meter.	1 sq. meter = 1.196 sq. yards.
1 sq. rod = 25.293 sq. meters.	1 are = 3.954 sq. rods.
1 acre = 40.47 ares.	1 hektare = 2.471 acres.
1 sq. mile = 259 hectares.	1 sq. kilometer = .3861 of a sq. mile.

Cubic Measure

1 cu. inch = 16.387 cu. centimeters.	1 cu. centimeter = .061 of a cu. inch.
1 cu. foot = 28.317 cu. decimeters.	1 cu. decimeter = .0353 of a cu. foot.
1 cu. yard = .7645 of a cu. meter.	1 cu. meter = 1.308 cu. yards.
1 cord = 3.624 steres.	1 stere = .2759 of a cord.

Measures of Capacity

1 liquid quart = .9463 of a liter.	1 liter = 1.0567 liquid quarts.
1 dry quart = 1.101 liters.	1 liter = .908 of a dry quart.
1 liquid gallon = .3785 of a dekaliter.	1 dekaliter = 2.6417 liquid gallons.
1 peck = .881 of a dekaliter.	1 dekaliter = 1.135 pecks.
1 bushel = .3524 of a hektoliter.	1 hektoliter = 2.8375 bushels.

Measures of Weight

1 grain, Troy = .0648 of a gram.	1 gram = .03527 of an ounce, Avoir.
1 ounce, Avoir. = 28.35 grams.	1 gram = .03215 of an ounce, Troy.
1 ounce, Troy = 31.104 grams.	1 gram = 15.432 grains, Troy.
1 pound, Avoir. = .4536 of a kilogram.	1 kilogram = 2.2046 pounds, Avoir.
1 pound, Troy = .3732 of a kilogram.	1 kilogram = 2.679 pounds, Troy.
1 ton (short) = .9072 of a tonneau.	1 tonneau = 1.1023 tons (short).

WRITTEN EXERCISE

1. What will be the cost in Paris of a cargo of 38,500 bu. United States wheat at 10 francs 60 centimes per hektoliter?

2. How many avoirdupois pounds in 10 myriagrams 4 kilograms?

3. Reduce 250 hectares to common units.

4. A pile of wood 56 meters long, 18½ meters wide, and 3¾ meters high was sold at $6 per cord. How much was received for it?

5. At 21¢ per liter, what will 150 quarts of olive oil cost?

6. If the cost of 50 liters of wine was 800 francs, what was the price per gallon in United States money?

7. A merchant bought silk at $1.20 per meter and sold it by the yard at a gain of 20%. What was the selling price per yard?

8. A man bought 50 kilograms of sugar for $5.51 and sold it at a gain of 20%. What did he receive per pound?

9. A pile of wood 8 meters long and 2 meters wide contains 56 steres. Find the height of the pile in meters; in feet and inches.

10. Reduce 954 miles to kilometers.

11. How many meters of carpet 70 centimeters wide will be required for a room 7 meters long and 5 meters wide, if the strips run crosswise and 4 meters are lost in matching the pattern? how many yards?

12. How many hectares in a field 150 meters on a side? how many acres?

POWERS AND ROOTS

1026. The **power** of a number is the product arising from multiplying the number by itself one or more times.

1027. A **perfect power** is a number that can be exactly produced by the involution of some number as a root.

Thus, 25 and 8 are perfect powers, since $5 \times 5 = 25$, and $2 \times 2 \times 2 = 8$.

1028. The **square** of a number is its *second power*.

1029. The **cube** of a number is its *third power.*

1030. An **exponent** is a small figure written at the right of a number to indicate how many times the number is to be used as a factor.

Thus, 2^2 is equivalent to 2 × 2 and is read *the second power of* 2; and 5^3 is equivalent to 5 × 5 × 5 and is read, *the third power of* 5.

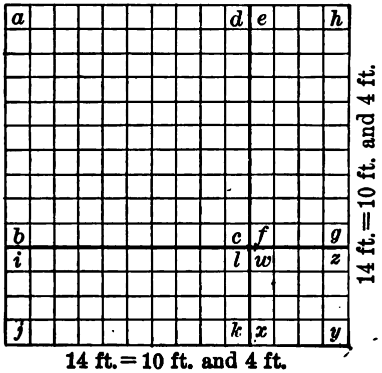
1031. The **root** of a number is one of the equal factors which multiplied together will produce the given number.

SQUARE ROOT

1032. The **square root** of a number is one of the *two equal* factors which multiplied together will produce the given number.

The accompanying diagram is a square 14 ft. on a side. Its area is, by inspection, found to be made up of:

1. The tens of 14, or 10^2, equal to 100 sq. ft., as shown by the square within the angles, a, b, c, d.
2. Twice the product of the tens by the units of the same number or 2 × (10 × 4), equal to 80 sq. ft., as shown by the surface within the angles e, f, g, h and i, j, k, l.
3. The square of the units, 4 ft., equal to 16 sq. ft., as shown by the square within the angles w, x, y, z.



Hence a square 14 ft. on each side will contain:

$$\begin{aligned} 10^2 &= 100 \text{ sq. ft.} \\ 2\times(10\times4) &= 80 \text{ sq. ft.} \\ 4^2 &= \underline{16} \text{ sq. ft.} \\ 14^2 &= 196 \text{ sq. ft.} \end{aligned}$$

1033. Therefore the following general principle may be stated:

The square of any number, composed of two or more figures, is equal to the square of the tens plus twice the product of the tens multiplied by the units plus the square of the units.

1034. In extracting the square root of a number, the first important step is to separate the figures of which the number is composed into groups.

The squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

From the above it is evident:

1. That the square of any number will contain at least one place or one order of units.

2. That the square of no number represented by a single figure will contain more than two places.

3. That if the number of which the square root is sought be separated into periods of two figures each, beginning at the units, the number of periods and partial periods so made will represent the number of unit orders in the root.

4. That the square of any number will contain twice as many places or one less than twice as many places as its root.

5. That where the product of the left-hand figure multiplied by itself is not greater than 9, then the square will contain one less than twice as many places as the root.

1035. Example. Find the square root of 625.

```
   6 25(25
   4
45)2 25
   2 25
```

Solution. The number consists of one full and one partial period; hence, its root will contain two places. The given number, 625, is the second power of the root to be extracted; therefore the first figure of the root, which will be the highest order of units in that root, must be obtained from the first left-hand period. The first, or tens' figure, of the root will be the square root of the greatest perfect square in 6. Hence, 2 is the tens' figure of the root. Subtracting the tens, the remainder, 225, must be equal to twice the tens multiplied by the units plus the square of the units. Twice the 2 tens is equal to 4 tens. 4 tens is contained in the 22 tens of the remainder 5 times; hence, 5 is the units' figure of the root. Twice the tens multiplied by the units plus the square of the units is equivalent to twice the tens plus the units multiplied by the units. Therefore, 5 units are annexed to the 4 tens and the result, 45, is multiplied by 5. Therefore, the square root of 625 is 25.

1036. From the foregoing explanations the following rule may be derived:

Beginning at the right, separate the given number into periods of two places each.

Take the square root of the greatest perfect square contained in the left-hand period for the first root figure; sub-

tract its square from the left-hand period, and to the remainder bring down the next period.

Divide the number thus obtained, exclusive of its units, by twice the root figure already found for a second quotient or root figure. Place this figure at the right of the root figure before found, and also at the right of the divisor.

Multiply the divisor thus formed by the new root figure. Subtract the result from the dividend, to the remainder bring down the next period, and so proceed until the last period has been brought down, considering the entire root already found as so many tens in determining subsequent root figures.

Whenever the divisor is greater than the dividend, place a cipher in the root, and also at the right of the divisor; bring down another period, and proceed as before.

When the root of a mixed decimal is required, form periods from the decimal point right and left, and if necessary supply a decimal cipher to make the decimal periods of two places each.

Any root of a common fraction may be obtained by extracting the root of the numerator for a numerator of the root, and the root of the denominator for the denominator of the root.

To find a root, decimally expressed, of any common fraction, reduce such fraction to a decimal, and extract the root to any number of places.

WRITTEN EXERCISE

Find the square root of:

1. 196.
2. 225.
3. 576.
4. 1225.
5. 5625.
6. 42436.
7. 15625.
8. 1048576.
9. 125.44.
10. 50.2681.
11. $\frac{4096}{5625}$.
12. $\frac{6561}{8464}$.

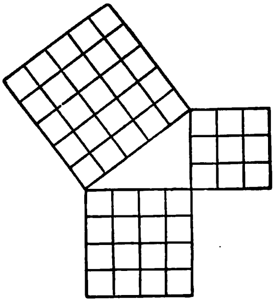
APPLICATIONS OF SQUARE ROOT

1037. It has been shown that the area of a square is the product of its two equal sides. Hence,

The side of any square is the square root of its area.

1038. The **hypothenuse** of a right-angled triangle is the side opposite the right angle.

1039. The square formed on the hypothenuse of a right-angled triangle is equal to the sum of the squares formed on the base and perpendicular. Hence,



The hypothenuse of a right-angled triangle is the square root of the sum of the squares of the other two sides; and

The base or perpendicular of a right-angled triangle is the square root of the difference between the square of the hypothenuse and that of the given side.

WRITTEN EXERCISE

1. The base of a figure is 60 ft. and the perpendicular 80 ft. What is the hypothenuse?

2. A farm of 80 acres is in the form of a rectangle, the length of which is twice its width. How many rods of fence will inclose it?

3. How many rods of fence will inclose a triangular field whose base is equal to its perpendicular and whose area is 20 acres?

4. If a farm is 1 mile square, how far is it diagonally across from corner to corner? Express the result in rods, feet, and inches.

5. What is the width of a street in which a ladder 60 ft. long can so be placed that it will reach the eaves of a building 40 ft. high on one side of the street, and of another building 50 ft. high on the opposite side of the street?

6. How many feet of fence will inclose a square field containing 16 acres?

7. How far apart are the opposite corners of a rectangular field having a width equal to $\frac{3}{4}$ of its length and containing 30 acres?

8. What is the length of one side of a square field, the area of which is one acre?

CUBE ROOT

1040. The **cube root** of a number is one of the *three equal* factors which multiplied together will produce the given number.

Thus, a cubic foot equals 12 × 12 × 12, or 1728 cubic inches, the product of its length, breadth, and thickness; and since 12 is one of the equal factors of 1728, it must be its cube root.

1041. The first point to be settled in extracting any root is the relative number of unit orders or places in the number and its root.

The cubes of	1,	2,	3,	4,	5,	6,	7,	8,	9,	10
are	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000

From the above it is evident:

1. That the cube of any number expressed by a single figure cannot have less than one nor more than three places or unit orders.
2. That each place added to the number will add three places to its cube.
3. That if a number be separated into periods of three figures each, beginning at the units, the number of places in the root will equal the number of periods and partial periods, if there are any.
4. That the cube of any number will contain three times as many places or one or two less than three times as many places as its roots.

1042. Since 57 equal 50 + 7, the cube of 57 may be determined in the following manner:

$$\begin{array}{r}
50+7 \\
50+7 \\ \hline
(50\times 7)+7^2 \\
50^2+(50\times 7) \qquad \\ \hline
50^2+2\times(50\times 7)+7^2 \\
50+7 \\ \hline
(50^2\times 7)+2\times(50\times 7^2)+7^3 \\
50^3+2\times(50^2\times 7)+(50\times 7^2) \qquad \\ \hline
50^3+3\times(50^2\times 7)+3\times(50\times 7^2)+7^3=185{,}193
\end{array}$$

$$\begin{array}{rr}
50^3= & 125{,}000 \text{ cu. ft.} \\
3\times(50^2\times 7)= & 52{,}500 \text{ cu. ft.} \\
3\times(50\times 7^2)= & 7{,}350 \text{ cu. ft.} \\
7^3= & 343 \text{ cu. ft.} \\ \hline
57^3= & 185{,}193 \text{ cu. ft.}
\end{array}$$

1043. Hence the following general principle may be stated:

The cube of a number is equal to the cube of the tens, plus three times the square of the tens, multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

1044. Example. Extract the cube of 15625.

$$\overset{t}{}\ \overset{u}{}$$

$t^3+3t^2u+3tu^2+u^3=15.625(2\ \ 5$

$t^3=\underline{8}$ or 8000

$3t^2u+3tu^2+u^3=\overline{7625}$=remainder.

$t^2=400$

$3t^2=1200$

$3t=\underline{60}$

$3t^2+3t=1260$ *trial divisor.*

$3t^2u=6000$

$3tu^2=1500$

$u^3=\underline{125}$

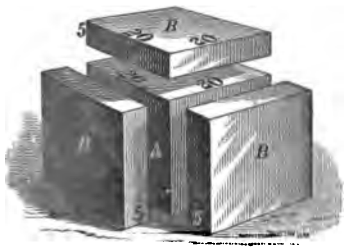
$3t^2u+3tu^2+u^3=7625.$

Solution. The given number consists of two periods of three figures each, therefore its cube root will contain two places.

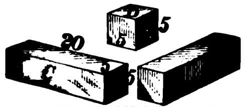
Since the given number is a product of the root taken three times as a factor, the first figure, or highest order of the root, must be obtained from the first left-hand period, or highest order of the power. The greatest cube in 15 is 8 and the cube root of 8 is 2; hence, 2 is the tens' figure of the root.

Subtracting the cube of the root figure thus found and bringing down the next figure, the entire remainder is found to be 7625.

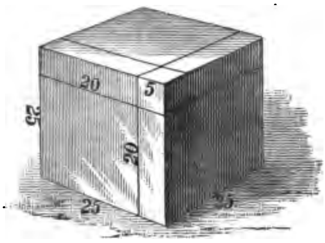
Referring to the general principle stated above, we find that having subtracted from the given number the cube of its tens, the remainder, 7625 must contain three times the product of the square of the tens by the units plus three times the product of the tens by the square of the units plus the cube of the units.



If a cube (*A*), 20 inches in length on each side, is formed, its solid contents will equal 8000 cubic inches, and it will be shown that the remaining 7625 cubic inches are to be so added to cube (*A*) that it will retain its cubical form. In order to do this, equal additions must be made to the three adjacent sides; and these three sides being each 20 inches in length and 20 inches in width, the addition to each of them in surface, or area, is 20^2, and to the three sides $3\times(20^2)$, as shown in the squares (*B*). It will also be observed that the three oblong blocks, as shown in (*C*), will be required to fill out the vacancies in the edges, and also the small cube (*D*), to fill out the corner.



Since each of the oblong blocks has a length of 2 tens, or 20 inches, the three will have a length of 3 x 20 inches. Observe now that the surface to be added to the cube (*A*), in order to include in its contents the 7625 remaining cubic inches, has been nearly, but not exactly, obtained; and since cubic contents divided by surface measurements must give units of length, the thickness of the three squares (*B*), and of the three oblong pieces (*C*), will be determined by dividing 7625 by the surface of the three squares plus the surface of the three oblong blocks. This division may give a quotient too large, owing to the omission in the divisor of the small square in the corner; hence, such surface measure taken as a divisor may with propriety be called a ***trial divisor.*** So using it, 5 is obtained as the second, or unit figure of the root.



Assuming this 5 to be the thickness of the three square blocks (*B*), and both the height and thickness of the three oblong blocks (*C*), gives for the solid contents of the three square blocks (*B*), 6000, and for the solid contents of the three oblong blocks (*C*), 1500; these two added together equal 7500. Again referring to the general principle stated above, we find that the only element required to complete the cube is the cube of the units.

Now, by reference to the illustrative blocks, observe that by placing the small cube (*D*) in its place in the corner, the cube is complete. And since (*D*) has been found to contain 5 × 5 × 5, or 125 cubic inches, add this sum to the 7500 obtained above, and the result is 7625. Subtracting 7625 from the remainder in the problem, nothing remains; hence, it has been shown that the cube root of 15,625 is 25. By the operation is also proved the correctness of the general principle as stated.

1045. From the foregoing explanations the following rule may be derived:

Beginning at the right, separate the given number into periods of three figures each.

Take for the first root figure the cube root of the greatest perfect cube in the left-hand period; subtract its cube from this left-hand period and to the remainder bring down the next period.

Divide this remainder, using as a trial divisor three times the square of the root figure already found, considered as tens, so obtaining the second or units' figure of the root; next subtract from the remainder three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

In examples of more than two periods proceed as above, and after two root figures are found, treat both as tens for finding the third root figure. For finding subsequent root figures treat all those found as so many tens.

In case the remainder, at any time after bringing down the next period, be less than the trial divisor, place a cipher in the root and proceed as before.

Should the cube root of a mixed decimal be required, form periods from the decimal point right and left. If the decimal be pure, point off from the decimal point to the right, and if need be annex decimal ciphers to make full periods.

The fourth root may be obtained by extracting the square root of the square root.

The sixth root is obtained by taking the cube root of the square root or the square root of the cube root.

WRITTEN EXERCISE

Extract the cube root of:

1. 1728.

2. 15625.

3. 110592.

4. 65939264.

5. $\frac{64}{125}$.

6. $\frac{1728}{148877}$.

7. $\frac{1331}{148877}$.

8. 1264.295441.

9. $\frac{91125}{274625}$.

APPLICATIONS OF CUBE ROOT

1046. It has been shown that the solid contents of a cube is the product of its three equal sides. Hence,

The side of any cube is the cube root of its solid contents.

WRITTEN EXERCISE

1. How many square inches in the six faces of a cubical block whose solid contents are 6400 cubic inches?

2. A cubical cistern contains 3375 cubic feet. What is its depth?

3. What must be the height of a cubical bin that will hold 1000 bushels of wheat?

4. A square cistern the capacity of which is 420 barrels, has a depth of only ½ its width. Find its dimensions.

5. A cubical cistern contains 630 barrels. How deep is it?

COMPOUND INTEREST TABLE FOR ANNUAL PAYMENTS

Showing how much $1.00 per annum will amount to, compounded annually, in any number of years from 1 to 50 years. (Compare with pages 251, 252.)

Years	3 per ct.	3½ per ct.	4 per ct.	4½ per ct.	5 per ct.	6 per ct.	Years
1	1.030	1.035	1.040	1.045	1.050	1.060	1
2	2.091	2.106	2.122	2.137	2.153	2.184	2
3	3.184	3.215	3.247	3.278	3.310	3.375	3
4	4.309	4.363	4.416	4.471	4.526	4.637	4
5	5.468	5.550	5.633	5.717	5.802	5.975	5
6	6.663	6.779	6.898	7.019	7.142	7.394	6
7	7.892	8.052	8.214	8.380	8.549	8.898	7
8	9.159	9.369	9.583	9.802	10.027	10.491	8
9	10.464	10.731	11.006	11.288	11.578	12.181	9
10	11.808	12.142	12.486	12.841	13.207	13.972	10
11	13.192	13.602	14.026	14.464	14.917	15.870	11
12	14.618	15.113	15.627	16.160	16.713	17.882	12
13	16.086	16.677	17.292	17.932	18.599	20.015	13
14	17.599	18.296	19.024	19.784	20.579	22.276	14
15	19.157	19.971	20.825	21.719	22.658	24.673	15
16	20.762	21.705	22.698	23.742	24.840	27.213	16
17	22.414	23.500	24.645	25.855	27.132	29.906	17
18	24.117	25.357	26.671	28.064	29.539	32.760	18
19	25.870	27.280	28.778	30.371	32.066	35.786	19
20	27.677	29.270	30.969	32.783	34.719	38.993	20
21	29.537	31.329	33.248	35.303	37.505	42.392	21
22	31.453	33.460	35.618	37.937	40.431	45.996	22
23	33.427	35.667	38.083	40.689	43.502	49.816	23
24	35.459	37.950	40.646	43.565	46.727	53.865	24
25	37.553	40.313	43.312	46.571	50.114	58.156	25
26	39.710	42.759	46.084	49.711	53.669	62.706	26
27	41.931	45.291	48.968	52.993	57.403	67.528	27
28	44.219	47.911	51.966	56.423	61.323	72.640	28
29	46.575	50.623	55.085	60.007	65.439	78.058	29
30	49.003	53.430	58.328	63.752	69.761	83.802	30
31	51.503	56.335	61.702	67.666	74.299	89.890	31
32	54.078	59.341	65.210	71.756	79.06	96.343	32
33	56.730	62.453	68.858	76.030	84.067	103.184	33
34	59.462	65.674	72.652	80.497	89.320	110.435	34
35	62.272	69.008	76.598	85.164	94.836	118.121	35
36	65.174	72.458	80.702	90.041	100.628	126.268	36
37	68.159	76.029	84.970	95.138	106.710	134.904	37
38	71.234	79.725	89.409	100.464	113.095	144.059	38
39	74.401	83.550	94.026	106.030	119.800	153.762	39
40	77.663	87.510	98.827	111.847	126.840	164.048	40
41	81.023	91.607	103.820	117.925	134.232	174.951	41
42	84.484	95.849	109.012	124.276	141.993	186.508	42
43	88.048	100.238	114.413	130.914	150.143	198.758	43
44	91.720	104.782	120.029	137.850	158.700	211.744	44
45	95.502	109.484	125.871	145.098	167.685	225.508	45
46	99.397	114.351	131.945	152.673	177.119	240.099	46
47	103.408	119.388	138.263	160.588	187.025	255.565	47
48	107.541	124.602	144.834	168.859	197.427	271.958	48
49	111.797	129.998	151.667	177.503	208.348	289.336	49
50	116.181	135.583	158.774	186.536	219.815	307.756	50

ANSWERS

Page 15. **1.** 43,400. **2.** 46,987. **3.** 48,099. **4.** 29,273. **5.** 21,771. **6.** 23,564. **7.** 25,606.

Page 19. **1.** 447,136,427. **2.** 387,213,476. **3.** 484,730,888. **4.** 503,980,350. **5.** 458,853,797. **6.** 475,095,610. **7.** 78,841. **8.** 84,551. **9.** 69,394. **10.** 79,473. **11.** 76,930. **12.** 53,143.

Page 20. **13.** Vertical totals: 34,134; 22,798; 26,057; 47,779; 34,788; 51,426. Horizontal totals: 23,764; 21,751; 15,089; 26,428; 31,480; 29,679; 30,061; 16,318; 22,412. Total, 216,982.

14. Vertical totals: Clothing, $4652.21; dry goods, $5500.32; furnishings, $849.08; millinery, $2357.92; household utensils, $4011.16. Horizontal totals: Monday, $2659.05; Tuesday, $2883.10; Wednesday, $2847.60; Thursday, $2613.38; Friday, $2937.30; Saturday, $3430.26. Total, $17,370.69.

Page 23. **1.** Vertical totals: Armories, $180,280.28; metropolitan sewer, $492,597.24; abolition of grade crossings, $393,663.73; metropolitan water, $1,946,951.37; highways, $1,157.89. Horizontal totals: 1895–1896, $392,774.63; 1896–1897, $404,963.27; 1897–1898, $386,627.68; 1898–1899, $425,974.50; 1899–1900, $647,621.81; 1900–1901, $756,688.62. Total, $3,014,650.51.

2. Vertical totals: Shoes, $2253.49; gloves, $1527.23; hats, $1409.44; dress goods, $3002.61; clothing, $3211.52. Horizontal totals: A to D Ledger, $1183.12; E to H Ledger, $1224.73; I to L Ledger, $1881.51; M to P Ledger, $1386.58; Q to T Ledger, $3078; U to Z Ledger, $2650.35. Total, $11,404.29.

3. Vertical totals: Domestics, $5760.89; notions, $3791.25; woolens, $6408.90; dress goods, $4961.63. Horizontal totals: Monday, $2056.71; Tuesday, $2481.41; Wednesday, $4749.31; Thursday, $2661.46; Friday, $4490.56; Saturday, $4483.22. Total, $20,922.67.

Page 24. Vertical totals: Registered letters, 11,420; ordinary letters, 94,667; postal cards, 9338; book packets, 3397; parcels, 1516; newspapers, 138,689. Horizontal totals: Monday, 45,717; Tuesday, 37,584 Wednesday, 42,788; Thursday, 47,162; Friday, 51,665; Saturday, 34,111. Total, 259,027.

5. 1,154,276,889. **6.** 662,377,884. **7.** 788,754,622. **8.** 837,865,199.

Page 29. **1.** 3380 lb. **2.** 1650 lb. **3.** 214 lb. **4.** 2520 lb.

Page 30. **5.** $1428.73. **6.** $7428.96. **7.** $654.97.

Page 31. **8.** $1722.32. **9.** $409.40. **10.** $463.73. **11.** $368.15.

Page 33. **1.** E. W. Allen, $1416.84; C. W. Briggs, $814.95; L. M. Comer, $1030.73; O. D. Day, $1477.43; A. L. Emery, $453.81; B. C. Foley, $907.83; J. I. Good, $1087.51; L. O. Hall, $1539.44; Chas. E. Irwin, $1929.54; Chas. H. Jones, $827.95. Total new balances, $11,486.03. Total old balances, $9054.87. Total checks, $3391.08. Total deposits, $5822.24.

2. D. T. Ames, $10,514.15; M. T. Ballou, $7602.30; W. T. Collins, $2663.89; Dorman & Co., $2985.63; Evans & Son, $1972.10; Farley Bros., $2162.01; Grant & Snow Co., $7574.90; Hall & Smith, $7578.60; J. T. Irwin, $1780.92; M. I. Jamison, $5338.33. Total new balances, $50,172.83. Total old balances, $36,732.21. Total checks, $8232.94. Total deposits, $21,673.56.

Page 35. **1.** $2000. **2.** $4225. **3.** $755. **4.** Gained $1638. **5.** Lost $96. **6.** $1056. **7.** 150 A.

Page 40. **1.** 462. **2.** 1584. **3.** 264. **4.** 14,190. **5.** 14,025. **6.** 1386. **7.** 4686. **8.** 7062. **9.** 11,000. **10.** 26,708. **11.** 3256. **12.** 2200. **13.** 8030. **14.** 4752. **15.** 6644. **16.** 8250.

Page 41. **1.** 1,480,016 ems. **2.** 6,856,080 men. **3.** 5,412,969 links. **4.** 106,272 pairs. **5.** $2,329,992. **6.** $53,816. **7.** 4,557,168 books. **8.** 11,751,810 lb. **9.** 125,244 lb. **10.** $154,716,975.

Page 43. **1.** 768. **2.** 1435. **3.** 67,680. **4.** 2921. **5.** 2226. **6.** 77,184. **7.** 47,082. **8.** 5220. **9.** 1222. **10.** 1776. **11.** 30,524. **12.** 10,679. **13.** 4250. **14.** 2088. **15.** 69,255. **16.** 136,000. **17.** 5248. **18.** 4644. **19.** 1734. **20.** 51,114. **21.** 8676. **22.** 3976. **23.** 4095. **24.** 135,030.

Page 44. **1.** 24,442. **2.** 46,748. **3.** 45,904. **4.** 66,825. **5.** 124,440. **6.** 133,179. **7.** 184,518. **8.** 291,450. **9.** 124,313. **10.** 112,420. **11.** 207,739. **12.** 379,080. **13.** 50,061. **14.** 197,290. **15.** 577,521. **16.** 145,754.

Page 45. **1.** 106,889. **2.** 23,150. **3.** 16,535.

Page 46. **1.** $3933. **2.** 549,120 ft. **3.** $498.96. **4.** 26,708 lb. **5.** 44,660 lb. **6.** Gained $1678.20.

Page 48. 1. 621 A. 2. 54.

Page 49. 3. 75,629. 4. 13. 5. 16 da. 6. 715 A. 7. $1105\frac{52}{125}$ cd. 8. 16,107. 9. 66. 10. Gained $2500.

Page 52. 1. 3, 3, 2, 2, 2, 2. 2. 2, 2, 31. 3. 11, 7, 3, 2, 2. 4. 17, 17. 5. 5, 3, 3, 3. 6. 5, 3, 3, 3, 191. 7. 7, 5, 5, 3, 3. 8. 3, 3, 7, 2, 2. 9. 3 and 317. 10. 3 and 509.

Page 53. 1. 11. 2. 12. 3. 7. 4. 16.

Page 54. 5. 12 ft. 6. 120 boards.

Page 55. 1. 480. 2. 450. 3. 624. 4. Jan. 1, 1904. 5. 252 A.

Page 56. 1. $92\frac{4}{7}$. 2. 25.

Page 57. 3. $15\frac{3}{4}$. 4. $37\frac{1}{3}$. 5. 45 bu. 6. 5 bbl. 7. 720 yd. 8. $5\frac{1}{4}$ pc. 9. 8800 bu. 10. $3\frac{1}{2}$ mi. 11. 5 sections. 12. 360 bbl. 13. 500 yd. 14. $120. 15. 100 bbl.

Page 62. 1. $11,693,823.17. 2. $1,417,548.52. 3. $1,193,532.63. 4. $1,384,147.19. 5. $24,320.65. 6. $1088.60. 7. $4073.76. 8. $384.51. 9. $2094.15.

Page 63. 1. $209.11. 2. $523.89. 3. $326.33. 4. $1256.81. 5. $5763.09.

Page 64. 1. Wheat, 350 bu.; oats, 425 bu.; corn, 175 bu. 2. $91.10.

Page 68. 1. 71,116,542. 2. 1,436,942,736. 3. 13,644,817,552. 4. $4265.91. 5. $299.64. 1. 195,448. 2. $2081.48.

Page 69. 3. $1387. 4. 10 yr. 5. 6,075,486. 6. 20 bbl. 7. $7500; $900. 8. $4500 and $4745. 9. $540.19. 10. 240 bbl. 11. $4960. 12. 1,450,950,624. 13. 2,046,757,518.

Page 73. 1. $\frac{67}{5}$. 2. $\frac{109}{4}$. 3. $\frac{20801}{20}$. 4. $\frac{2797}{15}$. 5. $\frac{88}{5}$. 6. $\frac{815}{4}$. 7. $\frac{130}{9}$. 8. $\frac{868}{33}$. 9. $\frac{609}{5}$. 10. $\frac{1701}{10}$. 11. $\frac{306}{7}$. 12. $\frac{6807}{16}$. 1. $142\frac{9}{16}$. 2. $17\frac{9}{28}$. 3. $27\frac{29}{73}$. 4. $13\frac{73}{79}$. 5. $38\frac{22}{181}$. 6. $8\frac{1}{29}$. 7. $24\frac{3}{17}$. 8. $53\frac{22}{23}$. 9. $23\frac{15}{47}$. 10. $16\frac{20}{91}$.

Page 74. 1. $\frac{3}{8}$. 2. $\frac{5}{12}$. 3. $\frac{13}{32}$. 4. $\frac{2}{25}$. 5. $\frac{168}{1457}$. 6. $\frac{11}{18}$. 7. $\frac{7}{9}$. 8. $\frac{3}{4}$. 9. $\frac{2}{3}$. 10. $\frac{3}{10}$.

Page 76. 1. $\frac{14}{126}, \frac{72}{126}, \frac{35}{126}$. 2. $\frac{180}{990}, \frac{440}{990}, \frac{693}{990}$. 3. $\frac{270}{390}, \frac{312}{390}, \frac{325}{390}$. 4. $\frac{24}{90}, \frac{40}{90}, \frac{75}{90}$. 5. $\frac{72}{102}, \frac{50}{102}, \frac{19}{102}$. 6. $\frac{60}{204}, \frac{136}{204}, \frac{17}{204}$. 7. $\frac{36}{81}, \frac{27}{81}, \frac{16}{81}$. 8. $\frac{20}{75}, \frac{27}{75}, \frac{45}{75}$.

Page 77. 1. $1\frac{65}{72}$. 2. $1\frac{496}{595}$. 3. $1\frac{1}{7}$. 4. $1\frac{51}{140}$. 5. $\frac{16}{17}$. 6. $1\frac{31}{48}$. 7. $\frac{131}{155}$. 8. $2\frac{37}{120}$. 9. $1\frac{137}{400}$. 10. $1\frac{32}{45}$. 11. $1\frac{709}{760}$. 12. $1\frac{40}{63}$.

Page 79. 1. 34,120. 2. $22,578\frac{5}{6}$. 3. $12,934\frac{39}{40}$. 4. $12,355\frac{8}{9}$.

Page 81. **1.** $151\frac{7}{80}$. **2.** $\frac{1}{156}$. **3.** $7\frac{23}{72}$. **4.** $10\frac{77}{408}$. **5.** 20. **6.** $11\frac{21}{40}$. **7.** $969\frac{5}{7}$. **8.** $29\frac{1}{4}$. **9.** $25\frac{53}{78}$. **10.** $47\frac{5}{16}$ acres.

Page 84. **1.** 25. **2.** $7\frac{5}{18}$. **3.** 44. **4.** 30. **5.** 248. **6.** $26\frac{5}{7}$. **7.** $462\frac{2}{9}$. **8.** $.94\frac{1}{2}$. **9.** $1\frac{8}{27}$. **10.** $\frac{11}{80}$. **11.** $\$84\frac{3}{8}$. **12.** $\$.63\frac{3}{4}$. **13.** $\$43\frac{3}{4}$. **14.** $\$1\frac{1}{2}$. **15.** $200. **16.** $\frac{7}{24}$.

Page 86. **1.** $2681\frac{1}{4}$. **2.** $505\frac{3}{4}$. **3.** $339\frac{3}{4}$. **4.** $679\frac{1}{4}$. **5.** $756\frac{1}{4}$. **6.** $316\frac{3}{4}$. **7.** $542\frac{1}{4}$. **8.** $635\frac{1}{4}$. **9.** $90\frac{3}{4}$. **10.** $1533\frac{3}{4}$. **11.** $1831\frac{3}{4}$. **12.** $939\frac{1}{4}$.

Page 89. **1.** $72. **2.** 5 shares. **3.** 40 families. **4.** $28\frac{1}{8}$ bu. **5.** 5 da. **6.** $\$4\frac{1}{2}$. **7.** $\frac{1}{5}$ A. **8.** $782\frac{2}{9}$ sacks. **9.** 18 da. **10.** 13 fields. **1.** $2\frac{25}{42}$. **2.** $7\frac{11}{30}$. **3.** $14,000. **4.** 3 bu. **5.** 2 lb. **6.** $135,000.

Page 90. **7.** $52,500. **8.** $70. **9.** $300. **10.** $2. **11.** $5. **12.** 336 trees. **13.** A, $6, B, $15, and C, $16. **14.** $35, $40. **15.** 1050 horses. **16.** $6\frac{3}{4}$ bu. **17.** $22\frac{1}{2}$ da. **18.** C. $\$38\frac{2}{9}$, J. $\$47\frac{7}{9}$. **19.** $114\frac{3}{4}$ ft. **20.** Colt, $94; cow, $30.

Page 91. **21.** $25\frac{1}{5}$ ft. **22.** 405 sheep. **23.** Carriage, $324; horse, $216. **24.** $6\frac{2}{3}$ da. **25.** $5\frac{95}{107}$ da. **26.** $42\frac{6}{7}$ da. **27.** A, $1260; B, $420; C, $840. **28.** Gained $10,106.56. **29.** A, $2800; B, $3500. **30.** 2161. **31.** 1200. **32.** 240 bu. **33.** 20 da.

Page 92. **34.** $\$2501\frac{7}{18}$. **35.** 28 da. **36.** 15 hr. **37.** A, $19.75; B, $15.80. **38.** Lose 52¢. **39.** 60 oranges. **40.** 28 bu. and 80 bu. **41.** $900. **42.** $255\frac{3}{4}$ bu. **43.** $11.91 gain. **44.** Gained $\$38\frac{1}{8}$. **45.** $\$438\frac{1}{8}$.

Page 93. **46.** Gained $98.32. **47.** $395.54; $9.17. **48.** Gained $3.14. **49.** $3277.40. **50.** $25,250.

Page 96. **1.** .26. **2.** .27. **3.** .0006. **4.** .04. **5.** 5.7. **6.** 500.05. **7.** .00022. **8.** 5000.005. **9.** 1,000,000.000001. **10.** .500 or .5. **11.** .00005.

Page 97. **12.** 7.7. **13.** 2.002. **14.** 2000.002. **15.** 11.000107. **16.** 83.0504. **17.** 710.00243. **18.** 54,054,054.0054054054. **19.** .37, .0004, 1.097, 3.0893, 9.17.

Page 98. **1.** $\frac{81}{50}$. **2.** $\frac{21}{200}$. **3.** $\frac{98}{250}$. **4.** $\frac{161}{400}$. **5.** $\frac{561}{2500}$. **6.** $\frac{989}{5000}$. **7.** $\frac{5818}{12500}$. **8.** $\frac{7207}{25000}$. **9.** $\frac{17}{40}$. **10.** $\frac{12021}{50000}$. **11.** $\frac{2001}{25000}$. **12.** $\frac{578}{5000}$. **13.** $\frac{1}{120}$. **14.** $\frac{49}{150}$. **15.** $\frac{47}{150}$. **16.** $5\frac{4}{25}$. **17.** $13\frac{41}{200}$. **18.** $11\frac{3}{4}$. **19.** $31\frac{27}{200}$. **20.** $16\frac{81}{500}$. **21.** $81\frac{118}{625}$. **22.** $65\frac{88}{250}$. **23.** $35\frac{3}{2500}$. **24.** $15\frac{2}{125}$. **25.** $28\frac{4}{9}$. **1.** .0625. **2.** .15. **3.** .275. **4.** .09375. **5.** .1375. **6.** .5652173+. **7.** .0525. **8.** .46875. **9.** .024. **10.** .9375. **11.** .015625. **12.** .96875. **13.** .028. **14.** .95. **15.** .94.

Page 99. 1. 848.1816. 2. 1652.461772. 3. 12,638.517852. 4. 1,000,608.012354001. 5. 57,697.358230005. 6. 385.8225 yd. 7. 41.885 cd. 8. 136.33 thousand feet. 9. 84.423 T. 10. Number of thousand feet, 101.184; total cost, $ 1403.75. 11. 376. 12. 926^2 or 926½.

Page 100. 1. .52977. 2. 1.27848. 3. 5.5264. 4. 1.546548. 5. .81. 6. .198. 7. 754.6005. 8. .3148. 9. 385,994.01246. 10. 1000.0099. 11. 2.99985. 12. 102.93702.

Page 101. 1. 0. 2. 117.843385. 3. .2375. 4. .6. 5. .0009. 6. 7231.98325125. 7. .0018044. 8. $ 554.63. 9. $ 336.33. 10. $ 14,856.56. 11. $ 3510.71. 12. $ 496.05.

Page 102. 13. $ 1068.57. 14. $ 3537.58.

Page 105. 1. 10,011,112.1010001. 2. 6,330,303.3333. 3. 40,448,404.48. 4. 1,056,000.605. 5. 5,655,500.1005. 6. 30,003.36303603.

Page 106. 1. 2.92125. 2. 751.383957246471. 3. $ 127.12.

Page 107. 4. 6.875 da. 5. 24.93+ thousand feet. 6. 7 yr. 7. Gained $ 5513.80. 8. Gained $ 400.75. 9. $ 1124.34. 10. 48 da. 11. Entire school, 1000 pupils; bookkeeping department, 500; shorthand and typewriting department, 375; English department, 125. 12. $ 2238.75.

Page 108. 13. Total gain, $ 5950; net gain, $ 5000. 14. $ 7500.

Page 112. 1. 371 lb. 2. 81.5 yd.

Page 113. 3. 115.2 A. 4. 689 yd. 5. 123 lb. 6. $ 40.96.

Page 115. 1. $ 2166. 2. $ 5163.97. 3. $ 1142.33.

Page 117. 1. $ 2643.52. 2. $ 6315.38. 3. $ 2462.43. 4. $ 8789.88. 5. $ 8256.38.

Page 118. 1. $ 20; $ 19.13. 2. $ 28.35; $ 28. 3. $ 30.17; $ 8. 4. $ 28.13; $ 60. 5. $ 121.50; $ 16. 6. $ 630.63; $ 180. 7. $ 31.04; $ 62.50. 8. $ 11.17; $ 12. 9. $ 148.28; $ 247.50. 10. $ 61.40; $ 125.

Page 119. 1. $ 8.25. 2. $ 14.66. 3. $ 158.76. 4. $ 123.18. 5. $ 53. 6. $ 38.75. 7. $ 19.80. 8. $ 100.32. 9. $ 140.81. 10. $ 399. 1. $ 2.68. 2. $ 4.71. 3. $ 78.70. 4. $ 1.42. 5. $ 1985.26. 6. $ 1609.78. 7. $ 903.96. 8. $ 21.06. 9. $ 12.65. 10. $ 19.51.

Page 120. 1. $ 32. 2. $ 29.71. 3. $ 52.52. 4. $ 39.29. 5. $ 24.66. 6. $ 22.63. 7. $ 55.67. 8. $ 58.33. 9. $ 174.44. 10. $ 132.36.

Page 128. 1. $11,607.65.

Page 129. 2. $322.67.

Page 130. 3. $452.57. 4. $540.26.

Page 131. 1. $458.33. 2. $524.03. 3. $168.68. 4. $1040.65.

Page 132. 5. $429.16. 6. $943.54. 7. $987.11. 8. $440.48. 9. $97.44.

Page 133. 10. (*a*) $170.29; (*b*) $136.23. 11. (*a*) $381.70; (*b*) $305.37.

12. (*a*) *10.* 20's, 1; 10's, 9; 5's, 7; 2's, 9; 1's, 4; halves, 4; quarters, 3; dimes, 3; nickels, 2; pennies, 14. (*b*) *10.* 10's, 9; 5's, 4; 2's, 9; 1's, 4; halves, 4; quarters, 6; dimes, 5; nickels, 4; pennies, 3. (*a*) *11.* 20's, 11; 10's, 7; 5's, 11; 2's, 13; 1's, 5; halves, 6; quarters, 6; dimes, 7; nickels, 7; pennies, 15. (*b*) *11.* 20's, 8; 10's, 7; 5's, 8; 2's, 11; 1's, 5; halves, 9; quarters, 9; dimes, 11; nickels, 7; pennies, 17.

Page 151. 1. 193,555 in. 2. 12,363*d.* 3. 709 pt. 4. 175 qt. 5. 155,243″. 6. 561 gi. 7. 180,002 oz. 8. 238,475 cu. in. 9. 6,860,715 sq. in. 10. 9543 l. 11. 296,065 oz. 12. 1373 pwt. 13. 3767 in. 14. 3859½ sq. ft. 15. 182,727.75 sq. ft. 16. 4,345,531 sec. 17. 194 cu. ft. 18. 40,396 gr. 19. 269 pt. 20. 129,832 in.

Page 152. 1. 7 wk. 1 da. 15 hr. 20 min; or 1 mo. 20 da. 15 hr. 20 min. 2. 24 bbl. 20 gal. 1 qt. 3. 112 bu. 2 pk. 5 qt. 4. 8 A. 66 sq. rd. 3 sq. yd. 4 sq. ft. 72 sq. in. 5. 3 mi. 124 rd. 2 yd. 8 in. 6. 1 T. 17 cwt. 95 lb. 7. 9 lb. 1 oz. 5 pwt. 20 gr. 8. 67 yr. 5 mo. 2 wk. 1 da. 9. 2 hr. 38 min. 57 sec. 10. 66 A. 72 sq. rd. 11. 5 cu. ft. 152 cu. in. 12. 17 T. 8 cwt. 32 lb.

Page 153. 1. 432 gr. 2. 10 pwt. 3. 213 rd. 1 yd. 2 ft. 6 in. 4. 110 sq. rd. 5. 112 A. 40 sq. rd. 29 sq. yd. 51.84 sq. in. 6. 43 sq. rd. 19 sq. yd. 2 sq. ft. 36 sq. in.

Page 154. 1. £$\frac{1}{800}$. 2. £$\frac{7}{160}$. 3. .12 T. 4. .017361+ yd.

Page 155. 1. .327 T. 2. .0625 A. 3. $\frac{809}{320}$ lb. 4. .29791+ lb. 5. $\frac{7089}{8000}$ T.

Page 156. 1. £90 3*s.* 2. 17 mi. 46 rd. 1 yd. 2 ft. 3. 33 A. 2 sq. rd. 17 sq. ft. 7 sq. in. 4. 672 lb. 1 oz. 12 pwt. 8 gr. 5. 211 lb. 11 oz. 18 pwt. 21 gr.

Page 157. 1. 14 gal. 2 gi. 2. 396 A. 78 sq. rd. 3. 3 yd. 2 ft. 1¼ in. 4. 6 lb. 9 oz. 7 pwt. 17 gr. 5. $44.31. 6. 6 T. 5 cwt. 7. Gained £507 5*s.*

Page 159. 1. 96 da. 2. 236 da. 3. 323 da. 4. 87 da. 5. 223 da. 6. 233 da. 7. 49 da. 8. 241 da. 9. 81 da. 10. 402 da. 11. 523 da. 12. 883 da. 13. 5 yr. 7 mo. 3 da. 14. 17 yr. 15. 4 yr. 4 mo. 5 da. 16. 2 yr. 10 mo. 25 da. 17. 3 yr. 1 mo. 3 da. 18. 7 yr. 4 mo. 23 da. 19. 3 yr. 1 mo. 1 da. 20. 18 yr. 8 da. 1. $1924.39. 2. $1958.25.

Page 160. 3. 54 T. 16 cwt. 47 lb.; $699. 4. $70.20. 5. $577.78; $650; $433.33. 6. $20,589.84. 7. $97.50 gain. 8. $73.88. 9. $122.80 by first method; $122.79 by second method.

Page 161. 1. 2 bu. 3 pk. 2. 1 T. 57 lb. 12 oz. 3. £4 15*s.* 3⅔ far. 4. 3 yr. 46 da. 7 hr. 30 min. 5. 1 lb. 3 oz. 5 pwt. 11 gr. 6. $13,050. 7. $19.97. 8. $5. 9. £1061 3*s.* 9*d.* 1 far. 10. £38 3*d.* 2.4 far.; £513 14*s.* 3*d.* 3.36 far. 1. 12. 2. £770 11*s.* 5*d.* 3 far. 3. 4 A. 83 sq. rd. 6 sq. yd. 64⅘ sq. in. 4. 101 sq. rd. 2 sq. yd. 21.6 sq. in. 5. 123 mi. 162 rd. 3 yd. 1 ft. 4 in.

Page 162. 6. $6.20 gain. 7. Gained 3¢. 8. $72.58 gain. 9. $19.16. 10. $54. 11. $72.81. 12. Gained $360.67. 13. $1209.60 gain. 14. $9.77. 15. $88.89.

Page 164. 1. 75 ft. 4.7808 in. 2. $335. 3. $17.33. 4. 440.6094 ft. 5. 1250 ft. 6. 168 ft. 4 in.

Page 166. 1. 4 A. 116 sq. rd. 2. 19 A. 122 sq. rd. 3. 34 A. 4. 10 A. 56 sq. rd. 5. 15 A. 6. 12 A. 12 sq. rd. 7. 446 A. 5 sq. ch., or 446 A. 80 sq. rd. 8. 249 A. 9 sq. ch., or 249 A. 144 sq. rd. 9. 390 A. 10. $3277.97. 11. 357 rd. 5 ft. 6 in. 12. 414 ft. $10\frac{2}{7}$ in. 13. Ark., 177 A. 80 sq. rd.; N. and S. Dak., 124 A. 40 sq. rd.; all other states, 133 A. 20 sq. rd. 14. $10.56. 15. $121,500. 16. 57 sq. rd. 21 sq. yd. 108 sq. in. 17. $100. 18. $262.35.

Page 168. 1. 63 yd. 2. $15.63. 3. 128⅛ yd.; 130 yd.; $327.25. 4. 65¼ yd. 5. $150. 6. 61⅚ yd.; 82½ yd.

Page 169. 7. $9.46. 8. $13.10.

Page 170. 1. 75 rolls. 2. $8.75. 3. 14 rolls. 4. $15.20. 5. $39.60.

Page 171. 1. $23.99. 2. $23.11. 3. $45.32. 4. $77.61. 5. $38.85. 6. $24.17.

Page 172. 1. $182.25. 2. $63. 3. $49.75. 4. 60,000 shingles. 5. 48 M.; $168. 6. $192.

Page 174. 1. $37.70. 2. 1458 cu. ft. 3. $284.28. 4. 1316¼ cu. ft. 5. 1570⅘ cu. ft. 6. 36 ft. 7. $16.29.

Page 176. **1.** 420 board ft. **2.** $14.04. **3.** $11.20. **4.** 27¢. **5.** $108.39. **6.** $23.42. **7.** $24.18. **8.** $21.78. **9.** $132. **10.** $132.44.

Page 178. **1.** 39$\frac{15}{64}$ cd. **2.** 26 ft. 9$\frac{3}{35}$ in. **3.** $227.11. **4.** 64 ft. **5.** $45. **1.** $6000. **2.** With asphalt; save $45,883.20. **3.** With brick; save $85.33. **4.** 147,840 blocks. **5.** $58,594.44.

Page 180. **1.** 345.6 bu. **2.** 1040 bu. **3.** 768 bu. **4.** 1920 bu. **5.** 480 bu. **6.** 1536 bu. **7.** 237.476+ bbl.

Page 181. **8.** 64 rd. 9.6+ ft. **9.** 1903.5648 gal. **10.** 22.3074 gal.

Page 182. **1.** 81$\frac{5}{11}$ or 81.45+ perches. **2.** 118 cu. yd. 14 cu. ft., or 118$\frac{14}{27}$ cu. yd. **3.** 836.1 perches. **4.** $1701.68. **5.** 284,460 bricks; 280,104 bricks. **6.** $283.50. **7.** 546,920 bricks; 535,040 bricks. **1.** $8100. **2.** 8 ft. **3.** $7200. **4.** $32.58.

Page 183. **5.** $25.34. **6.** $75.92. **7.** $57.60. **8.** $120. **9.** $140. **10.** 90 pupils. **11.** $21.36. **12.** $4937.50. **13.** $5. **14.** 149$\frac{1}{3}$ cu. ft.; 1792 board ft.; $\frac{1}{12}$ cu. ft. **15.** $3465. **16.** Cheaper to pave with asphalt; $10,260.80. **17.** $165.

Page 188. **1.** $60,000. **2.** $2700. **3.** Wheat, 8320 bu.; oats, 3120 bu.; barley, 13,520 bu.

Page 189. **4.** $17,820. **5.** $14.04. **6.** $4.40. **7.** $576.60; 558 bu. **8.** $40,500.

Page 190. **1.** 50%. **2.** 25%. **3.** 95%. **4.** 5$\frac{5}{9}$%. **5.** 54$\frac{11}{16}$%. **6.** 800%. **7.** 20%. **8.** 100%. **9.** 50%. **10.** 33$\frac{1}{3}$%.

Page 191. **1.** 24. **2.** 5%. **3.** $1900.

Page 192. **4.** 2040 qt. **5.** $40,300. **6.** $3750. **7.** 200 trees. **8.** $10.56. **9.** $54. **10.** 370$\frac{5}{8}$ bu.

Page 193. **1.** $600. **2.** $400. **3.** $20,000. **4.** $4.50. **5.** $28. **6.** 500 pupils. **7.** 36,080. **8.** $8000. **9.** 560 bu. **10.** 200,100.

Page 194. **1.** $10,000. **2.** $20,840. **3.** $3200.

Page 195. **4.** Horse, $300; carriage, $156. **5.** Monday, $500; Tuesday, $400; Wednesday, $600. **6.** $32.40. **7.** $175. **8.** A, $1000; B, $600. **1.** 23$\frac{17}{21}$%; 42$\frac{6}{7}$%; 33$\frac{1}{3}$%. **2.** $300. **3.** $1959.38. **4.** A, $7125; B, $2500. **5.** $7200; $12,000. **6.** 1$\frac{7}{8}$% gain. **7.** $13,650. **8.** 33$\frac{1}{3}$%.

Page 196. **9.** 122,048. **10.** 40 gal. **11.** 27$\frac{1}{37}$%; 21$\frac{23}{37}$%; 18$\frac{2}{111}$%. **12.** 25%. **13.** Amount of indebtedness, $2252.08; in bank, $9008.33. **14.** Lost, $666.67. **15.** $23,400. **16.** $64,000 **17.** $60,250.

Page 197. **18.** Carriage, $150; horse, $187.50; harness, $50. **19.** Cost, $308; lost, $28. **20.** $\frac{1}{40}\%$; .000125; 3642%. **21.** 5 cwt. 61.48 lb., or 5 cwt. 61 lb. 7.68 oz. **22.** First cost, $7480.50; $8\frac{104}{4987}\%$. **23.** Wife, $21,750; daughter, $10,000; younger son, $12,500; elder son, $13,750. **24.** $1750; $3062.50; $6125; and $8575. **25.** $3850. **26.** $20,000. **27.** $133\frac{1}{3}\%$. **28.** Grazing, 504 A.; grain, 420 A.; timber, 936 A.

Page 198. **29.** $7200. **30.** 50%.

Page 200. **1.** $826.67. **2.** $567. **3.** $2400. **4.** $252.80. **5.** 360 yd. **6.** $72. **7.** First; $$\frac{3}{4}$. **8.** Gained $18.

Page 201. **9.** $66\frac{2}{3}\%$; $420. **10.** $374.75.

Page 202. **1.** $680.24. **2.** $1246.80. **3.** $322.87. **4.** Amount of bill to render, $4863.75; amount to be remitted, $4620.56. **5.** Amount of bill to render, $2199; amount to be remitted, $2133.03.

Page 203. **6.** $222.30; trade discount, $91; cash discount, $11.70. **7.** $234.38. **8.** $270.

Page 205. **1.** 12%. **2.** $1000. **3.** $165.75. **4.** $920. **5.** $1000. **6.** $2\frac{2}{9}\%$.

Page 206. **1.** Lead pipe, 35¢; iron pipe, 13¢; bath tubs, $8.64, $7.20, and $5.76. **2.** $1317.50. **3.** $126. **4.** Second is $9 cheaper. **5.** $189.47.

Page 207. **1.** Gain, $24.75; selling price, $173.25. **2.** $6406.25. **3.** Gained, $74. **4.** $105 gain. **5.** $37.50 gain.

Page 208. **1.** 2000 bu. **2.** $29\frac{3}{13}\%$. **3.** $49\frac{1}{3}\%$. **4.** $6\frac{1}{4}\%$. **5.** $33\frac{1}{3}\%$. **6.** 50%. **7.** $77\frac{7}{9}\%$.

Page 209. **8.** $29\frac{9}{29}\%$. **1.** $5000. **2.** $2750. **3.** $40. **4.** $7085.71. **5.** $150.

Page 210. **1.** $10,500. **2.** $60 and $100. **3.** $1000. **1.** $2000. **2.** $22\frac{2}{9}\%$. **3.** $1218.75. **4.** 400 bbl.

Page 211. **5.** 1000 bu. **6.** 25%. **7.** $16\frac{2}{3}\%$. **8.** $20,010. **9.** 22% loss. **10.** $625. **11.** $11.25. **12.** $37\frac{1}{7}\%$. **13.** $2. **14.** $46\frac{2}{3}\%$. **15.** $$31\frac{7}{8}$. **16.** $33\frac{1}{3}\%$. **17.** $74.25.

Page 212. **18.** $9350. **19.** $16\frac{2}{3}\%$ loss. **20.** $10,000. **21.** $110. **22.** $16.80. **23.** $59,320. **24.** $3\frac{4}{7}\%$ loss. **25.** 15¢. **26.** $18\frac{16}{23}\%$ loss. **27.** $58.60.

Page 213. **28.** $9\frac{13}{15}$ gal. **29.** $14\frac{2}{7}\%$ gain. **30.** $300. **31.** 30¢. **32.** 50¢. **33.** 20%. **34.** $49\frac{1}{3}\%$. **35.** $231\frac{1}{29}\%$. **36.** 20%. **37.** $1080; $2\frac{6}{7}\%$. **38.** $250.

Page 214. **39.** 50%. **40.** 14½. **41.** $5985. **42.** $6.

Page 215. **1.** $\frac{\$E.RA}{\$I.BW}$; $\frac{\$E.IA}{\$I.BW}$. **2.** $\frac{\$S.TA}{\$T.MB}$; $\frac{\$I.EA}{\$M.LB}$. **3.** $\frac{\$.ST}{\$.TH}$; $\frac{\$EH.AG}{\$HA.LB}$. **4.** $\frac{\$I.EO}{\$M.LC}$; $\frac{\$Z.OA}{\$M.BS}$. **5.** $\frac{\$IER}{\$SHB}$; $\frac{\$EI.UA}{\$IO.TB}$. **6.** $\frac{\$SO}{\$TH}$; $\frac{\$E.IA}{\$I.WB}$.

Page 217. **1.** $1.67. **2.** 94¢. **3.** $2.43. **4.** $2.72. **5.** 40¢. **6.** $3.39. **7.** 74¢. **8.** $2.53. **9.** $1.75. **10.** $2.10. **11.** 15¢. **12.** $1.33. **13.** $3.75. **14.** $10.88. **15.** $3.55. **16.** $2.75. **17.** $2.67. **18.** $6. **1.** $40. **2.** $12.

Page 218. **3.** $64. **4.** 25%. **5.** 8⅓%. **1.** $7. **2.** Shoes, $5; boots, $4; rubbers, 85¢. **3.** Hosiery, 25¢; 26¢; 40¢; 50¢. Knit goods, 75¢; 43¢; $1.70; $1.08. **4.** Hosiery, $\frac{\$.AY}{\$.LK}$; $\frac{\$.AH}{\$.LH}$; $\frac{\$.NA}{\$.CE}$; $\frac{\$.DY}{\$.KE}$. Knit goods, $\frac{\$.OY}{\$.OK}$; $\frac{\$.ND}{\$.CA}$; $\frac{\$H.NO}{\$B.OE}$; $\frac{\$.EM}{\$B.ER}$. **5.** Hats, $2.22; gloves, $1.11; ties, 56¢. **6.** Hats, $1.67; $1.17; $2.78; $3.89. Gloves, $2; $2.33; $1.50. **7.** 9¢. **8.** 64¢. **Page 219.** **9.** 26% gain. **10.** 28⅘¢.

Page 221. **1.** Comm., $225; remitted to principal, $11,025. **2.** $93.75. **3.** $1110. **4.** $3937.50. **1.** 2½%. **2.** 2%.

Page 222. **1.** $90,000. **2.** 50 bales.

Page 223. **3.** 9000 bu. **4.** $16,910.40.

Page 224. **1.** 30,000 lb. **2.** 145 doz. **3.** 4000 lb.; comm., $30. **4.** 1750 lb.

Page 225. **1.** $2929.70. **2.** $4818.26. **3.** $394.48.

Page 226. **4.** $740.88. **5.** $1896.30. **6.** $67.50. **7.** 225 bbl. **8.** 4½%. **9.** 33⅓%. **10.** 2000 lb.; $16.20 comm. **11.** $1818.60.

Page 227. **12.** $1097.40. **13.** $1888. **14.** 1890 bbl.; $1.11. **15.** $1508.57 comm.; $16,091.43. **16.** 5½¢. **17.** 10,666⅔ yd. **18.** 5%. **19.** Insurance, ¼%; comm., 2%; net proceeds, $12,453. **20.** 14,400 bu.

Page 230. **1.** $9.13. **2.** $2.66. **3.** $3.68. **4.** $4.68. **5.** $6.44. **6.** $2.62. **7.** $7.55. **8.** $11.10. **9.** $9.05. **10.** 77¢. **11.** 58¢. **12.** $1.99. **13.** $10.19. **14.** $15.24. **15.** $4.99. **16.** $8.59. **17.** $5.84. **18.** $15.03. **19.** $1.36; $5.27. **20.** $11.08; $16.04. **21.** $34.99; $49.81. **22.** $81.92; $74.36. **23.** $68.06; $32.58. **24.** $24.40; $34. **25.** $12.04; $37.

Page 232. **1.** 9091. **2.** $1713.79. **3.** $2774.75. **4.** $2408. **5.** $1758.17. **6.** $4298. **7.** $2995.54. **8.** $1788.98. **9.** $2453.03. **10.** $1877.98. **11.** $2429.89. **12.** $7515.75. **13.** $2405.70. **14.** $1851.64. **15.** $2433.23. **16.** $1708.28.

Page 235. **1.** $41.95. **2.** $60.62. **3.** $110.15. **4.** $352.88.

Page 236. **1.** $461.40. **2.** $169.63. **3.** $110.58.

Page 237. **4.** $157.67.

Page 238. **1.** $7.95. **2.** $2.25. **3.** $21.74. **4.** $.20. **5.** $.32. **6.** $3.05. **7.** $2.20. **8.** $1.51. **9.** $5.58. **10.** $4.35. **11.** $14.72. **12.** $4.70. **13.** $1.57. **14.** $2.01. **15.** $1.83. **16.** $4.28. **17.** $2.80. **18.** $.88.

Page 239. **1.** $4.38. **2.** $5.25. **3.** $3.71. **4.** $17.44. **5.** $9.24. **6.** $1.13. **7.** $5.83. **8.** $3.33. **9.** $1.76. **10.** $5.70. **11.** $2.34. **12.** $8.02. **13.** $6.32. **14.** $3. **15.** $24.58. **16.** $1.15. **17.** $4.37. **18.** $11.65. **19.** $1.34. **20.** $10.96. **21.** $6.61. **22.** $19.71. **23.** $25.80. **24.** $10,60. **25.** $25.38. **26.** $2.39. **27.** $4.86. **28.** $8.22. **29.** $6.18. **30.** $2.82. **31.** $7.79. **32.** $1.79.

Page 240. **33.** $8.78. **34.** $14.30. **35.** $1.41. **36.** $1.47. **37.** $.42. **38.** $12.87. **39.** $10.50. **40.** $7.70. **41.** $79.20. **42.** $29.96. **43.** $80.84.

Page 242. **1.** $19.75. **2.** $7.84. **3.** $2.13. **4.** $11.67. **5.** $52. **6.** $28.80. **7.** $8. **8.** $1.11. **9.** $17.70. **10.** $12.67. **11.** $38.53. **12.** $8.80.

Page 243. **1.** $221.40. **2.** $2800.53. **3.** $300.28. **4.** $305.97. **5.** $1640.63. **6.** $1785. **7.** $4981.67. **8.** $98.90. **9.** $100.83.

Page 244. **10.** $2766.55. **1.** $11.53. **2.** $5.18. **3.** $3.62. **4.** $2.80. **5.** $5.19. **6.** $16.81. **7.** $3.95. **8.** $5.10. **9.** $16.95. **10.** $73.97. **11.** $15.28. **12.** $25.70. **13.** $68.24. **14.** $21.57. **15.** $21.37.

Page 245. **1.** 4%. **2.** 6½%. **3.** 7½%. **4.** 5%.

Page 246. **1.** 2 yr. 5 mo. 24 da. **2.** 3 yr. 10 mo. 12 da. **3.** April 25, 1881.

Page 247. **1.** $4408.16. **2.** $3204.23. **3.** $27,500.

Page 248. **1.** $3000. **2.** $4000. **3.** $5000. **4.** $2000.

Page 249. **1.** $202.08. **2.** $21.60. **3.** $366.60. **4.** $2422.30.

Page 250. **1.** $372.96. **2.** $96.44. **3.** $68.66. **4.** $588.46. **5.** $176.12.

Page 253. 1. $2921.83. 2. $2024.80. 3. $38.64. 4. $253.23. 5. $2766.76.

Page 254. 1. $1291.71. 2. 4%. 3. 6%; $600. 4. $51.20. 5. 4%.

Page 255. 6. 27 da. 7. 10%; 10 yr. 8. $74.82. 9. $23.61. 10. $537.97. 11. 12%. 12. $920.08. 13. $20.72. 14. $1\frac{1}{2}$%. 15. 3 yr. 7 mo. 24 da. 16. $7\frac{1}{2}$ yr. 17. $6\frac{41}{42}$%.

Page 256. 18. $4644.61. 19. May 18, 1905. 20. $2728.82.

Page 257. 1. Interest; $12.29.

Page 258. 2. Better to pay cash; 55¢. 3. No difference. 4. Lose $11.82. 5. $1523.15. 6. $7.48. 7. $6.40. 8. 4% loss. 9. $15,000. 10. $4000. 11. $29.79. 12. July 1, 1903. 13. $10,855.79.

Page 259. 14. 4%. 15. $9736.94.

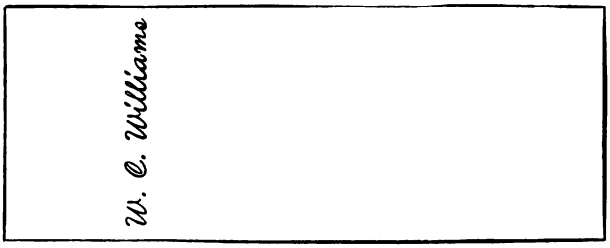
Page 266. 1. Apr. 15; 80 da. 2. Apr. 7; 23 da. 3. July 6; 64 da. 4. May 20; 49 da. 5. Dec. 20; 91 da. 6. Aug. 4; 47 da. 7. Sept. 7; 23 da. 8. Jan. 1; 73 da. 9. Feb. 22; 9 da. 10. Nov. 6; 25 da. 11. Feb. 27; 60 da. 12. Mar. 1; 84 da. 13. June 1; 85 da. 14. Sept. 29; 81 da. 15. Feb. 14; 28 da. 16. Mar. 14; 26 da.

Page 267. 1. Bank discount, $17.60; proceeds, $2382.40. 2. $9.80; $1190.20. 3. $18.28; $3231.72. 4. $3.13; $496.87. 5. $22.08; $2477.92. 6. $36; $3564. 7. $15; $1485. 8. $32; $2368. 9. $13.50; $4486.50. 10. $13.87; $2386.13. 11. Proceeds, $1188.

$1200.00. (Your place), Apr. 18, 19—.

Sixty days after date we promise to pay to the NATIONAL BANK OF REDEMPTION, or order, Twelve Hundred Dollars, value received.

J. M. Cox & Co.



Page 268. 12. Maturity, July 14, 1903; term of discount, 74 da.; discount, $11.10; proceeds, $1188.90. 13. June 1, 1903; 60 da.;

$45; $4455. **14.** Feb. 25, 1904; 55 da.; $11; $889. **15.** Aug. 3, 1903; 63 da.; $5.87; $664.94. **16.** Feb. 29, 1904; 177 da.; $97.55; $2382.45. **17.** Feb. 28, 1903; 18 da.; $2.41; $800.92.

Page 269. **18.** July 5, 1903; 65 da.; $9.95; $1214.05. **19.** Aug. 23, 1903; 85 da.; $36.50; $2539.75. **20.** Oct. 16, 1903; 56 da.; $1.54; $218.46. **21.** Nov. 5, 1903; 35 da.; $2.46; $419.04.

Page 270. **22.** July 14, 1904; 54 da.; $7.16; $787.54. **23.** Bank discount, $10; proceeds, $1190. **24.** $3483.28. **25.** Maturity, May 19, 1903; bank discount, $7.20; proceeds, $1792.80. **26.** Overdrawn, $123.98. **27.** $420.48, to their credit. **28.** $2.43; $500.07. **29.** $2.35; $647.76.

Page 271. **1.** $659.98. **2.** $600. **3.** $4000. **4.** $2400. **5.** $2400. **6.** $355.64.

Page 274. **1.** $550.40. **2.** $845.69. **3.** $1661.87. **4.** $344.45. **5.** $3089.10. **6.** $1181.24.

Page 276. **1.** $635.92. **2.** $122.29. **3.** $279.64. **4.** At 6%, $183.49; at 5%, $179.70; at 8%, $191.08. **5.** $180.93.

Page 282. **1.** Oct. 18, 1903. **2.** Nov. 15, 1903. **3.** June 23, 1903. **4.** Dec. 23, 1903.

Page 284. **1.** Apr. 11, 1903. **2.** June 5, 1903. **3.** July 6, 1903. **4.** Jan. 24, 1904. **5.** May 17, 1903. **6.** Nov. 18, 1903. **7.** July 22, 1903.

Page 288. **1.** Dec. 9, 1902.

Page 289. **2.** Feb. 6, 1903. **3.** Apr. 18, 1903. **4.** Sept. 10, 1903. **5.** Aug. 8, 1903.

Page 290. **1.** Dec. 7, 1903.

Page 291. **2.** Dec. 31, 1903.

Page 293. **1.** $313.18. **2.** $613.78. **3.** Equated date, Jan. 4, 1902; cash balance, $435.15.

Page 295. **1.** $2034.71. **2.** $1730.26.

Page 296. **3.** $1985.73.

Page 298. **1.** $752.50.

Page 299. **2.** $781.04. **1.** $10.

Page 300. **2.** $1264.74.

Page 304. **1.** $3637.50. **2.** $7275. **3.** $17,381.25. **4.** $8820. **5.** $26,400. **6.** $25,967.50.

Page 305. **7.** $92,868.75. **8.** $2917.50. **9.** $80,500, par value; $83,263.75, market value. **10.** 110. **1.** 400 shares; $40,000. **2.** 500 shares. **3.** 740 shares. **4.** 300 shares; $1200.

Page 306. 1. $ 64,837.50. 2. $ 29,718.75. 3. $ 180. 4. $ 51,700.

Page 307. 1. 5.268+ %. 2. N.Y. Air Brake, better by 1.481+ %. 3. 4.593+ %. 1. $ 1440. 2. $ 75,475.50.

Page 308. 3. Total dividend, $ 9,892,460; C's dividend, $ 110. 4. Total dividend, $ 251,250; F's dividend, $ 93.75. 5. Surplus fund, $ 47,085.79; dividend, $ 800,000; undivided profits, $ 498,222.46. 1. 9 %. 2. 6 % dividend; $ 11,135.60, undivided profits.

Page 309. 3. 13 %; $ 567,274.83 undivided profits. 1. $ 1352.06. 2. $ 415.38 loss.

Page 310. 3. $ 3782.55.

Page 312. 1. 4.347+ %. 2. $ 3600.62. 3. $ 62,625. 4. 3.897+ %. 5. $ 63,825 invested; 4.23 + %, rate of income. 6. $ 57.50. 7. 50 bonds. 8. 44 bonds; 3.936+ %, rate of income.

Page 313. 9. Mo. Pacific, better by .741+ %. 1. $ 81,940.63. 2. $ 1000. 3. $ 72,031.25. 4. Quotation, $121\frac{1}{2}$; brokerage, $ 62.50. 5. 5.024 + %. 6. 3.636+ %. 7. $ 1004.05 to make margin good; $ 1079.05 loss if sold out. 8. 127 bonds; unexpended balance, $ 485.62.

Page 314. 9. $ 36.25 gain; $ 3343.75 on hand. 10. Am. Ice Secs., 8.139+ %; Del. & Hud., 4.176+ %; Balt. & Ohio, 5.068+ %; Erie, 2d pf., 6.324+ %. 11. Surplus fund, $ 508,067.90; dividend, $ 3,500,000; undivided profits, $ 6,153,290.10. 12. $ 50,437.50. 13. $4\frac{1}{2}$ %. 14. Dividend, 10 %; undivided profits, $ 955,946. 15. Y's rate of income, 5.298+ %; Z's 5.263+ %; Y's dividend, $ 3000; Z's, $ 2500.

Page 318. 1. $ 117. 2. $ 840.

Page 319. 1. $1\frac{1}{8}$ %. 2. $\frac{2}{8}$ %. 1. $ 4450. 2. 475. 3. $ 37,500. 1. $ 300. 2. $ 8000. 3. $ 1576.50.

Page 320. 4. $ 523.75. 5. $\frac{3}{4}$ %. 6. $ 27,411.17. 7. $ 11,454.55. 8. $ 18,125. 9. $ 8.66. 10. $\frac{3}{4}$ %. 11. $ 1967.96. 12. $ 60. 13. G., $ 630; H., $ 150; and M., $ 337.50; $ 142.50 gain; $.7\frac{1}{8}$ %. 14. H., $ 26,522.73; M., $ 43,099.43; A., $ 23,207.39; Phœnix, $ 26,522.73; Provident, $ 26,522.73.

Page 321. 15. $ 47,500 and $ 39,375.

Page 325. 1. $ 158.10. 2. $ 280.40. 3. $ 333.40. 4. $ 6386.94

due from bank; $1386.94 more from bank than from insurance company. 5. $4610.64 more than premiums; $3947.08 more than the premiums and interest. 6. $680.05 loan; $1555.70 due from bank. 7. $6948 on ordinary policy; $24,885.60 on endowment policy. 8. $4265.92 more than premiums; $3985.33 loss on policy. 9. $29.60 less in premiums, per $1000; $1276.08 in principal and interest, per $1000.

Page 326. 10. $28,026.25. 11. $4011.05 more than premiums; $3891.27 less than amount received by A. 12. $440.55, loan value; 9 years 47 days, extended insurance; $925, amount of paid up policy. 13. $2596.60, loan value; 13 years, extended insurance; $3710, paid up policy.

Page 328. 1. $18.75. 2. $1134; B. $13.44.

Page 329. 1. Rate, $1\frac{1}{2}$ %; $120. 2. 50¢ per $100; $11.25. 1. Rate, $\frac{5}{8}$ %; $62.50. 2. $3550. 3. 4.5 mills; $70. 4. $35.35. 5. Rate, .012898; $82.39.

Page 330. 6. $2\frac{1}{2}$ mills; $2\frac{7}{8}$ mills. 7. 4.5 mills; $110.63. 8. Rate, $5\frac{3}{4}$ mills; $54.05. 9. 1.25669¢. 10. $24,519.60; $612.99.

Page 333. 1. $24,300. 2. $7273.56 total tax; $10,910.34 if he failed to make return. 3. $76,647, total city tax; $790, total suburban tax; $2182.50, total farm tax; $44.25, tax horses and cattle; $26,743.10, total tax money at interest; $5.60 tax vehicles to hire; grand total, $106,412.45. 4. $6687.18. 5. $66. 6. $94.48. 7. $243, discount; $24,057, total paid for real estate tax. 8. $4.74.

Page 337. 1. $1235.60. 2. $610.50. 3. $933.33. 4. $1238.64. 5. 40 gal.

Page 338. 1. $1178.80. 2. $5645.50. 3. $2010. 4. $27,031.80. 5. $10,656.40. 1. $866.80. 2. $576.30. 3. $1317.60.

Page 339. 4. $3644.55. 5. $155.90. 6. $2494. 7. $463.20. 8. $1168.80.

Page 340. 9. $210.08. 10. $1.80. 11. $16.82.

Page 344. 1. $848.87. 2. $1393. 3. $784.

Page 345. 4. $5000. 5. $8696.16. 6. $1378.30. 7. $1871.30 8. $1890.50. 9. $5080.98; $344.42 gain. 10. $2471.88.

Page 352. **1.** $1946. **2.** 11,160.89 francs, or 11,160 francs 89 centimes. **3.** $5103.48. **4.** £1195 17*s.* 6*d.* **5.** 25,000 marks. **6.** $1666.87. **7.** 1864 marks **8.** $25,876.26. **9.** 5.185 francs. **10.** $.40$\frac{3}{8}$.

Page 353. **11.** $2186.21. **12.** $10,558.96.

Page 355. **1.** $ 2003.72. **2.** $3790.30. **3.** $1501.87. **4.** $3506.93. **5.** $3056.63. **6.** $3594.11. **7.** Mk. 19,429.43. **8.** $2827.12.

Page 356. **9.** $2851.18. **10.** $119.37. **11.** $5824.17. **12.** $82.15 gained. **13.** $8.16. **14.** £15 14*s.* 6$\frac{1}{4}$*d.* more.

Page 358. **1.** $10,600. **2.** A, $2400; B, $1200; C, $800. **3.** A, $120; B, $300; C, $320. **4.** $12,000. **5.** $15,000. **6.** A, $150; B, $200; C, $300.

Page 359. **7.** $64,800. **8.** A, $3000; B, $6000; C, $9000; D, $12,000. **9.** 11$\frac{289}{551}$ da.; C, $32.67; H, $25.41; T, $22.87; L, $19.05.

Page 362. **1.** A, $126; B, $168. **2.** A, $6500; B, $6500; C, $3250.

Page 363. **3.** A's share of gain, $13,750; B's share of gain, $8250; A's share of selling price, $26,250; B's share of selling price, $15,750. **4.** A's gain, $398.88; C's gain, $319.10; B's gain, $239.32; A's present worth, $1648.88; C's present worth, $1319.10; B's present worth, $989.32. **5.** B, $6000; C, $4000; D, $2000. **6.** Net gain, $795.51; A's share of net gain, $530.34; B's share of net gain, $265.17; A's present worth, $5530.34; B's present worth, $2665.17. **7.** Net loss, $488.50; C's present worth, $3755.75; D's present worth, $1755.75. **8.** Net gain, $2255; E's present worth, $8405.66; F's present worth, $8751.67; G's present worth, $7751.67. **9.** $7290.

Page 365. **1.** A, $276.92; B, $346.15; C, $276.93.

Page 366. **2.** A, $596.53; B, $559.25; C, 994.22. **3.** A, $17.40; B, $29.70; C, $48. **4.** A, $36; B, $32; C, $20; D, $4. **5.** M's gain, $1737.93; E's gain, $1862.07; M's present worth, $3737.93; E's present worth, $6362.07. **6.** A, $4548.39; B, $2951.61. **1.** A's present worth, $12,025.17; B's present worth, $9525.18; firm's present worth, $21,550.35; firm's net gain, **$4050.35.**

Page 368. **2.** Net resources, $23,564.25; net solvency, $23,564.25; net gain, $1064.25. **3.** Burke, $17,533.33; Brace, $17,833.34; Baldwin, $17,633.33. **4.** Loss, $22,747.09; Brigg's loss, $5907.62; Parson's loss, $16,839.47; net insolvency, $2187.09; Brigg's present worth, $127.38; Parson's insolvency, $2314.47.

Page 369. **5.** A's investment, $3865.80; B's investment, $2577.20; firm's insolvency, $5557; A's insolvency, $3334.20; B's insolvency, $2222.80. **6.** Net gain, $2636.83; Mason's present worth, $5818.42; River's present worth, $5818.41. **7.** Net gain, $2865.14; D's present worth, $8632.57; E's present worth, $8432.57.

Page 370. **8.** $2100. **9.** Present worth of business, $16,766.57; net gain, $4673.09; A's present worth, $8371.78; L's present worth, $8394.79. **10.** Gain of each, $1825; E. H. Hill's present worth, $7425; N. P. Pond's present worth, $7825.

Page 373. **1.** $3200.25. **2.** $1910.40. **3.** $1565.

Page 374. **1.** 7th meeting, dues, $550; interest, $16.50; loan, $550; profit per share, $.03; book value of each share, $7.105. **2.** 13th meeting, dues, $400; interest, $26.40; loan, $400; profit per share, $.066; book value of each share, $13.429.

Page 375. **3.** 17th meeting, dues, $800; interest, $96; loan, $1600; profit per share, $.12; book value of each share, $18.02. 25th meeting, dues, $800; interest, $156; loan, $800; profit per share, $.195; book value of each share, $27.3175. **4.** $1 profit for 24th month, or $12.16 total profit for 24 months; $212.16, book value after paying 25th month's dues. **5.** $18.40. **6.** $22.

Page 376. **1.** L's profit, $475.20; M's, $206.25; N's, $158.40; O's, $110; P's, $16.50. Lowest terms, respectively, 36, 25, 16, 4, and 1. Profit per share, respectively, $3.96, $2.75, $1.76, $.44, and $.11. **2.** S's profit, $83.60; T's, $37.62; Q's, $104.50; U's, $12.54; V's, $9.50; R's, $14.25. Lowest terms, for S, 16; for T, 9; for Q, U, and V, 4; for R, 1. Profit per share, respectively, $1.52, $.855, $.38, $.095. **3.** $274.70.

Page 379. **1.** 733⅓ ft. **2.** 478 bu. **3.** $2812.50. **4.** 2 yr. 9 mo. 23 da. (22½ da.). **5.** 140 A.

Page 380. **1.** $567. **2.** 350 rd. **3.** 26⅔ acres. **4.** $384.75. **5.** 93 da.

Page 382. 1. $40. 2. $64. 3. $64.50.

Page 383. 1. $211. 2. 36.09.

Page 384. 3. $483.96. 4. $368.48. 5. $321.80.

Page 389. 1. 143,814.44 francs. 2. 229.2784 lb. 3. 617 A. 120 sq. rd. 4. $6431.23 5. $29.81. 6. 11.69. 7. $1.32. 8. 6¢. 9. $3\frac{1}{2}$ meters; 11 ft. 5.79 in. 10. 1535.2722 kilometers. 11. 54 meters; 59.0544 yd. 12. $2\frac{1}{4}$ hectares; 5.55975 A.

Page 392. 1. 14. 2. 15. 3. 24. 4. 35. 5. 75. 6. 206. 7. 125. 8. 1024. 9. 11.2. 10. 7.09. 11. $\frac{64}{75}$. 12. $\frac{81}{92}$.

Page 393. 1. 100 ft. 2. 480 rd. 3. 273.137 + rd. 4. 452 rd. 9 ft. .56 + in. 5. 77.88 + ft. 6. 3339.364 ft. 7. 100 rd. 8. 208.710 ft.

Page 397. 1. 12. 2. 25. 3. 48. 4. 404. 5. $\frac{4}{5}$. 6. $\frac{12}{55}$. 7. $\frac{11}{58}$. 8. 10.813 +. 9. $\frac{45}{65}$. 1. 2067.954 + sq. in. 2. 15 ft. 3. 10 ft. 9 + in. 4. 7 ft. 7 + in. deep; 15 ft. 2 + in. square. 5. 13 ft. 10 + in.

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